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ABSTRACT

The Comprehensive Instructional Management System (CIMS) is a unified mathematics curriculum for grades K-7. The curriculum is cyclical in design, and reflects the most recent New York City and New York State guidelines. Utilizing the experience of classroom teachers, a teacher-designed curriculum was developed that includes a rich selection of classroom-tested successful mathematics activities. The major goal of the CIMS mathematics project is to provide the classroom teacher with a well-structured, objective-based curriculum. All objectives are supported by a variety of strategies. Student mastery of these objectives is readily diagnosed through the system's testing component. The curriculum format is structured to allow the teacher to determine the ability level of the students in the topic under study, and to permit the students to move forward at their individual rates. Each module of instruction provides the teacher with a set of related performance objectives. These objectives are supported by activities which probe and develop the students' readiness to undertake new skills and concepts. Further reinforcement is given to master the objectives through accompanying student workbooks. This document is the manual developed for grade six and covers 14 topics. Tests are integrated into the manual sequence. (PK)

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CIMS

COMPREHENSIVE INSTRUCTIONAL MANAGEMENT SYSTEM

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A Cyclical Mathematics Curriculum

PART 1—TEACHER

NEW YORK CITY BOARD OF EDUCATION
OFFICE OF CURRICULUM DEVELOPMENT AND SUPPORT
DIVISION OF CURRICULUM & INSTRUCTION

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FOREWORD

The Comprehensive Instructional Management System (CIMS) is a unified mathematics curriculum for grades K-7. The curriculum is cyclical in design, and reflects the most recent New York City and New York State guidelines.

Utilizing the experience of our system's teachers, we have developed a teacher-designed curriculum that includes a rich selection of classroom-tested successful mathematics activities. The major goal of the CIMS mathematics project is to provide the classroom teacher with a well-structured, objective-based curriculum. All objectives are supported by a variety of strategies. Student mastery of these objectives is readily diagnosed through the system's testing component.

The curriculum format is structured to allow the teacher to determine the ability level of the students in the topic under study, and to permit the students to move forward at their individual rates. Each module of instruction provides the teacher with a set of related performance objectives. These objectives are supported by activities which probe and develop the students' readiness to undertake new skills and concepts. Further reinforcement is given to mastery of the objectives through accompanying student workbooks.

It is hoped that this curriculum will not only assist teachers in developing vital mathematics instruction, but will serve to expand and enrich students' mathematics skills.

Charlotte Frank

Charlotte Frank,
Executive Director
Division of Curriculum and Instruction

ACKNOWLEDGMENTS - 1988

It is the commitment of the Comprehensive Instructional Management System (CIMS) to update its curriculum, computer management system, and ongoing staff development program to be responsive to the needs of the teachers and supervisors who are using the system.

In response to the many requests from the teachers who have been using the CIMS curriculum, the materials have been revised by the Comprehensive Instructional Management System (CIMS) under the direction of Doreen D. Genkin and Arthur Flug, Assistant Directors.

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INTRODUCTION

The Comprehensive Instructional Management System (CIMS) is a teacher-designed educational delivery system composed of a curriculum, a testing component and a computer managed reporting system.

Designed to support mathematics instruction in the classroom, CIMS provides a comprehensive cyclical mathematics curriculum for grades K through 7. Each manual describes the curriculum for a single level or grade. Teacher curriculum developers from elementary, intermediate and senior high schools utilized the most current New York City and New York State guidelines in formulating the curriculum's modules and sequence.

Curriculum Topics

The CIMS mathematics curriculum has been organized into the following 14 topics:

.01	Numbers and Numeration
.02	Addition of Whole Numbers
.03	Subtraction of Whole Numbers
.04	Multiplication of Whole Numbers
.05	Division of Whole numbers
.06	Fractions
.07	Decimals
.08	Integers
.09	Geometry
.10	Measurement
.11	Algebraic Concepts
.12	Graphs, Probability, Statistics
.13	Problem Solving
.14	Review

Modules of Instruction

The CIMS curriculum is organized into modules or groupings of similar skills or understandings. Each module has a suggested period of duration for student mastery, a listing of new skills to be mastered by the student in the form of performance objectives, a section devoted to vocabulary, terms and symbols, getting started activities which serve as prerequisite skills in preparation for undertaking new objectives, strategies for teaching the new skills and student activities for providing reinforcement for the newly learned objective. Note that the lesson duration is included only as a guideline to the teacher for planning purposes. The actual time required for each module will vary from class to class and from child to child depending on the needs of the students and the teacher's judgment.

Priorities of Performance Objectives

There is an order of priority to be recognized in teaching the content presented in the CIMS NEW YORK CITY MATHEMATICS CURRICULUM. Performance objectives with the greatest priority are so basic that unless the student acquires a satisfactory degree of competency in them, the student cannot progress. Such objectives are marked A and mastery by all pupils is essential.

Performance objectives marked B are required to be presented to all students. Mastery is not expected at this time. These objectives will be developed further and reinforced later.

Performance objectives marked C are of special interest. These should be taught to increase positive attitudes toward mathematics.

Summary: A - Mastery of this objective is essential for progress.
Opportunities for reinforcement should be provided throughout the school year.
B - These objectives pertain to all pupils. Mastery is not essential at this time because the topics will be further developed and reinforced. Presentation of the concept in this grade is needed to facilitate mastery in the future.
C - Objectives of special interest.

Topic Tab Key

01. Numbers and Numeration

02. Addition of Whole Numbers

03. Subtraction of Whole Numbers

04. Multiplication of Whole Numbers

05. Division of Whole Numbers

06. Fractions

07. Decimals

08. Integers

09. Geometry

10. Measurement

11. Algebraic Concepts

12. Graphs, Probability, Statistics

13. Problem Solving

14. Review

Module Coding Key

6.13.02.01A

Grade level
(Grade 6)

Topic number
(.13 Problem Solving)

Module ranking within topic and level
(Use Trial and Error to Solve Problems)

Performance objective within this module
(The student will be able to
use trial and error to solve
a problem.)

Priority
(See page x)

INSTRUCTIONAL SEQUENCE DESCRIPTION

An instructional sequence is given in list form on the following pages for this level. The sequence has been arranged to reflect what is considered the most effective presentation of the mathematics curriculum. Each module in the list is described by its unit number, module number, title, and page location.

The CIMS tests are integrated within the manual sequence. The test number is located in the first column. Each test is designed to assess priority objectives from the preceding modules. The tests are used to determine student mastery of specific performance objectives within these modules and appear in a multiple choice format. Each performance objective code number preceded by an * indicates that the performance objective is not tested on a CIMS cumulative test.

INSTRUCTIONAL SEQUENCE - LEVEL 6

TEST NUMBER	UNIT NUMBER	MODULE NUMBER	TITLE	PAGE NUMBER
	1	6.14.01	Review: Problem solving................	1
	2	6.14.02	Review: Place value....................	2
	3	6.01.01	Place value through hundred billions...	3
	4	6.01.02	Round large numbers....................	5
6.01	5	6.02.01	Addition of large numbers..............	6
	6	6.03.01	Subtraction of large numbers...........	7
	7	6.01.03	Base five..............................	8
	8	6.01.04	Base two...............................	9
	9	6.14.03	Review: Customary measurement..........	10
6.02	10	6.10.01	Weight and linear customary measurement: Fractional relationships........................	11
	11	6.10.02	Measurement: Time......................	12
	12	6.10.03	Operations with customary measures.....	13
	13	6.14.04	Review: Equivalent fractions...........	14
	14	6.14.05	Review: Least common multiple..........	15
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	16	6.07.01	Decimals - ten-thousandths.............	18
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	20	6.14.07	Review: Multiplication of whole numbers............................	23

QUARTERLY TEST #1

TEST NUMBER	UNIT NUMBER	MODULE NUMBER	TITLE	PAGE NUMBER
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	23	6.05.01	Division by multiples of 100...........	26
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	29	6.09.02	Triangles and angles	34
	30	6.14.10	Review: Circle concepts	36
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INSTRUCTIONAL SEQUENCE - LEVEL 6

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6.14.06 Review: Metric measurement............... 20
6.14.07 Review: Multiplicaton of whole numbers... 23
6.14.08 Review: Division......................... 25
6.14.09 Review: Graphs........................... 29
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TO THE TEACHER:

Learning to solve problems is the principal reason for studying mathematics. The CIMS approach to problem solving (topic 13) offers students various strategies for solving problems. Whenever an opportunity for problem solving arises in any topics, either during instruction or when problems are used to motivate lessons, the objectives in the **TEACHER DIRECTED MODULE** should be employed.

A PROBLEM SOLVING MODULE FOR THE TEACHER

(ongoing)

PERFORMANCE OBJECTIVES

The teacher will be able to...

01A determine whether the students understand the problem;
e.g.,

Can the student...

- -identify the question?
- -identify the information given?
- -restate the problem in his/her own words?
- -identify whether there is too much information or not enough information?
- -draw a picture of the problem?

02A help the students design a plan to solve the problem;
e.g.,

Will the student solve the problem by...
- using a standard algorithm (adding, subtracting, dividing, multiplying)?
- drawing a picture?
- working backward?

- guessing and checking?
- making a chart?
- finding a pattern?
- locating information?

03A help the students carry out their plan;
e.g.,

Students follow through on their plans and verbalize their thinking.

04A help the students evaluate their solutions.
e.g.,

The student will look back and ask...
- is the answer reasonable?
- can I check my answer?
- are there other ways to solve the problem?
- is my plan the "best" way?

6.14.01 REVIEW: PROBLEM SOLVING

(1 lesson/ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

6.14.01.01 identify the prior knowledge needed to solve a problem and use the knowledge to solve the problem;

e.g.,

Problem: Maria was making eggnog for her club's party. She needed $\frac{2}{3}$ dozen eggs to make 2 quarts of eggnog. How many eggs will she need to make 6 quarts of eggnog?

Prior Knowledge:

1 dozen = 12

$\frac{2}{3}$ dozen = 8

6.14.01.02 solve problems by creating and solving similar, but simpler, problems.

e.g.,

Problem: There are 15 people at a party. If each person shakes hands once with every other person, how many handshakes will there be?

Simpler Problem: How many handshakes will there be if there are 3 people at the party?...4 people?...5 people?

6.14.01

PREVIOUS UNDERSTANDINGS:

- linear measurement

GETTING STARTED:

Display the following problems:

Problem 1: Artie worked 7 days this week. He worked four hours each day. How many hours did he work this week?

Problem 2: Suzanne is 5 feet tall. Maria is 63 inches tall. Who is taller?

Ask a student to read the first problem aloud. Say, "What information is given in this problem? (the number of days Artie worked and the number of hours he worked each day) What are we asked to find? (the total number of hours Artie worked that week) Is the information given in this problem sufficient to find the solution? (yes) How can we solve this problem?" (We can multiply the number of days times the number of hours each day.)

Ask a student to read the second problem aloud. Say, "What information is given in this problem? (Suzanne's height and Maria's height) What are we asked to find? (Who is taller, Suzanne or Maria?) Do we need any additional information to solve this problem? Explain." (Yes, we need to know the number of inches in a foot. We know the height of each girl, but we cannot compare them immediately because each girl's height was measured in different units.)

Record the students' conclusion in the following chart:

	Given Information	Asked to Find	Additional Information Needed
Problem 1:	- number of days Artie worked this week - number of hours he worked each day	- number of hours Artie worked	- none
Problem 2:	- Suzanne's height - Maria's height	- which girl is taller	- number of inches in a foot

GETTING STARTED: (cont'd)

Say, "How is the first problem different from the second problem? (The first problem gave us all the information needed to solve it. The second problem did not. However, the kind of information that is missing is a common fact which we would be expected to know.) If you have a problem which requires information that is not stated, you should ask youself, 'Is the missing information a common fact which I know?'"

6.14.01

6.14.01.01 **identify the prior knowledge needed to solve a problem and use the knowledge to solve the problem**

STRATEGY 1:

Display the second problem stated in the Getting Started. Say, "We have pointed out that this problem cannot be solved using only the information stated in the problem. In a problem such as this, you are expected to recall something you already know. This information we learned is referred to as prior knowledge. What prior knowledge will we use to find the solution to this problem? (We know that one foot equals 12 inches.) With this knowledge, we can change Suzanne's height to inches. With both units of measure the same, we can easily determine which girl is taller. Is there another way to use this prior knowledge to solve this problem? Explain." (We can change Maria's height to feet and inches and then easily find out who is taller.)

Ask a student to solve the problem at the chalkboard by changing Suzanne's height to inches. Ask another student to change Maria's height to feet and inches to solve the problem at the chalkboard. Ask the remaining students to solve the problem in their notebooks.

e.g.,

$$\begin{array}{r} 12 \text{ in.} \\ \underline{\times\ 5} \\ 60 \text{ in.} \end{array}$$

63 in. (Maria's height)
60 in. (Suzanne's height)
Maria is taller than Suzanne.

$$\begin{array}{r} 5\frac{3}{12}\text{ ft} \\ 12\overline{)63\text{ in.}} \\ \underline{-60} \\ 3 \end{array} = 5 \text{ ft } 3 \text{ in.}$$

5 ft (Suzanne's Height
5 ft 3 in. (Maria's Height)
Maria is taller than Suzanne.

Say, "What is the answer to the problem? (Maria is taller than Suzanne.) Did the method used to solve the problem make a difference in the answer? (No, the same answer was found with each method.) What prior knowledge was needed to solve this problem?" (We needed to know the number of inches in a foot.)

Say, "Here is a new problem situation: George had two hours before he had to leave for school. He read the newspaper for half an hour, then worked on his math homework for 45 minutes. What questions could we pose about this situation?" (Accept any reasonable responses, e.g.,

STRATEGY 1: (cont'd)

How much free time does he have left? How much more time did he spend on homework than on the paper? Did he finish his homework?) Write the students' questions on the chalkboard. Then ask of each question: What information is given? What are we asked to find? Is additional information needed? If so, is it special information or prior knowledge? Where prior knowledge is involved, elicit a statement of the common fact and the complete solution.

Repeat this procedure using other problems where prior knowledge is needed. Some of these problems should be created by the students.

6.14.01.02 solve problems by creating and solving similar, but simpler, problems

STRATEGY 1:

Present the following problem:

Find the sum of the first 100 odd numbers.

Ask a student to read the problem aloud. Say, "How might we begin to solve this problem? (We can list the first 100 odd numbers, 1, 3, 5, ... and then we can add them together to find the sum.) What difficulties might we encounter using this procedure? (It will be very time consuming. Also, we might make an error in addition with so many addends.) How else might we try to solve this problem? (We could use a calculator to find the sum. It would take less time.) What difficulties might we encounter using a calculator?" (We might unknowingly hit a wrong key. Also, even with the help of a calculator, we would still have to list the first 100 odd numbers so we will know when to stop adding, and that is time consuming.)

Say, "Sometimes the best way to solve a long and difficult problem is to solve a simpler, but similar, problem. What is the sum of the first two odd numbers? (1 + 3 = 4) ... the first three odd numbers? (1 + 3 + 5 = 9) ... the first four odd numbers?" (1 + 3 + 5 + 7 = 16) Ask a student to record this information on the chalkboard, and to extend it through the sum of the first six odd numbers. Ask the other students to do the same in their notebooks.

STRATEGY 1: (cont'd)

e.g.,

2 addends	1 + 3 = 4
3 addends	1 + 3 + 5 = 9
4 addends	1 + 3 + 5 + 7 = 16
5 addends	1 + 3 + 5 + 7 + 9 = 25
6 addends	1 + 3 + 5 + 7 + 9 + 11 = 36

Say, "Look at the sums. What pattern do you see? Explain. (Each sum is a square number.) What do we mean by a square number?" (A square number equals some smaller number times itself. For example, 4 is a square number because it equals 2 times 2.)

Write the following chart on the chalkboard:

Number of Addends	Odd Number Addends	Sum
2	1+3	4
3	1+3+5	9
4	1+3+5+74	16
5	1+3+5+7+9	25
6	1+3+5+7+9+11	36

Say, "What relationship can you see between the number of addends and the corresponding sums?" (If we multiply the number of addends by itself, the result will be the corresponding sum.) Ask a student to demonstrate this on the chalkboard as the other students write it in their notebooks.

e.g.,

Number of Addends	Odd Number Addends	Sum		Relationship
2	1+3	4	=	$2 \times 2 = 2^2$
3	1+3+5	9	=	$3 \times 3 = 3^2$
4	1+3+5+74	16	=	$4 \times 4 = 4^2$
5	1+3+5+7+9	25	=	$5 \times 5 = 5^2$
6	1+3+5+7+9+11	36	=	$6 \times 6 = 6^2$

STRATEGY 1: (cont'd)

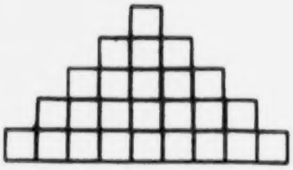
Say, "How can we use the simpler problem, finding the sum of the first six odd numbers, to help us find the sum of the first 100 odd numbers? (Since the sum in each case can also be thought of as the product of the number of addends times itself, then the sum of the first 100 odd numbers should be the same as the product of 100 times 100.) What is the product of 100 times 100?" (10,000)

Say, "What is the answer to the problem?" (The sum of the first 100 odd numbers is 10,000.) Ask a student to write the solution on the chalkboard as the other students write it in their notebooks.

Say, "How did we solve the problem?" (We first solved a similar, but simpler, problem. We found a pattern in its solution and then we used this pattern to help us solve the more difficult problem.)

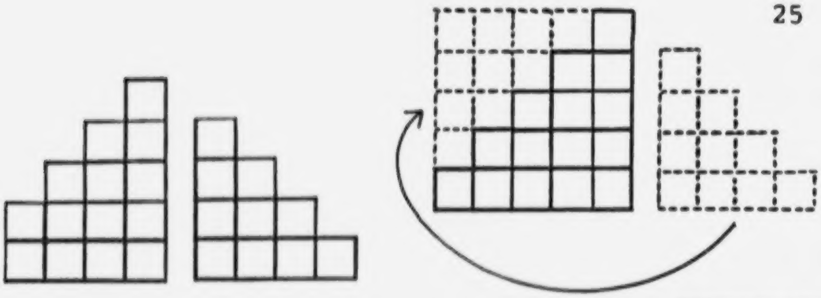
A geometric argument should be developed with the students to help them understand why the numerical pattern they discovered will continue beyond the cases examined.

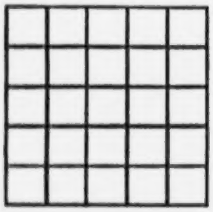
e.g.,



$$\left.\begin{array}{r} 1 \\ 3 \\ 5 \\ 7 \\ +9 \end{array}\right\} \text{the sum of the first five odd numbers}$$

25





5
5
5
5
+5
25

The stack can always be rearranged in a square in this fashion.

Repeat this procedure with other problems, some of which may be created by the students.

P.O. 6.14.01.01

NAME: ______________________ **CLASS:** _______ **DATE:** ______________

Identify the prior knowledge needed and solve each problem.

1. Harry was doing his homework. He spent 45 minutes on mathematics, 15 minutes on spelling, 40 minutes on his book report, and 20 minutes on his reading assignment. How many hours did he spend on his homework?

   Prior knowledge needed: ______________________________

   Answer: ______________________________

2. Sally was planning a big dinner for Saturday night. She invited 16 people. She intends to serve 8 ounces of meat per person. How many pounds of meat will she have to purchase to serve her guests?

   Prior knowledge needed: ______________________________

   Answer: ______________________________

3. Johnny made 15 quarts of lemonade for the class party. There are 28 children at the party. How many cups of lemonade were left after each child was served 2 cups of lemonade?

   Prior knowledge needed: ______________________________

   Answer: ______________________________

4. Brooke's grandmother said "I am 5 decades old." What is her grandmother's age in years?

   Prior knowledge needed: ______________________________

   Answer: ______________________________

5. Create a problem which requires the use of prior knowledge to find the solution.

   ____

   ____

   ____

P.O. 6.14.01.02

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

Solve the following problem.

There are forty vertical lines and twenty horizontal lines. Each horizontal line crosses each vertical line once, forming perpendicular lines. How many intersections are formed?

1. Create a problem that is similar to this one, but simpler.

2. Solve the problem you created.

3. What is the solution to the given problem?

4. Explain your thinking:

6.14.02 REVIEW: PLACE VALUE

(1 lesson/ongoing)

PERFORMANCE OBJECTIVE

The student will be able to...

6.14.02.01 identify place value through hundred millions;

e.g.,

In the numeral 427,065,983, the digit 7 is in the millions place and has a value of 7,000,000.

6.14.02.02 identify and write 7-, 8-, or 9-digit numbers in standard and expanded forms;

e.g.,

325,164,804 = 300,000,000 + 20,000,000
+ 5,000,000 + 100,000 + 60,000
+ 4,000 + 800 + 4

200,000,000 + 10,000,000 + 5,000,000 + 600,000
+ 50,000 + 4,000 + 200 + 60 + 8 = 215,654,268

6.14.02.03 compare two numbers through hundred millions using the correct relation symbol (<, >);

e.g.,

67,437,102 < 67,487,102
526,270,003 > 525,270,003

6.14.02.04 write the numeral for a given verbal phrase and write a verbal phrase for a given numeral.

e.g.,

fourteen million, two hundred seventeen thousand, one hundred eighty-five = 14,217,185

344,006,200 = three hundred forty-four million, six thousand, two hundred

PROBLEM EXPLORATION SKILLS:

- to generalize
- to state the goals of a problem clearly
- to compare and contrast

VOCABULARY	TERMS
digit	>
place value	<
units period	
thousands period	
millions period	
standard form	
expanded form	

PREVIOUS UNDERSTANDINGS:

- place value through hundred thousands
- writing a 6-digit number in standard and expanded form
- reading and writing a 6-digit verbal phrase
- comparing values of large numbers using symbols > and <

GETTING STARTED:

Materials:
- 9-column place value chart
- index cards numbered 0-9

Display a place value chart. Label each place in the units period and the thousands period.

e.g.,

Thousands			Units		
hundred thousands	ten thousands	thousands	hundreds	tens	ones

Shuffle the digit cards and ask a student to pick six cards. Say, "Let's make a numeral using these six digits." Randomly place each digit onto the place value chart.

e.g.,

Thousands			Units		
hundred thousands	ten thousands	thousands	hundreds	tens	ones
7	4	6	9	2	3

GETTING STARTED: (cont'd)

Ask a student to read this numeral aloud. (seven hundred forty-six thousand, nine hundred twenty-three) Underline the 6 in the numeral 746,923, e.g., 746,923. Say, "What is the total value of the 6? (6000) Does this digit have a greater or lesser total value than the 9? Explain. (The digit 6 has a greater value than the digit 9 because 6000 is greater than 900.) What is the total value of the 4? (4 ten thousands or 40,000) What does the 7 represent here? (700,000) What do the digits 7, 4, and 6 have in common in this numeral? (They are all in the thousands period.) What is the name of the period to its right? (the units period) What digits are in this period in this numeral?" (9, 2, and 3)

Remove the digit cards on the chart and replace them in the pack. Shuffle them and ask a student to select 6 index cards at random and to place each digit on the place value chart beginning in the left-most place. Ask another student to record the total value of each digit in this numeral in expanded form.

e.g.,

Thousands			Units		
hundred thousands	ten thousands	thousands	hundreds	tens	ones
3	4	8	1	6	5

300,000 + 40,000 + 8000 + 100 + 60 + 5

Say, "How do we read this numeral? (three hundred forty-eight thousand, one hundred sixty-five) Say, "This will be the sum of the total values." Demonstrate.

e.g.,

$$\begin{array}{lr} & 300,000 \\ & 40,000 \\ & 8,000 \\ & 100 \\ & 60 \\ + & 5 \\ \hline & 348\ 165 \end{array}$$

Say, "Where should we place the comma? (The comma should be placed between the 8 and the 1.) Why?" (It separates the thousands period from the units period.) Ask a student to place the comma in the correct location in the numeral, e.g., 348,165.

6.14.02

6.14.02.01 identify place value through hundred millions

STRATEGY 1:

Materials:
- place value chart and digit cards
- play money: $1 bill, $10 bill, $100 bill, $1,000 bill

Display the $1 bill and the $10 bill. Say, "What is the relationship between the values of these bills?" (Answers may vary.) Elicit that the $10 bill is ten times larger in value. Display the $100 bill with the $10 bill. Say, "What is the relationship between the hundred dollar bill and the ten dollar bill?" (The hundred dollar bill is ten times larger in value.) Repeat this procedure for the $1000 bill. Write the following on the chalkboard:

1 one-dollar bill = $ 1
1 ten-dollar bill = $10 = 10 one-dollar bills
1 hundred-dollar bill = $100 = 10 ten-dollar bills = 100 one-dollar bills
1 thousand-dollar bill = $1000 = 10 hundred dollar bills = 100 ten-dollar bills = 1000 one-dollar bills

Hold up a thousand-dollar bill and say, "What would ten of these be worth? (ten-thousand dollars) What would ten of those be worth? (a hundred thousand dollars) What would ten of those be worth?" (a million dollars). Ask two students to come to the chalkboard. One will enter the numerals you dictate on the place value chart while the other writes the word names of the numbers beside it.

e.g.,

MILLIONS			THOUSANDS			UNITS			
hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones	
		1	0	0	0	0	0	0	one million
	1	0	0	0	0	0	0	0	ten million
				1	0	0	0	0	ten thousand
			1	0	0	0	0	0	one hundred thousand
1	0	0	0	0	0	0	0	0	one hundred million
					1	0	0	0	one thousand
						1	0	0	one hundred
							1	0	ten
								1	one

STRATEGY 1: (cont'd)

Say, "What is the digit 1 called under the 'millions' column? (1 million) How do we say the number word for the digit in the 'ten millions' place? (10 million) How many times greater is 10 million than 1 million? Explain. (10 million is 10 times greater because each digit (of the same value) is always ten times greater in value as it moves one place to the left.) How can we show by multiplication that the digit 1 in the hundred millions place is ten times greater than the digit 1 in the ten millions place?" Elicit that 10,000,000 x 10 = 100,000,000.

Say, "When do people deal with numbers of this size?" (accept any reasonable response: winning the Lotto, counting the population in a large city or a state, paying all the soldiers in an army, measuring distances in outer space, etc...)

Erase the digits from the place value chart. Randomly choose other digits from the stack of digit cards. Ask students to place them in specific columns on the place value chart.

e.g.,

MILLIONS			THOUSANDS			UNITS		
hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
4	8	2	3	1	7	9	6	5

Say, "What is the value of the 8 in the ten millions place?" (80,000,000) Ask a student to write the number word on the chalkboard, e.g., eighty million. Say, "What is the value of the 4?" (The 4 is 400 million.) Ask a student to write the number word on the chalkboard, e.g., four hundred million. Say, "What three digits represent the thousands period? (317) ... the millions period?" (482)

Continue this procedure using other 9-digit numbers.

6.14.02

6.14.02.02 identify and write 7-, 8-, or 9-digit numbers in standard and expanded forms

STRATEGY 1:

> **Materials:**
> - 9-column place value chart

Prepare and display a place value chart.
e.g.,

MILLIONS			THOUSANDS			UNITS		
hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones

Ask students to name nine digits, other than zero, and record each digit on the place value chart in the following manner:

MILLIONS			THOUSANDS			UNITS		
hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
3								
	6							
		7						
			8					
				9				
					4			
						5		
							1	
								2

Say, "Which three digits are in the millions period? (3, 6, and 7) What number does 3 represent? (3 hundred million) How do we hold the 3 in the hundred millions place so that it is not mistaken for 3 thousands or 3 millions?" (Zeros must be written as place holders in

STRATEGY 1: (cont'd)

all the empty columns to the right of digit 3.) Ask a student to record the zeros on the place value chart so that 3 hundred million is expressed as the numeral 300,000,000.

e.g.,

MILLIONS			THOUSANDS			UNITS		
hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
3	0	0	0	0	0	0	0	0

Repeat this activity with the remaining digits.

Say, "Let's add the numbers on the chart." Ask a student to do the computation at the chalkboard.

e.g.,

	MILLIONS			THOUSANDS			UNITS		
	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
	3	0	0	0	0	0	0	0	0
		6	0	0	0	0	0	0	0
			7	0	0	0	0	0	0
				8	0	0	0	0	0
					9	0	0	0	0
						4	0	0	0
							5	0	0
								1	0
+									2
	3	6	7	8	9	4	5	1	2

Say, "What is the sum? (367,894,512) Another way of writing this numeral without using the place value chart is in a horizontal format." Demonstrate.

e.g.,

300,000,000 + 60,000,000 + 7,000,000 + 800,000 + 90,000 + 4000 + 500 + 10 + 2

STRATEGY 1: (cont'd)

Say, "What do we call a numeral written this way? (an expanded numeral or a number written in expanded form) What would be the sum or standard form of this expanded numeral?" (367,894,512) Ask a student to complete the math sentence.

e.g.,

300,000,000 + 60,000,000 + 7,000,000 + 800,000 + 90,000 + 4000 + 500 + 10 + 2 = 367,894,512

Repeat this procedure by rearranging the 9 digits on the place value chart.

6.14.02.03 **compare two numbers through hundred millions using the correct relation symbol (<, >)**

STRATEGY 1:

Materials:
- $\boxed{>}$ and $\boxed{<}$ symbol cards
- chart of planets' distances to the sun

Display the symbol cards and the following chart:

Distances from Planets to the Sun	
Planet's Name	Miles to the Sun
Mars	150,000,000
Venus	67,200,000
Saturn	937,500,000
Mercury	43,400,000
Jupiter	507,000,000
Earth	94,600,000

STRATEGY 2: (cont'd)

Say, "What does this chart show? (It shows the distances from some planets to the Sun.) How far is Earth from the Sun? (94,600,000 miles) How far is Mercury from the Sun? (43,400,000 miles) Which of these two planets is farther from the Sun?" (Earth is farther from the Sun than Mercury.) Ask a student to record the two distances on the chalkboard leaving a space in between them.

e.g.,

94,600,000 43,400,000

Ask another student to choose the correct symbol card and place it between these numerals to form a true number sentence.

e.g.,

$94,600,000 \boxed{>} 43,400,000$

Say, "What does the number sentence say? (94,600,000 is greater than 43,400,000.) How did we find the greater number?" (Since the numbers were the same length, we compared their left-most digits.)

Ask a student to rewrite the numbers on the chalkboard so that their positions are reversed.

e.g.,

43,400,000 94,600,000

Say, "How can we make a true number sentence?" (We can use the correct inequality symbol.) Ask a student to place the correct symbol card between the two numbers that will form a true sentence.

e.g.,

$43,400,000 \boxed{<} 94,600,000$

Say, "Can you read this number sentence? (43,400,000 is less than 94,600,000.) How would you compare two numbers of the same length which had the same left-most digit, such as 43,400,000 and 44,600,000?" (We would compare the second left-most digits to determine which is greater. Therefore, 44,600,000 > 43,400,000.) Can we compare the distances from Venus and Jupiter to the Sun in

STRATEGY 2: (cont'd)

the same way?" (No, the distance from Jupiter to the Sun has more digits. Therefore it is greater than the numeral representing the distance from Venus to the Sun.)

Repeat this procedure using other data from the Planet chart to compare distances of the planets from the sun.

6.14.04.04 write the numeral for a given verbal phrase and write a verbal phrase for a given numeral

STRATEGY 1:

Materials:
- chart of the population of the United States for different years
- chart separated into 3 periods

Prepare and display the following chart:

MILLIONS	THOUSANDS	UNITS
643	000	000
	643	000
		643

Say, "What are the differences between the groups of digits within each section? (They are in different periods and have different values.) What do we call the digits in the units period? (six hundred forty-three) What is the verbal phrase for the digits in the thousands period? (six hundred forty-three thousand) What is the verbal phrase for the digits in the millions period?" (six hundred forty-three million) Ask a student to record the word name for each set of numbers on the chalkboard while the other students do the same in their notebooks.

STRATEGY 1: (cont'd)

e.g.,

MILLIONS	THOUSANDS	UNITS	
643	000	000	six hundred forty-three million
	643	000	six hundred forty-three thousand
		643	six hundred forty-three

Say, "What is the sum of all three numbers?" (643,643,643) Ask a student to write the verbal phrase for 643,643,643 on the chalkboard.

e.g.,

643,643,643 = six hundred forty-three million, six hundred forty-three thousand, six hundred forty-three

Display the following population chart:

United States Population For Different Years	
Year	Population
1860	31,443,321
1900	76,212,168
1940	132,164,569
1980	226,542,580
2000	

Say, "If we were to say the verbal phrase for the 1860 population, how would we read the first period? (31 million) What told you to stop before moving to the next period? (the comma) What would you say next? (443 thousand) What would you say next?" (321)

Ask a student to come to the chalkboard and write the verbal phrase for the 1860 population while the other students write in their notebooks.

e.g.,

31,443,321 = thirty-one million, four hundred forty-three thousand, three hundred twenty-one

STRATEGY 1: (cont'd)

Repeat this procedure for the years 1900, 1940, and 1980. Say, "What population would you predict for the year 2000?" Ask students to write down their predictions both as numerals and verbal phrases. Then ask students to share their views with their neighbors. Call on several volunteers to explain their opinions to the whole group. Say, "Experts predict that in the year 2000, the population will reach 297,700,000." Write the verbal phrase for this number on the chalkboard.

e.g.,

> two hundred ninety-seven million, seven hundred thousand.

Ask a student to translate the verbal phrase to a numeral and record the numeral next to the year 2000 on the chart.

e.g.,

United States Population For Different Years

Year	Population
1860	31,443,321
1900	76,212,168
1940	132,164,569
1980	226,542,580
2000	297,700,000

Say, "How many of you made predictions higher than the experts' figure? How many of you made predictions lower than this? Did anyone make exactly the same prediction? In the year 2000, we'll find out who is right."

Continue in a similar fashion, writing numerals for given verbal phrases, and vice versa.

P.O. 6.14.02.01-.02

NAME: ______________________ **CLASS:** ______ **DATE:** ____________

1.A. Using the standard number 45,269,817 write the value of each digit.

4 means ______________
5 means ______________
2 means ______________
6 means ______________
9 means ______________
8 means ______________
1 means ______________
7 means ______________

B. Find the sum of the values written above.

C. Why does the sum match the standard numeral 45,269,817?

____
____
____
____

2. Write this expanded number in standard form.

30,000,000 + 4,000,000 + 600,000 + 80,000 + 5,000 + 900 + 60 + 7

____

3. Write this standard number in expanded form: 123,456,789

__________ + __________
+ ________ + ________ +
______ + ______ + ______
+ _____ + _____

4. How many times greater is the 4 in the hundred millions place than the 4 in the ten millions place?

440,356,071

ⓐ 100 million times ⓒ 10 times
ⓑ 10 million times ⓓ 100 times

5. Write a standard number using the following clues:
the 6 is in hundreds place
the 2 is in tens place
the 9 is in millions place
the 0 is in ones place
the 7 is in ten millions place
the 5 is in thousands place
the 4 is in hundred thousands place
the 8 is in ten thousands place

__ __, __ __ __, __ __ __

P.O. 6.14.02.02-.03

NAME: ______________________ **CLASS:** ______ **DATE:** ______________

Directions: For items 1-6, mark your answers on the Answer Grid below.

1. Which is the standard numeral for 400,000,000 + 80,000,000 + 600,000 + 30,000 + 400?

ⓐ 48,634 ⓑ 480,634,000 ⓒ 480,063,400 ⓓ 480,630,400

2. Which digit in this numeral has the greatest value?

67,890,000

ⓔ 9 ⓕ 6 ⓖ 8 ⓗ 7

3. 43,265,000 written in expanded form is:

ⓐ 40,000 + 3000 + 200 + 60 + 5
ⓑ 4 + 3 + 2 + 6 + 5
ⓒ 40,000,000 + 3,000,000 + 200,000 + 60,000 + 5000
ⓓ 40,000,000 + 3,000,000 + 20,000 + 6000 + 5000

4. Choose the correct statement.

ⓔ 489,000,000 > 498,000,000 ⓖ 270,050,709 < 270,050,710
ⓕ 807,632,000 < 807,623,000 ⓗ 199,999,999 > 200,000,000

5. Which symbol will make this statement true?

47,138,256 ○ 47,138,265

ⓐ < ⓑ = ⓒ > ⓓ +

6. Which numeral will complete this statement?

39,393,939 < __________

ⓔ 39,392,940 ⓕ 39,294,993 ⓖ 39,393,849 ⓗ 39,393,940

ANSWER GRID

1. ⓐ ⓑ ⓒ ⓓ 4. ⓔ ⓕ ⓖ ⓗ
2. ⓔ ⓕ ⓖ ⓗ 5. ⓐ ⓑ ⓒ ⓓ
3. ⓐ ⓑ ⓒ ⓓ 6. ⓔ ⓕ ⓖ ⓗ

P.O. 6.14.02.04

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

I. When we write a check for a certain amount of money, the money amount must be written both as a numeral and as a verbal phrase.

Example:

CIMS SAVINGS BANK 0204

May 4 1988

Pay to the order of Texas Oil Company $ 2,450

Two thousand, four hundred fifty DOLLARS

I. M. Rich

Fill in the missing money amount (numeral) on this check:

CIMS SAVINGS BANK 0205

__________ 19__

Pay to the order of Gettum Oil Company $__________

Forty-one million, six hundred fifty thousand, two hundred DOLLARS

I. M. Rich

Fill in the missing money amount (verbal phrase) on this check:

CIMS SAVINGS BANK 0206

__________ 19__

Pay to the order of S.O.C. Oil Company $ 508,017,010

______________________________________ DOLLARS

I. M. Rich

MATCH UP THE NUMERAL AND ITS VERBAL PHRASE

140,401,041	one hundred forty million, fourteen thousand, forty-one
104,410,401	one hundred four million, four hundred one thousand, fourteen
140,014,041	one hundred forty million, four hundred one thousand, forty-one
104,041,401	one hundred four million, four hundred ten thousand, four hundred one
140,401,014	one hundred four million, forty-one thousand, four hundred one

Unit 3 6.01.01 PLACE VALUE THROUGH HUNDRED BILLIONS

(3-4 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

6.01.01.01A identify place value through hundred billions; e.g.,

Billions			Millions			Thousands			Units		
hundred billions	ten billions	billions	hundred millions	ten millions	millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
6	3	2	1	0	4	9	8	6	6	2	5

*6.01.01.02A identify and write numbers through hundred billions in standard and expanded forms; e.g.,

$$482,307,562,914 = 400,000,000,000 + 80,000,000,000 + 2,000,000,000 + 300,000,000 + 7,000,000 + 500,000 + 60,000 + 2,000 + 900 + 10 + 4$$

*6.01.01.03A identify and write the exponential form of powers of ten through hundred billions place; e.g.,

$$1,000,000 = 10^6$$

$$1,000,000,000 = 10^9$$

$$10,000,000,000 = 10^{10}$$

6.01.01.04A	identify and write numbers through hundred billions in expanded form using exponential notation; e.g., $2{,}307{,}562{,}914 = (2 \times 10^{9}) + (3 \times 10^{8}) + (7 \times 10^{6}) + (5 \times 10^{5}) + (6 \times 10^{4}) + (2 \times 10^{3}) + (9 \times 10^{2}) + (1 \times 10^{1}) + (4 \times 10^{0})$
6.01.01.05A	compare two numbers through hundred billions using the correct relation symbol (<, >); e.g., 73,112,001,217 > 73,111,053,426 312,076,431 < 312,076,441
6.01.01.06A	write the short word name for numbers through hundred billions. e.g., 375,402,001,722 = 375 billion, 402 million, 1 thousand, 722

PROBLEM EXPLORATION SKILLS:

- to state the goals of a problem clearly
- to generalize

6.01.01

VOCABULARY

exponent
base
factor
expanded notation (form)
standard form
exponential form

SYMBOLS

>
<

PREVIOUS UNDERSTANDINGS:

- place value through hundred millions in standard, expanded, and exponential form
- comparing numerals through hundred millions using inequality symbols (<, >)

GETTING STARTED:

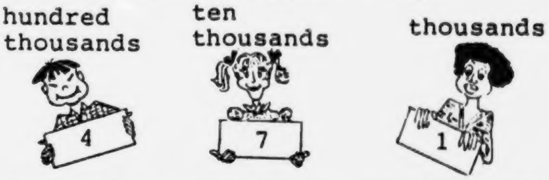
Materials:

- index cards labeled 1 through 9

Distribute one card to each of nine students. Say, "Let's play a game called Find Your Place. We will construct a very large numeral in front of the room with the digit cards you are holding."

Ask the student with the digit 7 to stand at the center of the chalkboard. Say, "We will call this the ten thousands place." Ask the students holding the digits 4 and 1 to stand in the hundred thousands and the thousands place, respectively.

e.g.,



Say, "In what period are these 3 students, or digits?" (thousands period) Repeat this procedure until a 9-digit number is formed. Ask a student to record the number on the chalkboard, along with the following information:

e.g.,

MILLIONS	THOUSANDS	UNITS
869,	471,	352

Say, "What is the value of the digit 6? (60 million)... the digit 8?" (800 million) Continue by asking students to change positions and to name the newly formed numerals as number words.

6.01.01

6.01.01.01A identify place value through hundred billions

STRATEGY 1:

Adapt Strategy 1, P.O. 6.14.02.01.

6.01.01.02A identify and write numbers through hundred billions in standard and expanded form

STRATEGY 1:

Adapt Strategy 1, P.O. 6.14.02.02.

6.01.01.03A identify and write the exponential form of powers of ten through hundred billions place

STRATEGY 1:

Display the following on the chalkboard:

Odometer Reading on Spaceship

1	0	0	0	0	0	0	0	0	0	0	0	km

Say, "An astronaut took a very long journey to a distant star. The odometer in his spaceship showed the distance he traveled from Earth in kilometers. It showed a one followed by eleven zeros. How can this number be written?" (It can be written using commas to separate periods or as 100 billion.) Demonstrate, e.g., 100,000,000,000 or 100 billion.

Say, "How do we write the number 100 in exponential form?" (10^2) Ask a student to write 10^2 on the chalkboard. Say, "How do we read 10^2? (10 squared) What does 10 squared mean?" (10 to the second power or (10 x 10) Repeat this discussion for 10^3.

Write the following chart on the chalkboard:

Exponential Form	Powers of Ten (10 as a factor)	Standard Form
10^2	10x10	100
10^3	10x10x10	1000

STRATEGY 1: (cont'd)

Say, "What does 'Powers of Ten' mean? (10 is used as a factor as many times as the exponent indicates.) What is the developing pattern?" (The number of zeros in the standard form is the same as the exponent.) Ask the students to complete the bottom part of the chart in their notebooks as one student works at the chalkboard.

e.g.,

Exponential Form	Powers of Ten (10 as a factor)	Standard Form
10^2	10x10	100
10^3	10x10x10	1000
10^4	10x10x10x10	10,000
10^5	10x10x10x10x10	100,000
10^6	10x10x10x10x10x10	1,000,000
10^7	10x10x10x10x10x10x10	10,000,000
10^8	10x10x10x10x10x10x10x10	100,000,000
10^9	10x10x10x10x10x10x10x10x10	1,000,000,000
10^{10}	10x10x10x10x10x10x10x10x10x10	10,000,000,000
10^{11}	10x10x10x10x10x10x10x10x10x10x10	100,000,000,000

Say, "What is the exponential form for 100,000,000,000? (10^{11}) How do we read 10^{11}? (10 to the eleventh power) How many zeros would there be in 10^{11} if it were written in standard form? (11 zeros) How many times is 10 used as a factor in 10^{11}? (11 times) What do we call this number in standard form? (100 billion) How far did the astronaut travel to get to that distant star?" (100 billion kilometers)

Say, "Let's complete the pattern for the first two rows on the chart."

e.g.,

Exponential Form	Powers of Ten (10 as a factor)	Standard Form
10^0		
10^1		
10^2	10x10	100

STRATEGY 1: (cont'd)

Say, "How are the exponents affected as we go up the chart? (The exponents decrease by 1.) How does the value of each power relate to the value of the power on the line below it? (one tenth as great) What value should be assigned to 10^1?" (10) Ask students to write this on their charts.

e.g.,

Exponential Form	Powers of Ten (10 as a factor)	Standard Form
10^0		
10^1	10	10
10^2	10x10	100

Say, " 10^0 is called 10 to the zero power, indicating that there are no factors of ten in the middle column." Elicit that 10^0 must equal 1 because as we move up the chart, the value of each number is one tenth as great as the number below it. Ask students to complete the chart in their notebooks.

e.g.,

Exponential Form	Powers of Ten (10 as a factor)	Standard Form
10^0		1
10^1	10	10
10^2	10x10	100

6.01.01.04A identify and write numbers through hundred billions in expanded form using exponential notation

STRATEGY 1:

Materials:

- place value chart through 100 billions

Display the following:

$10^0 = 1$ $10^3 = 1000$ $10^6 = 1{,}000{,}000$ $10^9 = 1{,}000{,}000{,}000$

$10^1 = 10$ $10^4 = 10{,}000$ $10^7 = 10{,}000{,}000$ $10^{10} = 10{,}000{,}000{,}000$

$10^2 = 100$ $10^5 = 100{,}000$ $10^8 = 100{,}000{,}000$ $10^{11} = 100{,}000{,}000{,}000$

Say, "What information is displayed on the chalkboard? (the exponential form and standard numeral for powers of ten through one hundred billion) Can we write any power of ten in exponential form?

STRATEGY 1: (cont'd)

Explain." (Yes, the exponent would indicate how many times ten occurs as a factor in the power.)

Write the following standard numeral and its expanded form on the chalkboard:

$$3420 = 3000 + 400 + 20$$

Say, "Today we will learn to write numbers, including very large numbers, in exponential form. What number is the base of a power of ten number? (10) When we expand 3420 to 3000 + 400 + 20, we can represent each expanded number as a product involving a power of ten." Demonstrate.

e.g.,

$$\begin{aligned} 3000 &= 3 \times 1000 = 3 \times 10^3 \\ 400 &= 4 \times 100 = 4 \times 10^2 \\ 20 &= 2 \times 10 = 2 \times 10^1 \\ 3420 &= (3 \times 10^3) + (4 \times 10^2) + (2 \times 10^1) \end{aligned}$$

Say, "Let's see how this works with a very large number. We will start by placing the digits of a very large number on a place value chart." Demonstrate.

e.g.,

Billions			Millions			Thousands			Units		
100 billion	10 billion	1 billion	100 million	10 million	1 million	100 thousand	10 thousand	1 thousand	100	10	1
4	2	3	5	6	7	2	6	9	8	2	5

STRATEGY 1: (cont'd)

Say, "How can we write 400 billion as a factor times a power of ten?" (4 x 100 billion) Ask a student to write 4 x 100 billion on the chalkboard. Say, "How is 100 billion written in exponential form?" (10 to the eleventh power) Demonstrate.

e.g.,

$$4 \times 100 \text{ billion} = 4 \times 10^{11}$$

Continue by developing the following chart on the chalkboard:

factor x power of ten	factor x exponential form
4 x 100 billion	4×10^{11}
2 x 10 billion	2×10^{10}
3 x 1 billion	3×10^{9}
5 x 100 million	5×10^{8}
6 x 10 million	5×10^{7}
7 x 10 million	7×10^{6}
2 x 100 thousand	2×10^{5}
6 x 10 thousand	6×10^{4}
9 x 1 thousand	9×10^{3}
8 x 1 hundred	8×10^{2}
2 x 1 ten	2×10^{1}
5 x 1 one	5×10^{0}

Ask a student to write the numeral in expanded form using exponential notation on the chalkboard.

e.g.,

$$\begin{aligned} 423{,}567{,}269{,}825 = {} & (4\times10^{11}) + (2\times10^{10}) + (3\times10^{9}) + \\ & (5\times10^{8}) + (6\times10^{7}) + (7\times10^{6}) + (2\times10^{5}) + \\ & (6\times10^{4}) + (9\times10^{3}) + (8\times10^{2}) + (2\times10^{1}) + \\ & (5\times10^{0}) \end{aligned}$$

Repeat this procedure for writing other large numbers in expanded form using exponential form.

6.01.01.05A compare two numbers through hundred billions using the correct relation symbol (<, >)

STRATEGY 1:

Materials:
- chart entitled "Billions of Burgers"

Display the following chart:

Billions of Burgers			
	Burger 'n Bun	Burger Hut	Burger Castle
Hamburgers	102,314,500,881	102,341,500,790	98,314,498,515
Fries	89,115,038,213	89,115,039,300	89,115,038,953

Say, "What information is shown on this chart? (the number of hamburgers and fries sold in one year by three different fast food restaurants) Which restaurant sold more hamburgers, Burger Hut or Burger Castle? (Burger Hut) How do we know this? (The number having the most digits is the largest number.) How can we express the comparison between these two numbers? (102,341,500,790 is greater than 98,314,498,515 or 98,314,498,515 is less than 102,341,500,790.) What symbols are used to indicate 'is greater than' and 'is less than' in a mathematics sentence?" (relation symbols or inequality symbols) Demonstrate.

e.g.,

> means is greater than
< means is less than

Ask a student to write this comparison on the chalkboard in two different ways using each relation symbol correctly.

e.g.,

102,341,500,790 > 98,314,498,515
or
98,314,498,515 < 102,341,500,790

STRATEGY 1: (cont'd)

Say, "Which restaurant, Burger Hut or Burger Castle, sold more hamburgers? (Burger Hut) When do we use relation symbols?" (We use relation symbols to compare numbers.)

Say, "Let's review how we compare numbers having the same number of digits. How can we find which restaurant, Burger Hut or Burger 'n Bun, sold fewer fries?" (We can compare each digit starting with the ten billions place.) Ask students to begin this procedure and record their observations.

Say, "What have you discovered?" (We found that all the digits match until the thousands place.) Draw a line under the digits in the thousands place.
e.g.,

89,115,039,300
89,115,038,213

Compare 8000 and 9000 using the inequality symbols.
e.g.,

8000 < 9000 or 9000 > 8000

Therefore: 89,115,038,213 < 89,115,039,300
or
89,115,039,300 > 89,115,038,213

Say, "Which restaurant sold fewer fries?" (Burger 'n' Bun sold fewer fries than Burger Hut because 89,113,038,213 is less than 89,115,039,300.)

Repeat this procedure by using correct relation symbols to compare other inequalities.

6.01.01.06A **write the short word name for numbers through hundred billions**

STRATEGY 1:

Materials:
- "Billions of Burgers" chart
- period chart through billions

Display the Billions of Burgers chart.
e.g.,

Billions of Burgers			
	Burger 'n Bun	Burger Hut	Burger Castle
Hamburgers	102,314,500,881	102,341,500,790	98,314,498,515
Fries	89,115,038,213	89,115,039,300	89,115,038,953

Say, "How many fries were sold by Burger Castle?" (89,115,038,953) Ask the students to write the verbal phrase for the number in their notebooks as one student writes it on the chalkboard.
e.g.,

eighty-nine billion, one hundred fifteen million,
thirty-eight thousand, nine hundred fifty-three

Say, "The phrase can be shortened by writing the digits in each period, followed by the name of the period." Demonstrate.
e.g.,

Verbal Phrase		Short Word Form
eighty-nine billion	=	89 billion
one hundred fifteen million	=	115 million
thirty-eight thousand	=	38 thousand
nine hundred fifty-three	=	953

Display a chart showing periods through billions.

STRATEGY 1: (cont'd)

e.g.,

BILLIONS	MILLIONS	THOUSANDS	UNITS

Ask a student to write the digits on the place value chart while another student reads the verbal phrase aloud.

e.g.,

BILLIONS	MILLIONS	THOUSANDS	UNITS
89	115	038	958

Write the short word form horizontally on the chalkboard as the students write it in their notebooks.

e.g.,

89 billion, 115 million, 38 thousand, 953

Elicit that the short word name drops the letter 's' from the period name, the units period name is not used, and each period is separated from the next period by a comma.

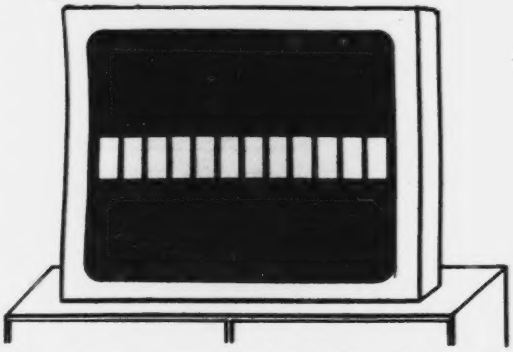
Practice writing other short word names using the "Billions of Burgers" chart.

P.O. 6.01.01.01-.02

NAME: ______________ **CLASS:** ________ **DATE:** ______________

THE HUNGRY COMPUTER

I. The student said, "Our computer had a large number of bytes because it was so hungry. On its screen are 12 places. Write the digits in their proper place value from the information below to find how many bytes the hungry computer ate?"



A. 5 in thousands place
B. 7 in ten billions place
C. 4 in ones place
D. 9 in hundred millions place
E. 0 in ten thousands place
F. 1 in tens place
G. 8 in hundred billions place
H. 3 in millions place
I. 6 in hundreds place
J. 2 in hundred thousands place
K. 1 in billions place
L. 4 in ten millions place

How many bytes did the hungry computer eat? ______________

II. In the computer above, 12 digits are on the screen. Write this very large number in expanded form.

Start here with the digit 8. 800,000,000,000

+ ______________ + ______________

+ ____________ + ____________ + ____________

+ ________ + ________ + ________ + ________

+ ________ + ________

III. Write the same number in expanded form using exponential notation.

8×10^{11} + ________ + ________ + ________

+ ________ + ________ + ________ + ________

+ ________ + ________ + ________ + ________

P.O. 6.01.01.01-.03

NAME: ____________________ **CLASS:** ______ **DATE:** ____________

Complete the following information by writing the standard numeral in expanded form.

A. 32,174,986,555 = ____________________ + 2,000,000,000

+ ______________ + ______________ + ______________

+ 900,000 + __________ + 6000 + ________ + ______ + _____

B. Write the standard form for the expanded numeral.

800,000,000,000 + 60,000,000,000 + 3,000,000,000 + 700,000,000
+ 10,000,000 + 9,000,000 + 400,000 + 50,000 + 8000 + 200 + 60 + 8 =

____

C. Write the standard numeral for this mixed-up expanded number.

90 + 4,000,000 + 80,000,000 + 700,000 + 2 + 3000 + 900

____

D. Write the exponential form (powers of ten) for each of these numbers.

1,000 = ______
10,000 = ______
100,000,000,000 = ______

10,000,000 = ______
100,000 = ______
10,000,000,000 = ______

10 = ______
1 = ______
1,000,000 = ______

E. Write each exponential number in standard form.

10^5 = __________ 10^2 = __________ 10^9 = __________

F. Find the product. Then express each product in exponential form.

10x10 = ________________ = ____________
10x10x10 = ________________ = ____________
10x10x10x10x10x10 = ________________ = ____________
10x10x10x10x10x10x10x10 = ________________ = ____________

P.O. 6.01.01.03-.04

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

I. **Riddle:**

If numbers are mixed in a bowl,
And the sum of the parts is a whole,
Put the numerical clues all together,
To discover the secret of weather.

To solve the riddle, match the value for three of the digits to its numerical clue. Place the corresponding letters in the boxes below.

UAF TBI MVR CEW
899,988,989,898

9×10^{10}	8×10^{6}	9×10^{3}
☐	☐	☐

II. Match each product with its standard number. Place the letter in the correct box to read the message.

☐	$(6\mathrm{x}10^{5})$	G	700,000,000,000
☐	$(9\mathrm{x}10^{8})$	N	40,000,000
☐	$(1\mathrm{x}10^{3})$	H	200
☐	$(2\mathrm{x}10^{2})$	M	600,000
☐	$(7\mathrm{x}10^{11})$	E	80
☐	$(8\mathrm{x}10^{1})$	S	80,000,000,000
☐	$(4\mathrm{x}10^{7})$	I	30,000
☐	$(3\mathrm{x}10^{4})$	U	5,000,000
☐	$(5\mathrm{x}10^{6})$	T	1000
☐	$(8\mathrm{x}10^{10})$	A	900,000,000

III. Which of the standard numerals names the same number as:

$(9\mathrm{x}10^{9})+(6\mathrm{x}10^{7})+(3\mathrm{x}10^{4})+(7\mathrm{x}10^{2})$

Ⓐ 9,060,030,700
Ⓑ 90,600,370,000
Ⓒ 9,600,300,700
Ⓓ 9,006,030,700

IV. Match the exponential form with the standard numeral.

Ⓐ	10^{9}	100,000,000
Ⓑ	10^{1}	1
Ⓒ	10^{6}	1,000,000,000
Ⓓ	10^{8}	10
Ⓔ	10^{0}	1,000,000
Ⓕ	10^{10}	10,000,000,000

P.O. **6.01.01.05-.06**

NAME: ______________________ **CLASS:** ______ **DATE:** ______________

This chart shows the number of shares of stock bought and sold on the CIMS Numerical Exchange during 6 months. Write the correct answer on the line provided or shade the circle next to the correct response.

CIMS Numerical Exchange

Month of Year	Bought	Sold
January	239,847,800,473	74,287,652,071
February	49,874,800,900	49,874,705,081
March	189,494,051,300	189,494,050,305
April	74,091,243,112	74,091,253,112
May	290,777,251,001	290,777,251,000
June	6,897,866,524	6,897,868,000

1. During which month were the least number of shares sold? _____

2. During which month were the greatest number of shares bought? _____

3. Which number sentence is true for March?
   ⓐ 189,494,051,300 = 189,494,050,305
   ⓑ 189,494,051,300 < 189,494,050,305
   ⓒ 189,494,051,300 > 189,494,050,305

4. Name the months in which more than 200 billion shares were bought.
   ____

5. Name the months in which less than 50 billion shares were sold.
   ____

6. Which statement is incorrect?
   ⓐ In May, the number of shares bought was greater than the number of shares sold.
   ⓑ In June, the number of shares bought was less than the number of shares sold.
   ⓒ In February, the number of shares sold was greater than the number of shares bought.

7. The short word form for 290,777,251,000 is _______.
   ⓐ 290 million, 777 thousand, 251 hundred
   ⓑ 290 billion, 777 million, 251 thousand
   ⓒ 290 billion, 777 million, 251

8. Which is the standard form for 49 billion, 874 million, 800 thousand, 900?
   ⓐ 49,874,800,900
   ⓒ 49,874,890,000
   ⓑ 49,874,809,000
   ⓓ 49,874,800,009

SHARPENING YOUR SKILLS

Units 1, 2, 3

Name: ____________________ **Class:** ____________ **Date:** ________

1. Tanya bought 6 pounds of chopped meat to make hamburgers for the barbecue she was planning. How many hamburgers, each weighing 4 ounces, can she make?

   Prior knowledge needed:

   ____

   Answer: ________________

2. In the numeral 260,517,300 the digit 6 is in the _______ place.

   ⓐ millions ⓒ ten millions
   ⓑ ten thousands ⓓ Not Here

3. The value of the 3 in 473,105,816 is __________________.

   ⓔ 3 thousand ⓖ 30 million
   ⓕ 3 million ⓗ Not Here

4. Use the correct relation symbol, < or >, to compare each of the following:

   75,408,399 ○ 75,480,399
   265,890,111 ○ 265,896,144
   6,740,005,920 ○ 6,740,000,928
   25,098,672,666 ○ 25,098,687,244

5. Twenty-two million, seven hundred fifty-six thousand, eleven in numeral form is ________.

   ⓐ 22,700,056 ⓒ 22,756,110
   ⓑ 22,756,011 ⓓ Not Here

6. In the numeral 35,476,921,003 the five is in the __________ place.

   ⓔ billions ⓖ ten billions
   ⓕ millions ⓗ Not Here

7. The value of the 8 in 483,127,519,600 is ____________.

   ⓐ 80 million ⓒ 80 billion
   ⓑ 8 billion ⓓ Not Here

8. In which numeral below does the 7 have a value of 70,000,000,000?

   ⓔ 375,629,084
   ⓕ 729,008,612,543
   ⓖ 267,405,981
   ⓗ Not Here

9. In exponential form, 10,000,000 is written as ________________.

   ⓐ 10^7 ⓒ 10^8
   ⓑ 10^6 ⓓ Not Here

10. 10^9 = ________________

   ⓔ 10,000,000
   ⓕ 1,000,000,000
   ⓖ 100,000,000
   ⓗ Not Here

11. Write the short word name for 17,682,098,165.

   ____

   ____

6.01.02 ROUND LARGE NUMBERS

(1 lesson/ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

*6.01.02.01A round large numbers to the nearest ten thousand; e.g.,

465,226 rounds to 470,000
2,782,816 rounds to 2,780,000

6.01.02.02A round large numbers to the nearest hundred thousand; e.g.,

364,195 rounds to 400,000
253,116,804 rounds to 253,100,000

6.01.02.03A create and solve word problems related to skills learned in this module. e.g.,

A 1982 census listed the populations of the largest cities in the kingdom of Morocco as follows: Casablanca, 2,139,204; Rabat, 518,616; Fez, 448,823; Marrakesh, 439,728. Express the population of each city to the nearest 10,000.

Note: When the digit to the right of the place to which we are rounding is exactly 5, we will use the convention of rounding up.

PROBLEM EXPLORATION SKILLS:

- to look for a pattern
- to generalize

VOCABULARY

round
rounding
estimate
actual

PREVIOUS UNDERSTANDINGS:

- place value through hundred billions
- rounding numbers to the nearest thousand

GETTING STARTED:

Materials:

- number line with increments of 50

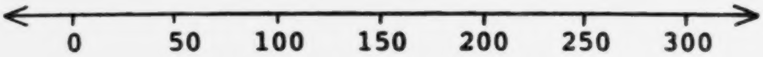
Say, "Estimate the number of legs on student desks and chairs in this room." (Accept all reasonable responses.) Write several responses on the chalkboard. Say, "How can we find the actual number of chair and desk legs. (We can count the legs on each student desk and chair or we can count the number of student desks and chairs and multiply by four.) Which method is more efficient?" (multiplication) Ask a student to count the number of student desks and chairs. Ask another student to record the count on the chalkboard and multiply the number by four.

e.g.,

29 desks x 4 legs = 116 legs
29 chairs x 4 legs = 116 legs
total = 232 legs

Display the number line.

e.g.,

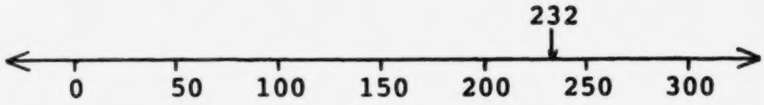


Say, "How can we round the total number of legs to the nearest hundred? (We must look at the digit in the tens place. If the digit is more than 5, we round up. If it is less than 5, the digit in the hundreds place stays the same. If it is exactly 5, the number is not 'nearer' the hundred on one side or the other. In this case, we follow a special rule or convention and round up, just as we do if the

GETTING STARTED: (cont'd)

digit is greater than 5.) What is 232 rounded to the nearest hundred?" (200) Ask a student to demonstrate rounding 232 to the nearest hundred on the number line.

e.g.,



Say, "How does the number line show us that 232 rounded to the nearest hundred is 200?" (The number line shows us that 232 is closer to 200 than 300.)

6.01.02

6.01.02.01A round large numbers to the nearest ten thousand

STRATEGY 1:

Write the following problem on the chalkboard:

> The Spott Manufacturing Company makes ball point pens. In 1987, it sold 4,256,554 fine point pens and 3,872,356 medium point pens. It is company policy that all yearly sales of one hundred thousand or more of the same item must be rounded to the nearest ten thousand in the company's records. How were the sales for fine and medium point pens entered into the company's yearly record books?

Say, "What are we being asked to find? (the amounts that were entered into the record books for each type of pen) What information were we given?" (the number of each type of pen sold in 1987, which we must round to the nearest ten thousand)

Ask a student to write on the chalkboard the number of fine point pens sold and to circle the digit in the ten thousands place, e.g., 4,2⑤6,554.

Say, "At what place must we look in order to round a number to the nearest ten thousand? (We must look at the thousands place.) What digit is in the thousands place? (6) What does the 6 in the thousands place tell us? Explain. (It tells us that the digit, 5, in the ten thousands place must be rounded up. When rounding a number to a specific place, we look at the digit to the right. If the digit to the right is 5 or more, we round up. If the digit to the right is less than 5, the digit in the specified place stays the same.) What happens to all of the other digits in the number? Explain." (The digits to the left of the place to which we are rounding stay the same. The digits to the right of the place to which we are rounding are replaced with zeros.)

Ask a student to write the rounded number on the chalkboard as the other students work in their notebooks.

STRATEGY 1: (cont'd)

e.g., 4,256,554 rounded to the nearest ten thousand is 4,260,000

Say, "What number was recorded in the company's record books for the sales of fine point pens? (4,260,000). How many medium point pens were sold in 1987? (3,872,356) What number was recorded in the company's record books for the sales of medium point pens? Explain." (3,870,000 was recorded in the record books. Since the digit in thousands place was less than five, the digit in the ten thousands place stayed the same. All other digits to the right were replaced with zeros.)

Repeat this procedure to round other large numbers to the nearest ten thousand.

6.01.02.02A round large numbers to the nearest hundred thousand

STRATEGY 1:

Adapt Strategy 1, P.O. 6.01.02.01.

6.01.02.03A create and solve word problems related to skills learned in this module

STRATEGY 1:

Display the following problem:

> The students of class 6-301 were asked to determine the actual population of London, England. They were given the following clues:
>
> (1) The actual population rounded to the nearest hundred thousand is 6,700,000.
> (2) The actual digit in the ten thousands place is 9.
> (3) The digit in the thousands place is the same as the digit in the hundred thousands place.
> (4) The digit in the ones place is equal to the product of 4 and 2.
> (5) The sum of the last three digits is 8.

STRATEGY 1: (cont'd)

Ask a student to read the problem aloud. Say, "What is the first clue?" (The population rounded to the nearest hundred thousand is 6,700,000.) Ask a student to record this information on the chalkboard as the other students work at their desks.

e.g., population rounded to the nearest hundred thousand is 6,700,000

Say, "Can we determine the actual population from the first clue? (No, we need additional information.) What does the second clue tell us? (The digit in the ten thousands place is 9.) How does this second clue help us? Explain. (We know that the population rounded to the nearest hundred thousand is 6,700,000. Since the actual digit in the ten thousands place is 9, the digit in the hundred thousands place must have been rounded up.) What digit belongs in the hundred thousands place?" (6) Ask a student to record this information on the chalkboard as the other students work in their notebooks.

e.g., population rounded to the nearest hundred thousand is 6,700,000
actual population is 6 , 6 9 ___ , ___ ___ ___

Say, "What does the third clue tell us? (The digit in the thousands place will be the same as the digit in the hundred thousands place.) What should we write in thousands place?" (6) Ask a student to record a 6 in the thousands place on the chalkboard as the other students work at their desks. Say, "What does the fourth clue tell us?" (The digit in the ones place will be 4 x 2, or 8.) Ask a student to record this information on the chalkboard as the other students work at their seats.

e.g., actual population is 6 , 6 9 6 , ___ ___ 8

Say, "What does the final clue tell us? Explain. (The sum of the last three digits is 8.) What number or numbers added to 8 give us 8 as the sum? (Zero is the only addend which does not change the sum. Therefore the digits in the hundreds place and in the tens place must be zeros.) What is the actual population of London, England?" (6,696,008)

Repeat this procedure with other word problems, some of which are created by the students. After students have had several problems modeled, they will enjoy solving or creating these in groups.

P.O. 6.01.02.01-.03

NAME: ______________________________ CLASS: _______ DATE: __________

I. Round the following numbers to the nearest ten thousand.

1 846,000	________________	6. 987,654	________________
2. 347,960	________________	7. 200,364	________________
3. 451,789	________________	8. 101,000	________________
4. 999,999	________________	9. 4,983,213	________________
5. 708,234	________________	10. 7,100,001	________________

II. Round the following numbers to the nearest hundred thousand.

1. 189,245,000	______________	6. 812,240	________________
2. 789,000	______________	7. 987,654,321	________________
3. 66,666,666	______________	8. 9,999,000	________________
4. 109,109,109	______________	9. 96,842	________________
5. 8,372,814	______________	10. 100,560	________________

III. Use the table to answer the following questions.

Circulations of Leading Magazines June-December, 1984	
Better Homes and Gardens	8,058,839
Family Circle	6,920,333
National Geographic	10,392,548
Reader's Digest	17,866,798
T.V. Guide	17,115,233
Woman's Day	6,517,684

1. When rounded to the nearest ten thousand, which magazine has a circulation of 6,520,000? ____________
2. What is the circulation of National Geographic, to the nearest hundred thousand? ________________
3. When rounded to the nearest hundred thousand, which magazine has a circulation of 17,100,000? ______________________________

6.02.01 ADDITION OF LARGE NUMBERS

(1 lesson/ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

6.02.01.01A estimate sums by rounding addends;

e.g.,

rounding to the nearest ten thousand:

610,291 rounds to 610,000

7,091 rounds to 10,000

+ 58,614 rounds to 60,000

680,000 is the estimated sum

rounding to the nearest hundred thousand:

2,517,234 rounds to 2,500,000

762,815 rounds to 800,000

3,300,000 is the estimated sum

6.02.01.02A add two or more large numbers and check;

e.g.,

610,291	$234.25	Check by adding in another way (change order, add up, find groups that add to ten).
7,019	48.69	
+ 58,614	+ 125.68	
675,924	$408.62	

6.02.01.03A create and solve word problems related to skills learned in this module.

e.g.,

On Tuesday, the morning edition of a metropolitan newspaper sold 187,180 copies. The evening edition sold 39,981 copies. How many copies of the newspaper were sold on Tuesday?

PROBLEM EXPLORATION SKILLS:

- to estimate (sums)

PREVIOUS UNDERSTANDINGS:

- rules for counting numbers
- place value to hundred thousand

GETTING STARTED:

Materials:

- number cards labeled 100, 200, 300, 700, 800, 900
- magnetic or masking tape

Write the following numbers in a column labeled A on the chalkboard:

A
750
99
872
725
301
199

Tape the number cards in a column labeled B on the chalkboard.
e.g.,

A	B
750	100
99	200
872	300
725	700
301	800
199	900

Say, "What place value do we look at in order to round 750 to the nearest hundred? (tens place) How do we know whether to round up or down?" (If the digit in the tens place is 5 or greater, we round up. If it is less than 5 we round down.) Ask a student to round 750 to the nearest hundred and draw a line to the matching number in Column B.

GETTING STARTED: (cont'd)

e.g.,

A	B
750	100
99	200
872	300
725	700
301	800
199	900

Have the students round each number in Column A to the nearest hundred and complete the matching in their notebooks.

6.02.01.01A estimate sums by rounding addends

STRATEGY 1:

Materials:
- chart with populations of western states

Display the following chart:

Populations of Western States	
State	1987 Population
Arizona	2,716,598
California	23,667,764
Idaho	944,127
Nevada	800,508
Oregon	2,633,156
Utah	1,461,037
Washington	4,132,253

Say, "Today we are going to find the total population of our western states. Before we add their populations, what can we do to help us ensure that our sum will be reasonable. (We can estimate the sum by rounding the populations.) Look at the populations for each of the western states listed. Does each state have at least one million people? Explain. (No, Idaho and Nevada have less than one million people.) Would it be reasonable to round our populations to the millions place? Explain." (Yes, but we will get a more precise estimate if we round each population to the one hundred thousands place.)

Review the rules for rounding numbers. Say, "If we round each population to the nearest hundred thousand, at what place must we look in order to determine whether the digit in the hundred thousands place will increase or stay the same?" (We will look at the digit in ten thousands place for each population. If the digit in ten thousands place is 5 or more, the digit in hundred thousands place will increase

STRATEGY 1: (cont'd)

by one. If the digit in the ten thousands place is less than five, the digit in the hundred thousands place will stay the same. In both cases, all digits to the right of the hundred thousands place will be replaced with zeros.)

Ask a student to come to the chalkboard and round each population to the nearest hundred thousand while the other students do the same in their notebooks.

e.g.,

Populations of Western States

State	1987 Population	Population Rounded to the Nearest Hundred Thousand
Arizona	2,716,598	2,700,000
California	23,667,764	23,700,000
Idaho	944,127	900,000
Nevada	800,508	800,000
Oregon	2,633,156	2,600,000
Utah	1,461,037	1,500,000
Washington	4,132,253	4,100,000

Ask another student to come to the chalkboard and find the sum of the rounded populations as the other students work in their notebooks.

e.g.,

```
  2,700,000
 23,700,000
    900,000
    800,000
  2,600,000
  1,500,000
 +4,100,000
 36,300,000
```

Say, "What is the estimated total population of the western states? (36,300,000) How accurate do we expect this estimate to be?" (Since each of the seven numbers might be off by as much as 50,000, the total might be off as much as 7 x 50,000, or 350,000. It is probably closer than that, since some numbers were rounded up and others were rounded down.)

6.02.01

6.02.01.02A add two or more large numbers and check

STRATEGY 1:

Materials:
- chart with populations of western states

Display the chart used in P.O. 6.02.01.01, Strategy 1. Say, "We rounded the populations to the nearest hundred thousand in order to estimate the sum of their populations. Before we find the actual sum of the populations of the seven western states listed on the chart, let's determine the total population of the three states farthest west, California, Oregon, and Washington." Ask a student to write the three states on the chalkboard along with the population of each rounded to the nearest hundred thousand.

e.g.,

California	23,667,764	→	23,700,000
Oregon	2,633,156	→	2,600,000
Washington	4,132,253	→	4,100,000

Ask another student to find the estimated sum of the populations of these three states while the other students work in their notebooks.

e.g.,

California	23,667,764	→	23,700,000
Oregon	2,633,156	→	2,600,000
Washington	+ 4,132,253	→	+ 4,100,000
			30,400,000
			estimated sum

Say, "What is the estimated sum of the populations of these three states?" (30,400,000) Ask a student to come to the chalkboard and find the actual sum of the populations of the three states while the other students do the same in their notebooks.

e.g.,

California	23,667,764	→	23,700,000
Oregon	2,633,156	→	2,600,000
Washington	+ 4,132,253	→	+ 4,100,000
	30,433,173		30,400,000
	actual sum		estimated sum

STRATEGY 1: (cont'd)

Say, "What is the actual sum of the three populations? (30,433,173) How do we know our answer is reasonable? Explain. (The estimated sum is close to our actual sum.) How can we be sure that our actual sum is correct." (We can check our answer by adding up or by changing the order of the addends.) A calculator may be used at this time to check the sum.

Repeat this procedure to find the total population for the seven western states listed on the chart.

6.02.01.03A create and solve word problems related to skills learned in this module

STRATEGY 1:

Write the following word problem on the chalkboard:

> In 1986, the United States produced 384,546 bushels of oats, 510,497 bushels of barley, 2,086,780 bushels of wheat, and 19,498 bushels of rye. How many bushels of these crops were produced altogether?

Ask a student to read the problem aloud. Say, "What are we asked to find? (How many bushels of these four crops were produced in the United States in 1986.) What information are we given? (The number of bushels produced for each crop.) How can we solve the problem? (We can add.) Before we actually solve the problem, how can we help ensure that we get a reasonable answer?" (We can estimate by rounding the addends.) Ask a student to write the number of bushels for each crop on the chalkboard while the other students do the same in their notebooks.

e.g.,

$$\begin{array}{r} 384{,}546 \\ 510{,}497 \\ 2{,}086{,}780 \\ \underline{+\quad 19{,}498} \end{array}$$

STRATEGY 1: (cont'd)

Say, "To what place should each addend be rounded in order to estimate the sum? Explain." (Since the smallest addend is expressed in ten thousands, it would be best to round each addend to ten thousands place.) Ask a student to come to the chalkboard and round each addend to the nearest ten thousand and compute the estimated sum as the other students work at their seats.

e.g.,

384,546	rounds to	380,000
510,497	rounds to	510,000
2,086,780	rounds to	2,090,000
+ 19,498	rounds to	+ 20,000
		3,000,000
		estimated sum

Say, "What is the estimated sum?" (3,000,000) Ask another student to come to the chalkboard and actually solve the problem as the other students solve the problem in their notebooks. Say, "How many bushels of the four crops were produced in the United States in 1986? (3,001,321) How do we know our answer is reasonable? (It is close to our estimated answer.) How can we make sure our answer is correct?" (We can check our answer by adding up, changing the order of the addends or finding groups that add to ten.)

Say, "How big is a bushel?" After students share their opinions, show them a bushel basket. Call their attention to the fact that they were able to answer the question without having this information.

Repeat this discussion with other problems involving addition of large numbers, some of which should be created by the students.

P.O. 6.02.01.01-.03

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

A. Estimate by rounding to the nearest hundred thousand. Then, find the actual sum.

1. 6,288,049 → ☐
   + 135,296 → ☐
   ☐ → ☐
   actual sum   estimated sum

2. 9,405,313 → ☐
   + 197,639 → ☐
   ☐ → ☐
   actual sum   estimated sum

B. Estimate by rounding to the nearest ten thousand. Then, find the actual sum.

3. 42,168 → ☐
   956,215 → ☐
   83,839 → ☐
   ☐ → ☐
   actual sum   estimated sum

4. 249,094 → ☐
   68,467 → ☐
   998,743 → ☐
   ☐ → ☐
   actual sum   estimated sum

C. Solve each of the following. Check by adding up, changing the order of the addends, or finding groups that add to ten.

5. During a six month period ending September 30, 1986, the New York Daily News circulated 1,270,926 newspapers daily. The New York Times circulated 1,001,694 newspapers and the New York Post circulated 731,668 newspapers. What was the total daily circulation for the three newspapers?

Estimate   Solve

6. Howard's older brother worked during summer vacation. During the month of June, he earned $411.15. In July he earned $1603.45 and in August he earned $902.90. What were his total earnings during summer vacation?

Estimate   Solve

6.03.01 SUBTRACTION OF LARGE NUMBERS

(1 lesson/ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

6.03.01.01A estimate differences by rounding the minuend and the subtrahend;

e.g.,

rounding to the nearest hundred thousand:

```
 343,116 rounds to  300,000
-185,024 rounds to  200,000
                    100,000 is the estimated
                                difference
```

6.03.01.02A subtract large numbers and check by adding;

e.g.,

```
                      check
 901,784             207,927
-693,857            +693,857
 207,927             901,784

                       check
 $2532.85            $2133.90
-  398.95            +  398.95
 $2133.90            $2532.85
```

6.03.01.03A create and solve word problems related to skills learned in this module.

e.g.,

The Ogden School and the Franklin School were collecting aluminum cans for recycling. The Ogden School collected 125,167 cans and the Franklin School collected 88,706 cans. How many more cans did the Ogden school collect than the Franklin School?

PROBLEM EXPLORATION SKILLS:

- to estimate (differences)

VOCABULARY

minuend
subtrahend
rounding
estimate
approximate(ly)

PREVIOUS UNDERSTANDINGS:

- rounding through billions
- subtraction through 6-digit minuends and 5-digit subtrahends
- estimating differences

GETTING STARTED:

Materials:

- chart (or on chalkboard) entitled "Large Islands" and their areas

Display the chart:

Large Islands

Island	Area (Sq Mi)
Jamaica	4,232
Trinidad	1,864
Iceland	39,769
Ireland	32,599
Philippines	115,830
Japan	142,727
New Zealand	103,883

Say, "An island can be very small, 10 feet across, or it can be the size of Manhattan Island in New York City. Some islands are much larger, such as Japan, which is also a nation. Study the chart. Let us compare the sizes of some large islands. When comparing numbers we can use subtraction to find how much larger one number is than another. In this way, we are also comparing islands. How can we compare the sizes of Jamaica and Trinidad? (By subtracting 1864 square miles from 4232 square miles, we can find how much larger Jamaica is in area.) Must we always use the exact numbers to compare things? Explain." (No, we can estimate by rounding numbers and subtract much faster if exact numbers are not that important.)

GETTING STARTED: (cont'd)

Ask a student to round 4232 and 1864 and subtract as the other students do the same in their notebooks.

e.g.,

$$\begin{array}{rlr} 4232 & \text{rounds to} & 4000 \\ \underline{-1864} & \text{rounds to} & \underline{2000} \\ & & 2000 \end{array}$$

Say, "What does this answer tell us? (that Jamaica is about 2000 square miles larger in area than Trinidad, or Jamaica is about twice as large as Trinidad.) How did we know what to round the numbers to? (Elicit that, unless stated otherwise, we always round the numbers we are working with to their left-most place in order to quickly estimate an answer.) When we subtract, what is the name of the number we are subtracting from? (the minuend) What is the name of the number we are subtracting? (the subtrahend) What is the answer called?" (the difference)

6.03.01

6.03.01.01A estimate differences by rounding the minuend and the subtrahend

STRATEGY 1:

> **Materials:**
> - the same chart (Large Islands) used in Getting Started

Display the chart. Say, "Let us estimate the differences in area (size) of Iceland and Ireland. How would we estimate the difference? Explain." (We would compare Iceland's area with Ireland's. Since they both have areas which are 5-digit numbers, we would round the numbers to their left-most places which is the nearest ten thousands place. Then we would subtract.) Ask a student to perform the indicated operation as the other students work in their notebooks.

e.g.,

```
  39,769   rounds to   40,000
 -32,599   rounds to  -30,000
                       10,000 is an estimated difference
```

Say, "What is the estimated difference in areas between Iceland and Ireland? Explain. (Iceland is approximately 10,000 miles larger in area than Ireland.) Let us now compare the areas of New Zealand and Iceland. Write the exact numbers from the chart which we must subtract, round them, and then subtract."

e.g.,

```
  103,883   rounds to   100,000
 - 39,769   rounds to  - 40,000
                         60,000 is an estimated difference
```

Say, "To which place was each number rounded? (The minuend and the subtrahend were rounded to the nearest ten thousand.) There are cases when we are rounding numbers and we must decide how accurate our approximation must be. Let us compare, for instance, the areas of Japan and the Philippines. Write the information into your notebooks exactly as it is in the chart."

STRATEGY 1: (cont'd)

e.g.,

Japan	142,727
Philippines	-115,830

Say, "If we were to round these 6-digit numbers to their left-most places, what would both numbers be? (100,000) If we subtract 100,000 from 100,000 our answer would be zero. This is not helpful; clearly, the islands are not the same size. How can we get a more accurate approximation?" Elicit that each number can be rounded to the nearest ten thousands place. Demonstrate and ask the students to solve the problem in their notebooks.

e.g.,

142,727	rounds to	140,000	
-115,830	rounds to	-120,000	
		20,000	is an estimated difference

Say, "Rounding numbers in this way still gives us an opoportunity to compare them. What does this estimated difference tell us?" (that Japan has an area which is approximately 20,000 square miles greater than the Philippines) Write the following problem on the chalkboard:

427,186
-196,380

Say, "There are instances when we must decide how accurate we wish our estimated differences to be. If we are not concerned about accuracy, yet wish to have a reasonable estimate, we can still round both of the above numbers to their left-most places." Demonstrate.

e.g.,

427,186	rounds to	400,000	
-196,380	rounds to	-200,000	
		200,000	is an estimated difference

Repeat this procedure to estimate differences between other large number pairs. Give the students opportunities to estimate the same pair of numbers by rounding to different places. Caution students that numbers being compared should always be rounded to the same place.

6.03.01.02A subtract large numbers and check by adding

Display the following subtraction problem in horizontal format:
$825{,}929 - 32{,}186 = \square$.

Say, "What would be the first step for solving the problem?" (Rewrite the subtrahend under the minuend.) Elicit the importance of placing the digits in the correct place value columns. Ask the students to write the subtraction example in their notebooks in vertical format as one student works at the chalkboard.
e.g.,

$$\begin{array}{r} 825{,}929 \\ \underline{-\ 32{,}186} \end{array}$$

Say, "How can we estimate the difference?" (We can round the minuend and the subtrahend and then subtract.) Demonstrate.
e.g.,

$$\begin{array}{rlr l} 825{,}929 & \text{rounds to} & 830{,}000 & \\ \underline{-\ 32{,}186} & \text{rounds to} & \underline{-\ 30{,}000} & \\ & & 800{,}000 & \text{is an estimated difference} \end{array}$$

Ask a student to find the actual difference at the chalkboard as the other students work in their notebooks. Say, "What is the actual difference? (793,743) Is it reasonable? (Yes, 800,000 is a close approximation to the actual difference of 793,743.) How can we check our actual answer to confirm its accuracy?" (We can add the difference to the subtrahend. The sum should be equal to the minuend.) Ask a student to model it on the chalkboard.
e.g.,

$$\begin{array}{rr} & \underline{\text{check}} \\ 825{,}929 & 793{,}743 \\ \underline{-\ 32{,}186} & \underline{+\ 32{,}186} \\ 793{,}743 & 825{,}929 \end{array}$$

Students may also use a calculator to check their work.

Display the following problem: $\begin{array}{r} 732{,}730 \\ \underline{-\ 89{,}910} \end{array}$

STRATEGY 1: (cont'd)

Ask students to find the estimated difference, the actual difference, and check.

e.g.,

732,730 rounds to 730,000
- 89,910 rounds to - 90,000
640,000 is an estimated difference

732,730
- 89,910
642,820 is the actual difference

check
642,820
+ 89,910
732,730

List the following procedures on the chalkboard:

A. Round and estimate the difference.
B. Find the actual difference.
C. Compare the actual difference to the estimated difference.
D. Check by adding up.

Say, "How does following these procedures help us?" (They provide us with a method by which we can be certain that our final answer is correct.) Compute other subtraction examples including money problems, having minuends and subtrahends to 6-digits.

6.03.01.03A create and solve word problems related to skills learned in this module

Write the following word problem on the chalkboard:

> The ABC Department Store collected $4278.17 during a Columbus Day sweater sale. On the same day, the XYZ Department Store earned $3456.21 during their sweater sale. About how much more money did the ABC Department Store earn than the XYZ Department Store?

Say, "What are we being asked to find? (an estimated difference in earnings between the two stores) What word tells us that it's an estimated answer we are looking for? (the word 'about') What must we do to find the answer?" (Estimate by rounding the two amounts.) How might we round the numbers?" (Considering the amounts of money

STRATEGY 1: (cont'd)

involved, we could round each amount to the nearest thousand dollars.)

e.g.,

$$\begin{array}{lll} \$4278.17 & \text{rounds to} & \$4000.00 \\ \underline{-3456.21} & \text{rounds to} & \underline{-3000.00} \\ & & \$1000.00 \end{array}$$

Say, "What is the estimated difference? (The ABC Department Store earned about $1000 more.) What if we wanted to estimate the difference a bit more accurately? How might we round the numbers?" (We might round to the nearest hundred dollars for both the minuend and the subtrahend.) Demonstrate.

e.g.,

$$\begin{array}{lll} \$4278.17 & \text{rounds to} & \$4300.00 \\ \underline{-3456.21} & \text{rounds to} & \underline{-3500.00} \\ & & \$\ 800.00 \end{array}$$

Say, "In this case, $800 is still reasonably close to $1000 but is somewhat more accurate as the difference in money that the two stores earned. How would we have answered this question if the word 'about' were eliminated?" (We would subtract the numbers just as they appear in the word problem, and check the difference by adding up.) Ask a student to solve this problem at the chalkboard as the others work in their notebooks. Compare and discuss this difference with the estimated differences found before.

e.g.,

$$\begin{array}{rcr} & & \underline{\text{check}} \\ \$4278.17 & \qquad & \$\ 821.96 \\ \underline{-3456.21} & & \underline{+3456.21} \\ \$\ 821.96 & & \$4278.17 \end{array}$$

Provide, or have the students create, additional addition and subtraction word problems involving 6-digit numbers, including some which do not involve money.

84

P.O. 6.03.01.01-.02

NAME: ______________________ **CLASS:** ________ **DATE:** ____________

I. Use the data found in this table to answer questions 1-5.

Distances from the Sunstar in the Chocvan Planetary System	
Planet	Miles from Sunstar
Strawnilla	432,578
Rasolate	751,062
Fudgina	294,611
Ribbleberry	87,293
Heavenly	346,755

1. How much farther from the Sunstar is Rasolate than Fudgina?

____

2. Heavenly is how much closer to Sunstar than Strawnilla?

____

3. Approximately how much farther from Sunstar is Strawnilla than Fudgina?

____

4. Exactly how much difference is there in miles (from Sunstar) between the farthest planet and the closest planet?

____

5. Which 2 planets are closest in distance to each other?

____________ and ____________

II. Solve each problem by;

A) Rounding to estimate

B) Finding the actual difference

C) Checking by adding up

1. $\begin{array}{r} 684{,}973 \\ -\ 96{,}273 \\ \hline \end{array}$

Ⓐ estimate

Ⓑ subtract

Ⓒ check

2. $\begin{array}{r} 415{,}219 \\ -343{,}790 \\ \hline \end{array}$

Ⓐ estimate

Ⓑ subtract

Ⓒ check

III. Round 473,275 and 42,705 to the nearest ten thousand and find the estimated difference.

P.O. 6.03.01.03

NAME: ______________________________ **CLASS:** _______ **DATE:** ____________________

MIXED PROBLEM SOLVING

1. A four-engine plane weighs 775,416 pounds. How much greater is its weight than a two-engine plane that weighs 214,387 pounds?

____

2. On Monday, the Coblenz Courier sold 317,538 newspapers. On that same day, the Wilson Gazette sold 458,997 newspapers. How many newspapers were sold on Monday?

____

3. The 6th grade students of the Woodside School collected 8400 aluminum cans for recycling. If each student collected 80 cans, how many students are there in the 6th grade?

____

4. Rocky Star earned $264,220 doing concerts during the year. Tom Singer earned $641,560 during the same year. What is the difference in their earnings?

____

5. By January, the Woodside School had collected 352,425 cans. How many more cans must be collected to reach its goal of 700,000 cans?

____

6. Saul drove 475 miles each week making deliveries. During the year he worked 48 weeks. How many more miles did he drive this year than last year when he drove only 19,350 miles?

____

7. Create your own problem using subtraction of 6-digit numerals.

____

____

SHARPENING YOUR SKILLS

Units 4, 5, 6

Name: ____________________ **Class:** ____________ **Date:** ________

1. Round each of the following to the nearest ten thousand.

386,920 rounds to ____________
1,609,265 rounds to ____________
7,233,824 rounds to ____________

2. Round each of the following to the nearest hundred thousand.

295,865 rounds to ____________
4,806,291 rounds to ____________
3,754,003 rounds to ____________

3. Which of the following would round to 680,000 when rounded to the nearest ten thousand?

ⓐ 685,297 ⓒ 681,269
ⓑ 674,865 ⓓ Not Here

4.

$$\begin{array}{r} 275{,}416 \\ 86{,}904 \\ \underline{+315{,}649} \end{array}$$

ⓔ 676,969 ⓖ 677,969
ⓕ 667,969 ⓗ Not Here

5.

$$\begin{array}{r} 175{,}488 \\ 216{,}911 \\ 45{,}623 \\ 6{,}195 \\ \underline{+104{,}263} \end{array}$$

ⓐ 558,480 ⓒ 548,480
ⓑ 448,480 ⓓ Not Here

6.

$$\begin{array}{r} \$2749.85 \\ \underline{-\ 165.99} \end{array}$$

ⓔ \$2583.86 ⓖ \$2684.86
ⓕ \$2584.96 ⓗ Not Here

7.

$$\begin{array}{r} 700{,}931 \\ \underline{-\ 8{,}744} \end{array}$$

ⓐ 791,287 ⓒ 692,187
ⓑ 681,187 ⓓ Not Here

8. On Friday, 284,154 cars drove across the Manhattan Bridge. On Saturday, 69,788 cars drove across. How many more cars drove across the bridge on Friday than on Saturday?

____________ cars

9. Ms. Denton, a salesperson for the Bradley Corporation, earned \$2460.08 in May and \$2157.94 in June. How much did she earn during the two months.

____

10. Mr. Meyers wants to buy a car that costs \$9875.90. He has saved \$2699.89. How much more does he need to buy the car?

____

6.01.03 BASE FIVE

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

6.01.03.01A identify the basic symbols in base five;

e.g.,

0, 1, 2, 3, 4

6.01.03.02A read a base five place value chart:

e.g.,

twenty - fives	fives	ones
5^2	5^1	5^0
1	3	2

$= (1x5^2) + (3x5^1) + (2x5^0)$

6.01.03.03A find the base ten equivalent of a base five numeral;

e.g.,

$$\begin{aligned} 132_{five} &= (1 \times 5^2) + (3 \times 5^1) + (2 \times 5^0) \\ &= 25 + 15 + 2 \\ &= 42_{ten} \end{aligned}$$

6.01.03.04B find the base five equivalent of a base ten numeral.

e.g.,

$46_{ten} = 141_{five}$

$25_{ten} = 100_{five}$

PROBLEM EXPLORATION SKILLS:

- to use analogy
- to generalize
- to adopt a different point of view

VOCABULARY

place value	total value	factor
face value	exponent	base

PREVIOUS UNDERSTANDINGS:

- place value in base ten
- exponential notation
- any number to the zero power is equal to one

GETTING STARTED:

Materials:

- large place value chart
- deck of numeral cards (0-9)

Display the large place value chart and the deck of numeral cards (0-9).

e.g.,

Large Place Value Chart

thousands	hundreds	tens	ones
10^3	10^2	10^1	10^0

Ask one student to shuffle the deck and another student to pick a card. Say, "Read the numeral aloud." Elicit that the value of a number on the numeral card can be identified as its face value. Ask a student to place the numeral card in any column on the place value chart.

e.g.,

Large Place Value Chart

thousands	hundreds	tens	ones
10^3	10^2	10^1	10^0
	6		

Say, "What is the face value of the card? (6) In which place is the 6 located? (the hundreds place) How can we determine the total value of the six?" (We can multiply the face value, 6, by the place value, 10^2). Demonstrate that $6 \times 10^2 = 6 \times 100 = 600$.

Repeat this procedure by finding the total value of the same face value card placed in different columns of the place value chart.

6.01.03

6.01.03.01A identify the basic symbols in base five

STRATEGY 1:

Display the following on the chalkboard:

0,1,2,3,4,5,6,7,8,9   , , , ,

Ask a student to read the symbols which appear in the first chart aloud. Say, "What do these symbols represent? (These symbols are used to write any number in base ten.) How many symbols are there? (10) What can we label this number chart?" (Symbols in Base Ten) Ask a student to label the chart.

e.g.,

Symbols in Base Ten

0,1,2,3,4,5,6,7,8,9   , , , ,

Say, "The second chart will show the symbols we use in base five. How many symbols do we have in a base five number system? Explain. (There are 5 symbols in the base five system, just as there are 10 symbols in base ten.) What does the base tell us? (The base tells us how many symbols exist in the system.) What are the 5 symbols in the base five system?" (0,1,2,3,4) Ask a student to list the 5 symbols on the chart.

e.g,

0,1,2,3,4

Say, "What is an appropriate label for the chart?" (Symbols in Base Five) Ask a student to label the chart.

e.g..,

Symbols in
Base Five

0,1,2,3,4

STRATEGY 1: (cont'd)

Say, "Since we only have five symbols to use for all the numbers in base five, how can we name the number after 4? (If we follow the same pattern as we did in base ten, we will use a combination of digits to represent the number after 4 in base five.) What is the correct combination of digits?" (10 in base five) Write the following on the chalkboard:

1_{five} 2_{five} 3_{five} 4_{five} 10_{five}...

Say, "Why do we use the word five in subscript when we are recording our numbers in base five?" (to differentiate between base ten numbers and base five numbers.)

6.01.03.02A read a base five place value chart

STRATEGY 1:

Materials:
- place value charts
- interlocking cubes

Distribute interlocking cubes and say, "Today we are going to work with sets of fives in order to develop a place value chart based on groups of five. How many symbols did we use in base ten? (We used ten symbols.) What are the symbols used in base ten? (0,1,2,3,4,5,6,7,8,9) How is the value of ten represented? (We use a 1 and a 0 to represent ten.) How does the base five system differ from the base ten system?" (Instead of groups of ten we are going to use groups of five.) On your own place value chart write the word 'ones' in the first column on the right and 5^0 directly beneath it."
e.g.,

	ones
	5^0

STRATEGY 1: (cont'd)

Say, "What value do we give 5^0? (1) Why? (Any number to the zero power equals one.) Since we do not write a base five numeral using the digit 5, the number which comes after 4 is represented by the digits one and zero. One-zero base five is written 10_{five}. What is the value of 10_{five}? (It is given the value of one set of five and zero ones.) Let's use our place value chart and the interlocking cubes to show 6 in base five." Place the train of five cubes in the place value column second from the right. Label the column 'fives' with 5^1 beneath it and put one cube in the 5^0 column.

e.g.,

fives	ones
5^1	5^0

Say, "How many cubes have you used? (6) How could you write this number in base five?" (one group of five and 1 one or 11_{five}) Ask students to continue to add one cube at a time to their place value charts and record the base five numerals as they add, e.g., 7 cubes = 12_{five}, 8 cubes = 13_{five}, 9 cubes = 14_{five}. Say, "How can ten cubes be arranged so that we can record the number in base five? (We can make 2 sets of 5 and place both sets in the fives column and use a zero as a place holder in the ones column.) How would we write 10 cubes as a base five number?" (20_{five}) Continue to expand the place value chart.

e.g.,

twenty-five	fives	ones
5^2	5^1	5^0

STRATEGY 1: (cont'd)

Write the following base ten chart on the chalkboard:

thousands	hundreds	tens	ones
10^3	10^2	10^1	10^0

Say, "We have followed the same pattern that is used in base ten to develop a base five place value chart. What name would we give to the next column in the base five place value chart?" (one hundred twenty-fives place or 5^3) Ask students to record this information on the base five charts.

e.g.,

one hundred twenty-fives	twenty-fives	fives	ones
5^3	5^2	5^1	5^0

Write 333_{five} on the chalkboard and ask the students to record this number in the correct places on their base five charts. Say, "Is each digit equal in total value? Explain." (No, the actual value of each digit can be determined by multiplying the digit by the place it occupies.) Write the following on the chalkboard and ask the students to write it into their notebooks:

$$333_{five} = (3 \times 5^2) + (3 \times 5^1) + (3 \times 5^0)$$

Repeat this procedure using other numbers on the base five place value chart.

6.01.03.03A **find the base ten equivalent of a base five numeral**

STRATEGY 1:

Materials:
- base ten place value chart
- base five place value chart

STRATEGY 1: (cont'd)

Display a base ten place value chart and a base five place value chart. On the chalkboard write the following numbers:

413_{ten} 413_{five}

Say, "Why do we need a subscript with these numbers? (so we can see what value each number has according to the base) Does it make a difference what the base is when the numbers are the same? (Yes, although the face value of the digit is the same, the place value changes according to the base.) If there is no subscript we assume the number is a base ten number." Ask a student to write the base ten and base five numbers in the correct places on their respective charts.

e.g.,

hundreds	tens	ones
10^2	10^1	10^0
4	1	3

twenty-fives	fives	ones
5^2	5^1	5^0
4	1	3

Say, "What is the value of 413_{ten}? (4x100) + (1x10) + (3x1) How did you find your answer? (I multiplied face value x place value in base ten.) Let's look at the same digits in base 5. How can we find the total value of each digit? (We can multiply face value times place value in base five.) What value would we get for 413_{five}? (We would have (4x25) + (1x5) + (3x1). The value of 413_{five} = 100 + 5 + 3.) What is the sum of the three products?" (108) On the chalkboard write:

$$\begin{aligned} 413_{five} &= (4\times25) + (1\times5) + (3\times1) \\ &= 100 + 5 + 3 \\ &= 108_{ten} \end{aligned}$$

Say, "In order to rename any number from a base five to a base ten value, we must find the total value of each digit and add to find the total value of the number."

Repeat this procedure to find the base ten equivalent of other base five numbers.

6.01.03.04B find the base five equivalent of a base ten numeral

STRATEGY 1:

Write the following on the chalkboard: 323_{five} 69_{ten}

Say, "I would like to arrange these numbers in order. How do I know which is the greater number? (I cannot compare them unless both numbers are in the same base.) Let's change the base ten number to a base five equivalent in order to compare the two numbers." Draw a base five place value chart on the chalkboard.

e.g.,

twenty-fives	fives	ones
5^2	5^1	5^0

Say, "I have 69 objects. I would like to show this amount correctly on the base five place value chart. What is the largest base five group of objects I can subtract from 69? (twenty-five) How many groups of 25 can I subtract from 69? (2 groups of 25) How can I record this on my place value chart?" (I can write a 2 in the twenty-fives place.) Demonstrate.

e.g.,

twenty-fives	fives	ones
5^2	5^1	5^0
2		

Ask a student to come to the chalkboard and demonstrate the division algorithm while the other students work in their notebooks.

e.g.,

$$\begin{array}{r} 2 \\ 25\overline{)69} \\ \underline{-50} \\ 19 \end{array}$$

Say, "Do I have any more groups of 25 to subtract? (No, the remainder to be divided is 19.) What is the next largest set in base five that can be subtracted from 19? (I can make groups of 5.) How many groups

STRATEGY 1: (cont'd)

of five can be subtracted from 19?" (3) Ask a student to record the 3 in the fives column and demonstrate the division algorithm.

e.g.,

twenty-fives	fives	ones
5^2	5^1	5^0
2	3	

$$\begin{array}{r} 3 \\ 5\overline{)19} \\ -15 \\ \hline 4 \end{array}$$

Say, "Let's complete this example. How many objects are left? (4) Where should I record this digit?" (in the ones column) Ask a student to record the 4 in the appropriate column and demonstrate this division algorithm.

e.g.,

twenty-fives	fives	ones
5^2	5^1	5^0
2	3	4

$$\begin{array}{r} 4 \\ 1\overline{)4} \\ -4 \\ \hline 0 \end{array}$$

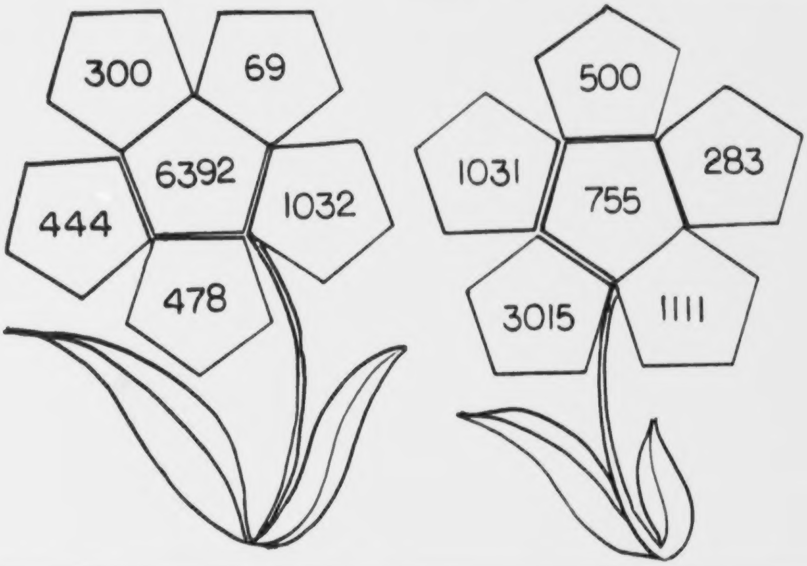
Say, "What is the base five equivalent of 69_{ten}? (234_{five}) Now we can order the numbers. Which is greater?" (323_{five} is greater than 234_{five}.)

Repeat this procedure to find other base five equivalents of base ten numbers.

P.O. 6.01.03.01-.03

NAME: ____________________ **CLASS:** ______ **DATE:** ____________

I. Circle the petals that could be base five numerals.
Place an X on the petals that cannot be base five numerals.



II. Complete the base five place value chart. Place each base five numeral in the correct place on the chart.

a. 320_{five}
b. 1032_{five}
c. 441_{five}

	one hundred twenty-fives	twenty-fives	fives	ones
		5^2		5^0
a				
b				
c				

III. Complete the expanded notation using exponents in base five. Find the base ten equivalents of the base five numerals.

a. $320_{\text{five}} = (\quad\quad) + (2 \times 5^1) + (\quad\quad) = \underline{\qquad\qquad}$ ten

b. $1032_{\text{five}} = (\quad\quad) + (0 \times 5^2) + (\quad\quad) + (\quad) = \underline{\qquad}$ ten

c. $441_{\text{five}} = (4 \times 5^2) + (\quad\quad) + (\quad\quad) = \underline{\qquad\qquad}$ ten

P.O. **6.01.03.03-.04**

NAME: ______________________ **CLASS:** _______ **DATE:** ______________

I. Match each base five numeral with its base ten equivalent.

a.	203_{five}	10_{ten}
b.	4_{five}	156_{ten}
c.	4444_{five}	624_{ten}
d.	20_{five}	194_{ten}
e.	1234_{five}	53_{ten}
f.	1111_{five}	4_{ten}

II. Find the base five equivalent for the following base ten numerals. Use the base five place value chart to help you.

one hundred twenty-fives	twenty-fives	fives	ones
5^3	5^2	5^1	5^0

a. 85_{ten} = ______ five

b. 20_{ten} = ______ five

c. 438_{ten} = ______ five

d. 76_{ten} = ______ five

CHALLENGE!

Develop a place value chart for base eight through four places.

Base Eight Place Value Chart

6.01.04 BASE TWO

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

6.01.04.01B read a base two place value chart;

e.g.,

fours	twos	ones
2^2	2^1	2^0
1	1	1

$= (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0)$

6.01.04.02B find the base ten equivalent of a base two numeral;

e.g.,

$$\begin{aligned} 111_{two} &= (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\ &= 4 + 2 + 1 \\ &= 7_{ten} \end{aligned}$$

6.01.04.03B find the base two equivalent of a base ten numeral.

e.g.,

$23_{ten} = 10111_{two}$

PROBLEM EXPLORATION SKILLS:

- to use analogy
- to generalize
- to adopt a different point of view

6.01.04

VOCABULARY

base
exponent
binary

PREVIOUS UNDERSTANDINGS:

- place value (base ten)
- place value (base five)

GETTING STARTED:

Materials:

- chart representing hotel rooms done in pencil or erasable ink

Display the following chart:

	5	1	1	0	0	1	1	0
	4	1	0	0	1	1	0	1
Floors	3	1	1	1	0	0	0	1
	2	1	0	1	1	1	0	0
	1	0	1	0	0	1	0	1
		1	2	3	4	5	6	7

Rooms

Say, "We are going to play a game called "Hotel". Imagine that you work in a small hotel with 5 floors and 7 rooms on each floor. You are in charge of using the hotel's computer to keep track of which rooms are occupied or unoccupied. This chart represents the display on the computer's screen. A zero indicates that a room is not occupied. A one indicates that a room is occupied. A family enters the hotel and asks for two rooms next to each other, on the fifth floor. Will you be able to accomodate them? Explain. (Yes. The computer displays 1100110 for the fifth floor. I can place the family in the 3rd and 4th rooms on the fifth floor.) After you have entered

GETTING STARTED: (cont'd)

this information into the computer, what will the computer display show for the fifth floor?" Ask a student to make these changes on the chart.

e.g.,

	5	1	1	1	1	1	1	0
	4	1	0	0	1	1	0	1
Floors	3	1	1	1	0	0	0	1
	2	1	0	1	1	1	0	0
	1	0	1	0	0	1	0	1
		1	2	3	4	5	6	7

Rooms

Say, "A person occupying the fifth room on the second floor checks out of the hotel. What must you do? Explain." (The computer display reads 1011100 for the second floor. I must change the display so that it will read 1011000.)

Repeat this procedure to change the computer display for various floors as other people check in and out of the hotel.

6.01.04

6.01.04.01B read a base two place value chart

STRATEGY 1:

Materials:
- place value chart

Say, "We have worked with numerals in base ten and base five. How many digits are used to write numbers in base ten (10) What are these digits? (0,1,2,3,4,5,6,7,8,9) How many digits are used to write numbers in base five? (5) What are these digits? (0,1,2,3,4) Today, we will work with numerals in base two which is also called the binary system. Using our knowledge involving base ten and base five, how many digits will we use to write base two numerals (2). What are these digits? (0,1) Base two numerals look like the rows of digits shown on the computer display in our Getting Started activity."

Say,"In order to work with base two numerals, we will need to develop a base two place value chart. Let's use our knowledge of base ten and base five to develop this chart. What do we call the place farthest to the right in base ten and base five? (ones) What will we call the place farthest to the right in base two?" (ones) Display a place value chart and ask a student to record this information on the chart as the other students work in their notebooks.
e.g.,

Base Two Place Value Chart

	ones

Say, "How did we express ones place using an exponent in base ten and base five? Explain. (In base ten, ones place is 10^0 using an exponent. In base five, it is 5^0.) How will we express ones place using an exponent in base two?" (2^0) Ask a student to record this information on the chart as the other students work in their notebooks.

STRATEGY 1: (cont'd)

e.g., **Base Two Place Value Chart**

	ones
	2^0

Say, "Moving to the place left of the ones place on the chart, what name is given to this place in base ten and in base five? (In base ten, it is called tens place. In base five, it is called fives place.) If we follow the same pattern, how will we name this place on the base two place value chart? (We will call it the twos place.) How did we express this place using an exponent in base ten and base five? (In base ten, it is 10^1. In base five, it is 5^1.) How will we express this place using exponents in base two?" (2^1) Ask a student to record this information on the chart as the other students work in their notebooks.

e.g.,

	twos	ones
	2^1	2^0

Say, "If we continue to follow the same pattern that is used to develop place value charts for base ten and base five, how can we determine each place as we move left on the base two place value chart? Explain." (In base ten, each place is ten times greater than the previous place as we move left on the base ten chart. In base five, each place is five times greater than the place to the right of it. In base two, each place will be two times greater than the place to the right of it.) Ask a student to develop the base two chart up to sixteens place as the other students do the same in their notebooks.

e.g.,

sixteens	eights	fours	twos	ones
2^4	2^3	2^2	2^1	2^0

STRATEGY 1: (cont'd)

Say, "What does the base two place value chart show us? (The first place in base two is the ones place. Each succeeding place is found by multiplying by two as we move to the left.) What would the next place be if we wanted to continue developing the chart? Explain." (The next place would be 16x2 or thirty-twos place.)

6.01.04.02B find the base ten equivalent of a base two numeral

STRATEGY 1:

Materials:
- base two place value chart

Write a base two numeral on the chalkboard.
e.g.,

$$10111_{two}$$

Say, "What do we know about base two or binary system numerals?" (The digits 0 and 1 are used to write all base two numerals.) Display the base two place value chart. Say, "We read this base two numeral as one-zero-one-one-one base two. How would we write this numeral on the Base Two Place Value Chart?" (The digit 1 farthest to the right would go in the ones place. We would then fill in the digits moving to the left on the chart.) Ask a student to demonstrate this information on the chart while the other students do the same in their notebooks.

e.g.,

Base Two Place Value Chart

sixteens	eights	fours	twos	ones
1	0	1	1	1

Say, "What is the value of the 1 in the sixteens place? (1 x 16 or 16) ... the zero in the eights place? (0 x 8 or 0) If we find the value of each digit and add the products, the result will be the base ten equivalent of our base two numeral." Demonstrate.

STRATEGY 1: (cont'd)

e.g.,

$$\begin{array}{rcrcr} 1 & \times & 16 & = & 16 \\ 0 & \times & 8 & = & 0 \\ 1 & \times & 4 & = & 4 \\ 1 & \times & 2 & = & 2 \\ 1 & \times & 1 & = & \underline{1} \\ & & & & 23 \end{array}$$

Say, "What is the base ten equivalent of 10111_{two}?" (23). Record this information on the chalkboard as the students do the same in the notebooks.

e.g., $10111_{two} = 23_{ten}$

Continue this procedure by eliciting other base two numerals from the students and then finding their base ten equivalents.

6.01.04.03B find the base two equivalent of a base ten numeral

STRATEGY 1:

Materials:
- base two place value chart up to sixty-fours place

Display the Base Two Place Value Chart.

e.g.,

Base Two Place Value Chart

sixty-fours	thirty-twos	sixteens	eights	fours	twos	ones

Say, "How did we find the base ten equivalent of a base two numeral?" (We wrote the base two numeral on the base two chart, multiplied each digit by the place it occupied, and then added the products.) Write these steps on the chalkboard and ask the students to write them into their notebooks.

STRATEGY 1: (cont'd)

e.g.,

Finding the Base Ten Equivalent of a Base Two Numeral

(1) Write the base two numeral on the base two chart.

(2) Multiply each digit by the place it occupies.

(3) Find the sum of the products.

Say, "How can we find the base two equivalent of a base ten numeral? Explain." (We can reverse the procedure. Instead of multiplying and adding, we will divide and then subtract.) Write a base ten numeral on the chalkboard, e.g., 27. Say, "Look at the Base Two Place Value Chart. We will begin by dividing at the highest possible place on the chart. Our chart has been developed up to the sixty-fours place. Can we begin dividing at the sixty-fours place? Explain. (No. Sixty-four is larger than 27.) At what place can we begin to divide?" (We can begin at the sixteens place.) Demonstrate.

e.g.,

$$\begin{array}{r} 1 \\ 16\overline{)27} \\ \underline{-16} \\ 11 \end{array}$$

Say, "How many sixteens are in 27." (1) Ask a student to place the 1 in the sixteens place on the base two chart.

e.g.,

Base Two Place Value Chart

sixteens	eights	fours	twos	ones
1				

$$\begin{array}{r} 1 \\ 16\overline{)27} \\ \underline{-16} \\ 11 \end{array}$$

Say, "We have divided 27 by sixteen and subtracted 1 x 16 or 16 from 27. How much is left? (11) What should we do with the 11." (We should move to the next place on the chart and divide 11 by that place.) Ask a student to demonstrate at the chalkboard as the other students work in their notebooks.

STRATEGY 1: (cont'd)

e.g.,

Base Two Place Value Chart

sixteens	eights	fours	twos	ones
1	1			

$$\begin{array}{r} 1 \\ 16\overline{)27} \\ -16 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 1 \\ 8\overline{)11} \\ -\ 8 \\ \hline 3 \end{array}$$

Say, "We have divided eight into 11. How much is left over? (3) What should be done with the 3? (We should move to the fours place on the chart and divide.) Can we divide four into 3? (no) What should we put in the fours place?" (zero) Ask a student to continue dividing and subtracting until the ones place is reached.

e.g.,

Base Two Place Value Chart

sixteens	eights	fours	twos	ones
1	1	0	1	1

$$\begin{array}{r} 1 \\ 16\overline{)27} \\ -16 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 1 \\ 8\overline{)11} \\ -\ 8 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 1 \\ 2\overline{)3} \\ -2 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 1 \\ 1\overline{)1} \\ -1 \\ \hline 0 \end{array}$$

Say, "What is the base two equivalent of 27_{ten}?" (11011_{two})

Repeat this procedure to find the base two equivalent of other base ten numerals.

P.O. 6.01.04.01-.02

NAME: ______________________________ **CLASS:** ________ **DATE:** ______________

I. Complete the base two place value chart. Place the base two numerals in the correct places on the chart.

					fours		
			2^4			2^1	
a. 11010_{two} a.							
b. 100111_{two} b.							
c. 1010101_{two} c.							
d. 11111_{two} d.							

II. Express each base two numeral using exponents.

a. 11010_{two} = (1×2^4) + ________ + ________ + ________ + ________

b. 100111_{two} = ________ + ________ + ________ + ________ +
________ + ________

c. 1010101_{two} = ________ + ________ + ________ + ________ +
________ + ________ + ________

d. 11111_{two} = ________ + ________ + ________ + ________ +
____

III. Express each base two numeral as its base ten equivalent.

a. 11010_{two} = ________ ten

c. 1010101_{two} = ________ ten

b. 100111_{two} = ________ ten

d. 11111_{two} = ________ ten

P.O. 6.01.04.02-.03

NAME: ______________________ **CLASS:** ________ **DATE:** ____________

I. Match each base two numeral with its base ten equivalent.

a. 10011_{two}	36_{ten}
b. 1010_{two}	23_{ten}
c. 100100_{two}	108_{ten}
d. 11100_{two}	19_{ten}
e. 10111_{two}	10_{ten}
f. 111110_{two}	28_{ten}
g. 1011110_{two}	62_{ten}
h. 1101100_{two}	94_{ten}

II. Express the following base ten numerals as base two equivalents.

a. 49_{ten} = ________$_{two}$

b. 72_{ten} = ________$_{two}$

c. 96_{ten} = ________$_{two}$

d. 33_{ten} = ________$_{two}$

e. 58_{ten} = ________$_{two}$

f. 89_{ten} = ________$_{two}$

g. 101_{ten} = ________$_{two}$

h. 66_{ten} = ________$_{two}$

Do you remember?

1 yard = ________ inches

1 foot = ________ inches

1 yard = ________ feet

1 pound = ________ ounces

1 quart = ________ pints

1 gallon = ________ quarts

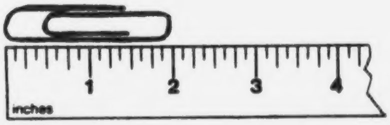
6.14.03 REVIEW: CUSTOMARY MEASUREMENT

(1 lesson/ongoing)

PERFORMANCE OBJECTIVES

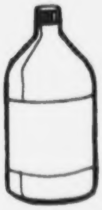
The student will be able to...

6.14.03.01 estimate and measure length using inches, feet, and yards;
e.g.,



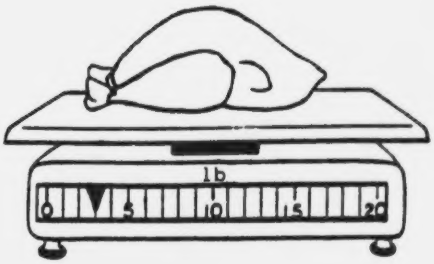
The paper clip is about 2 inches long.

6.14.03.02 estimate and measure capacity using cups, pints, quarts, and gallons;
e.g.,



The bottle holds 2 quarts of liquid.

6.14.03.03 estimate and measure weight using ounces and pounds.
e.g.,



The chicken weighs about 3 pounds.

PROBLEM EXPLORATION SKILLS:

- to estimate (length, capacity, weight)
- to adopt a different point of view

VOCABULARY

inches
feet
yards
cups
pints
quarts
gallons
ounces
pounds
linear
capacity
weight

PREVIOUS UNDERSTANDINGS:

- customary measurement
- selection of appropriate unit of measure

GETTING STARTED:

Say, "Imagine the problems which would exist in a world which has neither units of measure nor tools of measurement." Ask students to name and list examples of customary units of measure and tools of measurement to insure that everyone understands the problem.

e.g.,

	CUSTOMARY UNITS OF MEASURE	TOOLS OF MEASUREMENT
LINEAR	inch, foot, yard	ruler, yardstick
CAPACITY	cups, pints, quarts, gallons	measuring cups and containers
WEIGHT	ounces, pounds	scales

Say, "What kinds of problems might occur if these units and tools did not exist? (Accept all reasonable responses with regard to consumers' purchases, packaging, construction, etc.) How do you think this problem could be solved? (People might use physical standards such as the width of a handspan to demonstrate "how wide" or the weight of a rock to demonstrate "how heavy".) Why do you think using a person's physical measurements, such as a handspan, would create problems?" (Handspans vary in size.)

Say, "Ancient civilizations faced the same problems since they too had neither units of measure nor tools of measurement. How do you think they began to develop a system of measurement?" (They began to develop a system of measurement based on units that were the length of

GETTING STARTED: (cont'd)

certain parts of the body.) Write the following on the chalkboard:

cubit (Egyptians) - the length of a man's forearm from his elbow to the tip of his middle finger

uncia (Romans) - the width of a man's thumb

Say, "Which measurement unit do you think comes from the Roman uncia?" (The word inch comes from uncia.) How do you think it is related to a foot? (Approximately 12 uncias equal a foot.) To what other unit of measure is the foot related? (There are 3 feet in a yard.) How can the physical measurements of a person be used to demonstrate a yard? (Three feet, or one yard, is the approximate distance from a man's nose to the tip of the middle finger of his outstretched arm.) Why did the use of these non-standard units of measure make it necessary to develop standard units? (Since people had different length cubits (elbow to finger tips) and uncias (thumb width), measurements were inaccurate.) How have standard units changed that?" (The use of a standard unit insures the uniformity of measurements.)

Interested students may wish to investigate/research the development of other areas of measurement, such as weight and capacity, as well as the origins of the vocabulary used as standard units of measure.

6.14.03

6.14.03.01 estimate and measure length using inches, feet, and yards

STRATEGY 1:

Materials:
- worksheet shown below
- ruler
- yardstick
- objects to be measured

Ask students to think about a time when they learned how to roller skate or to ride a bicycle. Say, "Did you learn by reading about it in a book or by practicing? (We could read about it in books, but we really learned how to do it by practicing.) What happened while you were practicing? (We made mistakes, we fell, but we eventually learned how to skate or ride.) How can we improve our measurement skills?" (by measuring objects)

Distribute the following worksheet to small groups of students along with several objects, e.g., pencils, pens, crayons, sticks, adding machine tape. Write the worksheet headings on the chalkboard. e.g.,

MEASUREMENT WORKSHEET

OBJECT OR DISTANCE TO BE MEASURED	UNIT OF MEASURE	1ST ESTIMATE	2ND ESTIMATE	ACTUAL MEASURE	DIFFERENCE 1ST ESTIMATE	DIFFERENCE 2ND ESTIMATE

Say, "Let's complete the first activity together. In order to measure the length of the classroom, what is an appropriate unit of measure? (feet or yards) Which measuring tool should we use? (a ruler or a yardstick) What is a reasonable estimate of the length of the classroom?" List all responses on the chalkboard. Write the consensus agreed to on the chart.

Ask several students to come to the front of the room and begin measuring the length of the classroom. At approximately the midpoint ask the students to stop. Say, "Let's re-evaluate our first

STRATEGY 1: (cont'd)

estimate." Allow the students time to reappraise their first estimate and record their second estimate on the Measurement Worksheet. Say, "Why have you revised your initial estimate?" (Accept any reasonable responses.)

e.g.,

MEASUREMENT WORKSHEET

OBJECT OR DISTANCE TO BE MEASURED	UNIT OF MEASURE	1ST ESTIMATE	2ND ESTIMATE	ACTUAL MEASURE	DIFFERENCE 1ST ESTIMATE	DIFFERENCE 2ND ESTIMATE
classroom	feet	50 feet	30 feet			

Ask the students to complete the actual measurement, record it on the chart, and compute the differences. Say, "How close was your first estimate to the actual measurement?... the second estimate? Why was the second estimate more reasonable than the first?" (We were able to refine our first estimate using new information about the situation.)

Repeat this procedure using other objects and distances. Several students can work at the chalkboard while the other students work in small groups at their desks.

6.14.03.02 estimate and measure capacity using cups, pints, quarts, and gallons

STRATEGY 1:

Materials:
- cup, pint, quart, and gallon containers
- empty containers of various sizes
- water
- worksheet shown below

Display the cup, pint, quart, and gallon containers along with various empty containers, e.g., large plastic pitcher, plastic punch bowl, small plastic container. Say, "We are going to estimate and determine the actual capacity of these containers." Distribute the following

STRATEGY 1: (cont'd)

worksheet to each student or group of students:

CAPACITY WORKSHEET

OBJECT TO BE MEASURED	UNIT OF MEASURE	ESTIMATE	ACTUAL CAPACITY

Write the worksheet headings on the chalkboard. Hold up the plastic pitcher and the quart container and say, "About how many quarts of water will it take to fill this pitcher?" Draw a consensus from the student responses and ask a student to record the estimate on the chalkboard as the other students do the same on their worksheets. e.g.,

CAPACITY WORKSHEET

OBJECT TO BE MEASURED	UNIT OF MEASURE	ESTIMATE	ACTUAL CAPACITY
Pitcher	Quart	3 Quarts	

Hold up the pint container and say, "About how many pints of water will it take to fill the pitcher?" Draw a consensus from the student responses and ask a student to record the estimate on the chalkboard as the other students do the same on their worksheets. e.g.,

CAPACITY WORKSHEET

OBJECT TO BE MEASURED	UNIT OF MEASURE	ESTIMATE	ACTUAL CAPACITY
Pitcher	Quart	3 Quarts	
Pitcher	Pint	6 Pints	

Ask a student to fill the quart container with water to determine how many quarts the pitcher will hold. Ask another student to record the quart capacity of the pitcher on the chalkboard as the other students do the same on their worksheets. Repeat this procedure with the pint container.

STRATEGY 1: (cont'd)

e.g.,

CAPACITY WORKSHEET

OBJECT TO BE MEASURED	UNIT OF MEASURE	ESTIMATE	ACTUAL CAPACITY
Pitcher	Quart	3 Quarts	$2\frac{1}{2}$ Quarts
Pitcher	Pint	6 Pints	5 Pints

Say, "What is the actual capacity of the pitcher? ($2\frac{1}{2}$ quarts or 5 pints) How does $2\frac{1}{2}$ quarts compare to 5 pints? Explain." (They are equivalent capacities. Since there are 2 pints in 1 quart, 5 pints equals $2\frac{1}{2}$ quarts.)

Repeat this procedure to find the capacity of the punch bowl and the small plastic container.

6.14.03.03 estimate and measure weight using ounces and pounds

STRATEGY 1:

Materials:
- a balance scale
- a one-pound weight
- 16 one-ounce weights

Distribute the following chart to each student or group of students:

MEASUREMENT WORKSHEET

OBJECT TO BE WEIGHED	UNIT OF MEASURE	1ST ESTIMATE	2ND ESTIMATE	ACTUAL WEIGHT	DIFFERENCE 1ST ESTIMATE	DIFFERENCE 2ND ESTIMATE

STRATEGY 1: (cont'd)

Say, "We will use the balance scale and weights to weigh various objects. Our data can be recorded on the Measurement Chart." Ask the students to select a light book to be weighed. Say, "Which unit of measure would be appropriate to determine the weight of the book? (ounces) What is your best estimate for the weight of the book?" (Draw a consensus from the responses.) Fill in the chart with the information gathered thus far.

e.g.,

MEASUREMENT WORKSHEET

OBJECT TO BE WEIGHED	UNIT OF MEASURE	1ST ESTIMATE	2ND ESTIMATE	ACTUAL WEIGHT	DIFFERENCE 1ST ESTIMATE	DIFFERENCE 2ND ESTIMATE
book	ounces	14 oz				

Ask a representative from each group to place the book on one side of the balance scale and start adding one-ounce weights to the other side. When the balance scale begins to move, say, "Would you like to revise your original estimate? Explain." (Accept all reasonable responses.) Record a consensus amount, such as 10 ounces, under the heading SECOND ESTIMATE.

Ask another student to continue adding one-ounce weights until the actual weight is discovered. Say, "What is the difference between the first estimate and the actual weight?... the second estimate and the actual weight?" Ask students to complete the Measurement Chart.

e.g.,

MEASUREMENT WORKSHEET

OBJECT TO BE WEIGHED	UNIT OF MEASURE	1ST ESTIMATE	2ND ESTIMATE	ACTUAL WEIGHT	DIFFERENCE 1ST ESTIMATE	DIFFERENCE 2ND ESTIMATE
book	ounces	14 oz	10 oz	12 oz	2 oz	2 oz

Repeat this procedure using other objects.

P.O. 6.14.03.01

NAME: ______________________________ **CLASS:** ________ **DATE:** ____________

Measure the items indicated on the worksheet. Some of the measurements can be completed in class. Others must be completed at home or in the school yard. Give your answer to the nearest inch, foot, or yard.

Object/Distance To Be Measured	Unit of Measure	Estimate	Actual Measurement	Difference Between Estimate and Actual Measure
1. The distance from your classroom to the nearest exit				
2. A pencil				
3. The length of the chalkledge				
4. The height of the classroom door				
5. The distance across the classroom on a diagonal				
6. The width of your handspan				
7. The width of your refrigerator				
8. The length of your room				
9. The length of your kitchen table				
10. The length of the schoolyard or gym				

***THINK: What are two different strategies you could use to measure the schoolyard or the gym?**

1 ____

2 ____

P.O. **6.14.03.02**

NAME: ______________________ **CLASS:** ________ **DATE:** ____________

Here are activities for you to complete at school and at home.

A. Use a cup, pint, quart, and gallon container to find the equivalent measures.

3 pints = ________ cups
1 quart = ________ pints
1 quart = ________ cups

1 gallon = ________ pints
4 quarts = ________ pints
4 quarts = ________ gallon

B. Find five items at home which are sold in units of cups, quarts and gallons.

1. Apple juice is sold in a quart bottle.
2. ______________________________
3. ______________________________
4. ______________________________
5. ______________________________

C. Pretend you are planning to sell these items but you want to change the size of the container. List the items and the new size and tell why you would make the change.

	Item	Old Size	New Size	Reason
1.	apple juice	1 quart	8 ounce	easier to carry as a snack
2.				
3.				
4.				
5.				

D. **Class Project**

1. Collect 3 different containers used for milk. How much does each one hold? ______________________
2. Collect 3 different containers used for soda. How much does each one hold? ______________________
3. Collect 3 different size containers used for juice. How much does each one hold? ______________________

P.O. 6.14.03.03

NAME: ______________________ **CLASS:** _______ **DATE:** ________

Here are some activities for you to complete at home and at school.

I. Complete this activity in school.

A. Use ounce and pound weights on a balance scale to weigh five different objects. Make a (✓) to indicate the unit being used.

	Object to be Weighed	Unit oz	Unit lb	Estimate	Actual Weight	Difference
1.						
2.						
3.						
4.						
5.						

II. Complete these activities at home.

A. Record and list from lightest to heaviest, the weights of five different food items.

1. ______________________
2. ______________________
3. ______________________
4. ______________________
5. ______________________

B. List five supermarket items that are sold by the ounce. List their weights two different ways.

a. ______________________
b. ______________________
c. ______________________
d. ______________________
e. ______________________

C. List five items sold by the pound. List their weights two different ways.

a. ______________________
b. ______________________
c. ______________________
d. ______________________
e. ______________________

120

SHARPENING YOUR SKILLS

Units 7, 8, 9

Name: ____________________ **Class:** ____________ **Date:** ________

1. 23_{five} = __________

ⓐ $(2x5^{1})+(3x5^{0})$
ⓑ $(2x5^{2})+(3x5^{1})$
ⓒ $(3x5^{1})+(2x5^{0})$
ⓓ Not Here

2. 103_{five} = __________

ⓔ 33_{ten}
ⓕ 13_{ten}
ⓖ 28_{ten}
ⓗ Not Here

3. 37_{ten} = __________

ⓐ 135_{five}
ⓑ 122_{five}
ⓒ 132_{five}
ⓓ Not Here

4. 1011_{two} = __________

ⓔ 111_{ten}
ⓕ 13_{ten}
ⓖ 21_{ten}
ⓗ Not Here

5. 10001_{two} = __________

ⓐ 33_{ten}
ⓑ 9_{ten}
ⓒ 17_{ten}
ⓓ Not Here

6. 35_{ten} = __________

ⓔ 10010_{two}
ⓕ 10011_{two}
ⓖ 11001_{two}
ⓗ Not Here

7. The width of this page is about _______________.

ⓐ 5 in.
ⓑ 8 in.
ⓒ 1 ft
ⓓ Not Here

8. A full kitchen sink will hold _______________ of water.

ⓔ less than 1 gal
ⓕ more than 3 gal
ⓖ less than 1 qt
ⓗ Not Here

9. The classroom door is about _______________ high.

ⓐ 2 ft
ⓑ 2 yd
ⓒ 36 in.
ⓓ Not Here

10. A newborn baby would probably weigh _______________.

ⓔ more than 15 lb
ⓕ less than 2 lb
ⓖ less than 10 lb
ⓗ Not Here

Answer Grid

1.	ⓐ	ⓑ	ⓒ	ⓓ
2.	ⓔ	ⓕ	ⓖ	ⓗ
3.	ⓐ	ⓑ	ⓒ	ⓓ
4.	ⓔ	ⓕ	ⓖ	ⓗ
5.	ⓐ	ⓑ	ⓒ	ⓓ
6.	ⓔ	ⓕ	ⓖ	ⓗ
7.	ⓐ	ⓑ	ⓒ	ⓓ
8.	ⓔ	ⓕ	ⓖ	ⓗ
9.	ⓐ	ⓑ	ⓒ	ⓓ
10.	ⓔ	ⓕ	ⓖ	ⓗ

Unit 10 6.10.01 WEIGHT AND LINEAR CUSTOMARY MEASUREMENT: FRACTIONAL RELATIONSHIPS

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

6.10.01.01A use fractions to describe relationships concerning specified measures of ounces, pounds, and tons;

e.g.,

1 lb = 16 oz
$\frac{1}{4}$ lb = 4 oz
$\frac{1}{2}$ lb = 8 oz
$\frac{3}{4}$ lb = 12 oz

1 T = 2000 lb
$\frac{1}{4}$ T = 500 lb
$\frac{1}{2}$ T = 1000 lb
$\frac{3}{4}$ T = 1500 lb

6.10.01.02A use fractions to describe relationships concerning specified measures of inches, feet, and yards;

e.g.,

1 ft = 12 in.
$\frac{1}{3}$ ft = 4 in.
$\frac{1}{2}$ ft = 6 in.

1 yd = 3 ft
$\frac{1}{3}$ yd = 1 ft
$\frac{2}{3}$ yd = 2 ft

1 yd = 36 in.
$\frac{1}{2}$ yd = 18 in.

6.10.01.03A create and solve word problems related to skills learned in this module.

e.g.,

Jason bought $\frac{1}{4}$ pound of Swiss cheese and $\frac{1}{4}$ pound of American cheese. How many ounces of cheese did Jason buy?

PROBLEM EXPLORATION SKILLS:

- to adopt a different point of view

VOCABULARY	SYMBOLS
ounces	oz
pounds	lb
tons	T
inches	in.
feet	ft
yards	yd

PREVIOUS UNDERSTANDINGS:

- customary measurement

GETTING STARTED:

Ask 16 students to form a group at the front of the room. Say, "How can this group be divided in half? (We can separate the students into two equal groups.) How many students will be in each group?" (8) On the chalkboard write: $\frac{1}{2}$ of 16 = 8. Say, "Let's explore another way to solve this problem." Below the equation on the chalkboard write: $\frac{1}{2}$ of 16 = $\square$.

Say, "What operation shall we perform? (multiplication) How do we know to multiply?" (The word "of" in a mathematical sentence indicates multiplication is required. For example, if crayons come in boxes of 12, and you have five boxes, you then have 60 crayons.) Ask a student to come to the chalkboard and record the number sentence for the multiplication "half of sixteen".

e.g.,

$$\frac{1}{2} \times 16 = \square$$

Say, "How much is $\frac{1}{2} \times 16$? ($\frac{16}{2}$) How can we rename $\frac{16}{2}$ as a whole number?" ($\frac{16}{2} = 8$) Ask a student to complete the open sentence at the chalkboard as the other students write it in their notebooks.

e.g.,

$$\frac{1}{2} \times 16 = \frac{1}{2} \times \frac{16}{1} = \frac{16}{2} = 8$$

Say, "How did we divide the group in half?" (We used two different strategies. One way was to divide the group into 2 equal parts. The other was to multiply the number in the group by $\frac{1}{2}$.)

Continue this procedure by multiplying other whole numbers by a fraction.

6.10.01

6.10.01.01A use fractions to describe relationships concerning specified measures of ounces, pounds, and tons

STRATEGY 1:

Materials:
- balance scale and weights
- one pound scale

Distribute a balance scale, a one-pound weight and other weights calibrated in ounces. Ask a student to come to the balance scale and place the one-pound weight on the left side of the balance. Explore and discover how many of each of the same smaller weights must be used to balance the heavier one-pound weight. Ask a student to record the results.

e.g., One pound is balanced by two eight-ounce weights.
One pound is balanced by four four-ounce weights.
One pound is balanced by sixteen one-ounce weights.

Ask another student to come to the chalkboard to draw a representation of the discoveries made with the weights on the balance scale.

e.g.,

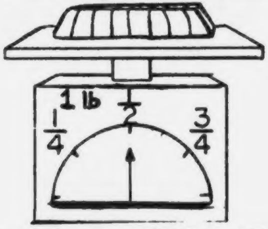
$$1\text{ lb} = \boxed{8\text{oz}} + \boxed{8\text{oz}}$$

$$1\text{ lb} = \boxed{4\text{oz}} + \boxed{4\text{oz}} + \boxed{4\text{oz}} + \boxed{4\text{oz}}$$

$$1\text{ lb} = \begin{array}{l} \boxed{1\text{oz}} + \boxed{1\text{oz}} + \boxed{1\text{oz}} + \boxed{1\text{oz}} + \boxed{1\text{oz}} + \boxed{1\text{oz}} + \boxed{1\text{oz}} + \boxed{1\text{oz}} \\ \boxed{1\text{oz}} + \boxed{1\text{oz}} + \boxed{1\text{oz}} + \boxed{1\text{oz}} + \boxed{1\text{oz}} + \boxed{1\text{oz}} + \boxed{1\text{oz}} + \boxed{1\text{oz}} \end{array}$$

Say, "Let's use the one-pound scale to find the number of ounces in $\frac{1}{2}$ pound." Ask a student to place enough one-ounce weights on the scale until $\frac{1}{2}$ lb is indicated.

e.g.,



Say, "How many ounces equal a half-pound?" (There are 8 ounces in $\frac{1}{2}$ pound.)

Repeat this procedure for finding the weight of other fractional parts of a pound.

STRATEGY 1: (cont'd)

e.g.,

$\frac{1}{4}$ pound equals four ounces

$\frac{1}{16}$ pound equals one ounce

Say, "Can we find the number of ounces in $\frac{1}{2}$ pound another way?" (We can multiply $\frac{1}{2}$ times 16.)

Ask a student to show this calculation at the chalkboard as the others write it in their notebooks.

e.g.,

$$\frac{1}{2} \times 16 = \frac{1}{2} \times \frac{16}{1} = \frac{16}{2} = 8 \text{ ounces}$$

Say, "How much is $\frac{1}{2}$ of 16 ounces?" (8 ounces) Write this relationship on the chalkboard and ask students to write it in their notebooks, e.g., $\frac{1}{2}$ lb = 8 oz.

Say, "How can we find the number of ounces in $\frac{1}{4}$ pound?" (We can find the product of $\frac{1}{4}$ and 16.) Ask a student to show this calculation at the chalkboard as the other students write it in their notebooks.

e.g.,

$$\frac{1}{4} \times 16 = \frac{1}{4} \times \frac{16}{1} = \frac{16}{4} = 4 \text{ ounces}$$

Say, "How much is $\frac{1}{4}$ of 16 ounces?" (4 ounces) Write this relationship on the chalkboard and ask students to write it in their notebooks, e.g., $\frac{1}{4}$ lb = 4 oz.

Repeat this procedure using multiplication of a fraction and a whole number to find other fractional parts of a pound.

Say, "How can we find fractional parts of other units of measure, such as $\frac{1}{4}$ ton?" (We can multiply $\frac{1}{4}$ by 2000.) Ask a student to show this calculation at the chalkboard as the others write it in their notebooks.

e.g.,

$$\frac{1}{4} \times 2000 = \frac{1}{4} \times \frac{2000}{1} = \frac{2000}{4} = 500 \text{ pounds}$$

Say, "How much is $\frac{1}{4}$ ton?" (One-quarter ton is equal to 500 pounds.) Write the relationship on the chalkboard and ask students to write it in their notebooks, e.g., $\frac{1}{4}$ T = 500 lb.

Repeat this procedure using multiplication of a fraction and a whole number to find other fractional parts of a ton.

6.10.01.02A use fractions to describe relationships concerning specified measures of inches, feet, and yards

STRATEGY 1:

Materials:
- twelve-inch rulers
- yardsticks
- 12-inch lengths of string

Distribute a yardstick and several pieces of 12-inch string to each student or group of students. Say, "How many feet does a yardstick measure? (3) How can we rename 3 feet using another unit of measure? (one yard) What fractional part of a yard is one foot?" (Since one yard equals three feet, one foot equals $\frac{1}{3}$ yard.) Ask the students to record the following in their notebooks:

e.g.,

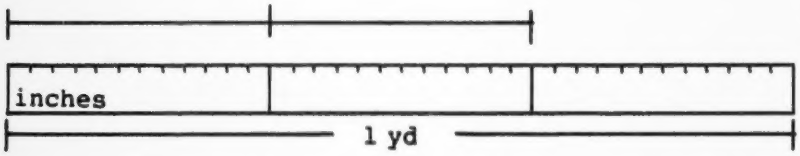
$$\frac{1}{3} \text{ yd} = 1 \text{ ft}$$

Say, "What fractional part of a yard does two feet represent? Explain." (Since one foot equals $\frac{1}{3}$ yard, 2 feet equals $\frac{2}{3}$ yard.) Ask the students to record the following in their notebooks:

$$\frac{2}{3} \text{ yd} = 2 \text{ ft}$$

Say, "Verify this statement by using a piece of one foot string and/or a 12-inch ruler."

e.g.,



Say, "How many inches equal 1 yard? (36) How can this information be used to find the number of inches in $\frac{1}{3}$ yard?" (We can multiply $\frac{1}{3}$ times 36.) Ask a student to come to the chalkboard to show the calculation as the other students write it in their notebooks.

e.g.,

$$\frac{1}{3} \times 36 = \frac{1}{3} \times \frac{36}{1} = \frac{36}{3} = 12 \text{ inches}$$

STRATEGY 1: (cont'd)

Say, "How much is $\frac{1}{3}$ of 36 inches?" (12 inches) Write the following relationships:

$$\frac{1}{3}\text{ yd} = 1\text{ ft} \qquad \text{or} \qquad \frac{1}{3}\text{ yd} = 12\text{ in.}$$

Say, "How can we find the number of inches equal to $\frac{2}{3}$ yard? (Since 12 inches equal $\frac{1}{3}$ yard, 2 times 12 inches equal $\frac{2}{3}$ yard.) Can this be demonstrated another way?" (We can multiply $\frac{2}{3}$ times 36.) Ask a student to show the calculation at the chalkboard as the other students write it in their notebooks.

e.g.,

$$\frac{2}{3} \times 36 = \frac{2}{3} \times \frac{36}{1} = \frac{72}{3} = 24\text{ inches}$$

Say, "How much is $\frac{2}{3}$ of 36 inches?" (24 inches) Write the following relationships on the chalkboard and ask the students to write them in their notebooks:

$$\frac{2}{3}\text{ yd} = 2\text{ ft} \qquad \text{or} \qquad \frac{2}{3}\text{ yd} = 24\text{ inches}$$

Continue this procedure, using both strategies, to investigate other relationships among inches, feet, and yards.

6.10.01.03A **create and solve word problems related to skills learned in this module**

STRATEGY 1:

Display the following statement:

Mr. Charles weighs 2592 ounces.

Ask a student to read the sentence aloud. Say, "What is unusual about this statement? (A person's weight is usually expressed in pounds.) Using large numbers is cumbersome, the possibility of making errors is increased and, in many cases, the extensive calculations would be pointless."

Say, "There are situations which allow for the selection of a different measuring unit to describe equivalent relationships." Write the following word problem on the chalkboard:

Kevin bought $\frac{1}{2}$ pound walnuts and $\frac{1}{4}$ pound pistachio nuts. How many ounces of nuts did he buy?

STRATEGY 1: (cont'd)

Say, "What are we being asked to find? (the total number of ounces of nuts which Kevin bought) What is the prior knowledge needed to solve the problem? (the number of ounces in a pound) How many ounces equal 1 pound?" (16) Ask a student to write this relationship on the chalkboard, e.g., 1 lb = 16 oz.

Ask a student to find the number of ounces equal to each of the fractional amounts.

e.g.,

$\frac{1}{2}$ lb = 8 oz $\qquad$ $\frac{1}{4}$ lb = 4 oz

Say, "How can we find the total number of ounces Kevin bought?" (We can add 8 oz and 4 oz.) Ask a student to write this on the chalkboard.

e.g., 8 oz + 4 oz = 12 oz

Say, "How many ounces of nuts did Kevin buy?" (12 oz) Is there another way to find the solution?" (Yes, find the sum of $\frac{1}{2}$ lb and $\frac{1}{4}$ lb.) Demonstrate.

e.g.,

$$\begin{array}{r}
\frac{1}{2}\text{ lb} = \frac{2}{4}\text{ lb} \\
+\frac{1}{4}\text{ lb} = \frac{1}{4}\text{ lb} \\
\hline
\frac{3}{4}\text{ lb}
\end{array}$$

and

$\frac{3}{4}$ lb = 12 oz

Ask students to create and solve a word problem incorporating the following data: $\frac{2}{3}$ yd ribbon and $\frac{1}{2}$ yd ribbon

e.g.,

Ms. Williams bought $\frac{2}{3}$ yd red ribbon for a baby gift. Mr. Stone bought a $\frac{1}{2}$ yd blue ribbon for a birthday gift. Who bought the longer piece of ribbon?

Ask a student to solve the problem at the chalkboard while others work at their seats. Continue in the same fashion to create and exchange problems related to skills learned in this module.

P.O. 6.10.01.01&.03

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

I. Complete.

$\frac{1}{4}$ lb = ______ oz $\frac{1}{4}$ T = ______ lb

$\frac{1}{2}$ lb = ______ oz $\frac{1}{2}$ T = ______ lb

$\frac{3}{4}$ lb = ______ oz $\frac{3}{4}$ T = ______ lb

II. Say it another way.

1. Ricky bought $\frac{3}{4}$ lb of cheese at the grocery store. Ricky bought ________ ounces of cheese.

2. Marcy bought a 1 pound bag of raisins. She ate $\frac{1}{4}$ pound of raisins. Marcy ate ________ ounces of raisins.

III. Create and solve a problem using the information given.

1. Storage Bin A has $\frac{3}{4}$ T of grain.
2. Storage Bin B has $\frac{1}{2}$ T of grain.
3. Storage Bin C has $\frac{1}{4}$ T of grain.

____

____

____

____

Do you remember?

Round the following to the nearest ten thousand.

1. 127,562 rounds to __________   3. 296,230 rounds to __________
2. 61,080 rounds to __________   4. 87,615 rounds to __________

P.O. **6.10.01.02-.03**

NAME: ______________________ **CLASS:** ______ **DATE:** ______________

I. Complete.

$\frac{1}{3}$ ft = ______ in. $\frac{1}{3}$ yd = ______ ft $\frac{1}{2}$ yd = ______ in.

$\frac{1}{2}$ ft = ______ in. $\frac{2}{3}$ yd = ______ ft $\frac{2}{3}$ yd = ______ in.

II. Solve.

1. Which is longer, a piece of wood $\frac{5}{6}$ yard long or a piece of wood $\frac{2}{3}$ yard long?

   ____

2. Julio has a chain of paper clips $\frac{1}{3}$ yard long. His brother has a chain of paper clips $\frac{1}{2}$ yard long. Whose chain of paper clips is longer?

   ____

III. Create and solve a problem using the information given.

Mickey has a stack of paper plates $\frac{3}{4}$ ft in height. His sister has a stack of paper plates $\frac{5}{6}$ ft in height. ______________________

____

____

____

____

____

Do you remember?

Round the following numbers to the nearest hundred thousand. Place < or > in the circles provided to make a true statement.

	Number		Rounded Number		Number		Rounded Number
1.	3,654,123	○	______	4.	987,654	○	______
2.	451,789	○	______	5.	200,264	○	______
3.	708,234	○	______	6.	8,569,432	○	______

6.10.02 MEASUREMENT: TIME

(2-3 lessons/ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

6.10.02.01A state equivalents for given measures of time;

e.g.,

75 min = 1 hr 15 min
90 sec = 1 min 30 sec
30 hr = 1 day 6 hr

6.10.02.02A use fractions to describe relationships concerning numbers of months and years;

e.g.,

6 months = $\frac{1}{2}$ year
9 months = $\frac{3}{4}$ year

*6.10.02.03A define a decade and a century;

e.g.,

1 decade = 10 years
1 century = 100 years

6.10.02.04A create and solve word problems related to skills learned in this module.

e.g.,

Jan worked for 2 hours 20 minutes on Thursday, and 3 hours 45 minutes on Friday. How long did she work during both days?

PROBLEM EXPLORATION SKILLS:

- to adopt a different point of view
- to look for hidden assumptions
- to make an analogy

6.10.02

VOCABULARY	**SYMBOL**
seconds	sec
minutes	min
hours	hr
day	d
week	wk
month	mo
year	yr
decade	
century	

PREVIOUS UNDERSTANDINGS:

- the number of seconds in a minute, minutes in an hour, hours in a day
- the relationships among days in months, months in a year

GETTING STARTED:

Materials:
- clock, calendar

Separate the students into small groups. Display or draw pictures of a clock and a calendar. Ask each group to compile a vocabulary list related to time on the clock and on the calendar. Ask a student to come to the chalkboard to write the words suggested by each group to create a master list of 'time' words. The group with the greatest number of 'time' words is the winner.

e.g.,

seconds
minutes
hours

days
weeks
months

years
decade
century

Note to Teacher: While words such as decade and century are not usually thought of as on a calendar, allow students to include any 'time' words on their list.

Ask students from one group to ask time related questions of students in another group. As an example, say, "How many weeks are in a year?" (There are 52 weeks in a year.)

Ask the students to continue this procedure with other time (clock) or calendar related questions.

6.10.02

6.10.02.01A state equivalents for given measures of time

STRATEGY 1:

Materials:
- a set of cards which state measures of time

Display the set of cards and shuffle them. Ask a student to come to the chalkboard to match the cards which show equivalent measures of time.

e.g.,

12 months = 1 year
60 seconds = 1 minute
24 hours = 1 day

Write the following list on the chalkboard:

65 sec
90 sec
125 sec

Say, "How many seconds equal one minute? (60) Would 65 seconds be more or less than one minute? (5 seconds more) Explain. (I can change 60 seconds for one minute and still have 5 seconds left.) What operation did we use?" (We used division.)

Continue this procedure and develop the relationship between minutes and seconds.

e.g.,

65 sec ÷ 60 = 1 min 5 sec
90 sec ÷ 60 = 1 min 30 sec
125 sec ÷ 60 = 2 min 5 sec

Write the following list on the chalkboard:

85 min
95 min
245 min

STRATEGY 1: (cont'd)

Say, "Look at the list. If we want to make a card showing the equivalent measures of time in hours and minutes, how can we do it?" (We would divide the given time by 60 since there are 60 minutes in an hour. We are really answering the question, 'How many groups of 60 can we make?')

e.g.,

85 min + 60 = 1 hr 25 min
95 min + 60 = 1 hr 35 min
245 min + 60 = 4 hr 5 min

Write the following list on the chalkboard.

1 week = 7 days
4 weeks = ? days
2 weeks = ? days
3 weeks = ? days

Say, "Look at the list. How can we make a card showing the equivalent measures of time in days?" (We could multiply the given time by 7 since there are 7 days in a week.)

e.g.,

4 weeks x 7 = 28 days
2 weeks x 7 = 14 days
3 weeks x 7 = 21 days

Repeat this procedure changing hours to minutes.

e.g.,

5 hr x 60 min = 300 min
2 hr x 60 min = 120 min
4 hr x 60 min = 240 min

Say, "Can you think of a rule(s) which we can use to find equivalents for given measures of time?" (Yes, to rename smaller units with larger units, divide. To rename larger units with smaller units, multiply.) Write the following on the chalkboard and ask the students to write it in their notebooks:

STRATEGY 1: (cont'd)

1. Read the problem.
2. Find the key words.
3. Recall the appropriate fact(s).
4. Decide if we are changing from smaller units to larger units (÷), or from larger units to smaller units (x).

Ask students to THINK of these steps as they complete the following:

A. 126 days = _____ weeks

key words: days and weeks

fact: 7 days = 1 week

change smaller units to larger units, and divide

```
   18
7)126   = 18 weeks
  7↓
  56
  56
   0
```

126 days = <u>18</u> weeks

B. 8 weeks = _____ days

fact: 7 days = 1 week

change from larger units to smaller units, therefore we multiply

8 weeks x 7 days = 56 days

8 weeks = 56 days

6.10.02

6.10.02.02A use fractions to describe relationships concerning numbers of months and years

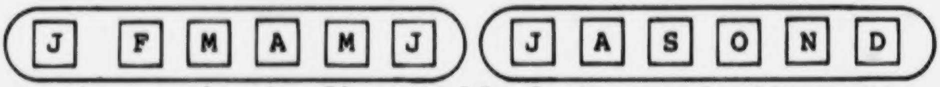
STRATEGY 1:

Materials:
- a 12 month calendar

Display the calendar. Say, "How many months are there in one year? (There are 12 months in one year.) This calendar shows the 12 months of the year."

Ask a student to divide the 12 months of the year in half.

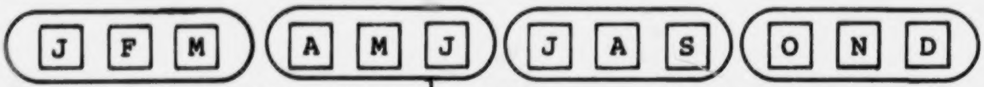
e.g.,



Say, "Which months are in the first half of the year? (January, February, March, April, May, June) Which months are in the second half? (July, August, September, October, November, December) Into how many groups has the year been divided? (two) What part of the year is six months?" (Since there are 12 months in the year, six months is $\frac{6}{12}$ or $\frac{1}{2}$ of a year.)

Ask a student to repeat this procedure dividing the months of the year into four equal groups.

e.g.,



Say, "Which months are in the first $\frac{1}{4}$ of the year?" (January, February, March) Repeat this procedure naming months in other quarters of the year. Say, "Into how many groups has the year been divided? (four) How many months are in each group? (three) What part of the year is 3 months?" (Since there are 12 months in the year, 3 months is $\frac{3}{12}$ or $\frac{1}{4}$ of a year.)

Repeat this same procedure dividing the months of the year into three equal groups.

Say, "How can we find the number of months in a fractional part of a year?" (We can find the number of months in a fractional part of a year by finding the fractional part of 12.)

STRATEGY 1: (cont'd)

e.g.,

A. $\frac{1}{2}$ yr = _____ mo

$\frac{1}{2}$ of 12 = _____ mo

$\frac{1}{2} \times 12 = \frac{1}{2} \times \frac{12}{1} = \frac{12}{2} = 6$

$\frac{1}{2}$ yr = $\underline{\ 6\ }$ mo

B. $\frac{1}{3}$ yr = _____ mo

$\frac{1}{3}$ of 12 = _____ mo

$\frac{1}{3} \times 12 = \frac{1}{3} \times \frac{12}{1} = \frac{12}{3} = 4$

$\frac{1}{3}$ yr = $\underline{\ 4\ }$ mo

Say, "Let's try it the other way. Let's see if we can find the fractional part of a year represented by a number of months?" Demonstrate.

e.g., 8 months = _______ year

$\frac{8 \text{ months}}{12 \text{ months}}$ = _______ year

$\frac{8 + 4}{12 + 4} = \frac{2}{3}$

8 months = $\frac{2}{3}$ year

Say, "What fractional part of a year is 9 months?" Demonstrate.

e.g.,

9 months = _______ year

$\frac{9 \text{ months}}{12 \text{ months}}$ = _______ year

$\frac{9 + 3}{12 + 3} = \frac{3}{4}$

9 months = $\frac{3}{4}$ year

STRATEGY 1: (cont'd)

Say, "How did the steps we used earlier (see Strategy 1, 6.10.02.01A) help us? (We learned to multiply when changing from a larger to a smaller unit.)

e.g.,

$\frac{1}{2}$ year = ____ months

$\frac{1}{2}$ of $12 = \frac{1}{2} \times 12 = \frac{1}{2} \times \frac{12}{1} = \frac{12}{2} = 6$

$\frac{1}{2}$ year = 6 months

Say, "How else did the steps help us? (We learned to divide when changing from a smaller unit to a larger unit.)

e.g.,

8 months = ____ year

$\frac{8 + 4}{12 + 4} = \frac{2}{3}$

8 months = $\frac{2}{3}$ year

Repeat this procedure using the steps to discover equivalent relationships between months and years.

6.10.02.03A define a decade and a century

STRATEGY 1:

Materials:
- 5"x7" cards with units of time
- adding machine tape for a time line

Say, "When measuring distance (linear measurement), it is necessary to choose the appropriate unit to avoid working with very large numbers."

e.g.,

How do we measure the distance from New York City to Washington D.C.? (miles or kilometers)

STRATEGY 1: (cont'd)

How do we measure the weight of an automobile? (tons/metric tons)
How do we measure the amount of water in a pool? (kiloliters/gallons)

Say, "How do we measure time?" (For very short periods of time, we may select seconds as our unit of measure while for longer periods of time, we may select minutes, hours, days, weeks, months, or years.) Ask students to give an example of an activity/event measured in these units of time.

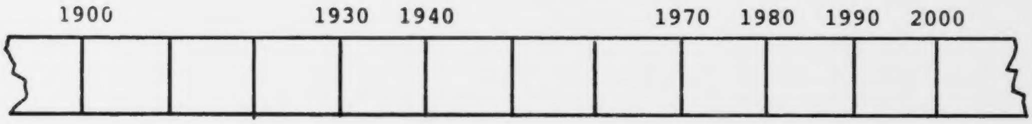
e.g.,

Unit	Activity/Event
seconds	100 yard dash race
minutes	cook an egg
hours	
days	
weeks	
months	
years	

Say, "How should we measure *very* long periods of time? (We should use units of measure that equal longer periods of time. We measure periods of ten years in decades.) The prefix *deca* comes from the Greek word *deka* and the Latin word *decs* meaning ten." Ask students to list other words with the same prefix, e.g., decimeter, decameter (dekameter), decimal.

Display the time line made from the adding machine tape.

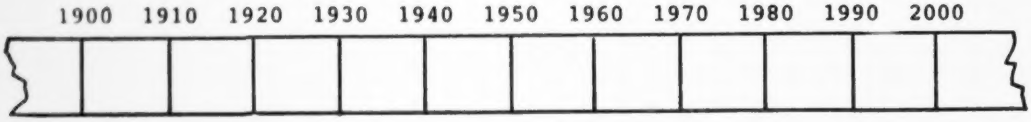
e.g.,



Ask a student to come to the chalkboard to add missing decades to the line at each fold of the tape *prior* to 1990.

STRATEGY 1: (cont'd)

e.g.,



Say, "By definition, a decade is the passage of any ten year period." Ask a student to tell the class his/her date of birth, e.g., January 6, 1976. Say, "What is the date a decade before you were born? (January 6, 1966) What was the decade after you were born?" (January 6, 1986) Ask other students to give significant dates and to name the date a decade before or after.

e.g.,

Decade Before	Event/Date	Decade After
	First American in Space May 5, 1961	
	First American Space Walk June 3, 1965	
	First American on Moon July 20, 1969	

Say, "In less than a dozen years, we will arrive at a very special time in the passage of time. How will you record this date on the time line for the decade after 1990? (That is a special date, 2000, because it marks the end of the last decade of this century.) What years will have passed in that decade? (1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1998, 1999) In what other way is the year 2000 very special. (It also marks the end of a century.) What is a century?" (A century marks the passage of 100 years.)

Say, "The Declaration of Independence was signed on July 4, 1776. On what date did we celebrate the 100th birthday of the signing? (July 4, 1876) What was the date a century before the signing?" (July 4, 1676)

Elicit other significant dates and name the date that is one century before and one century after.

STRATEGY 1: (cont'd)

e.g.,

Century Before	Event/Date	Century After
____________	Columbus' discovery of America 1492	____________
____________	The Declaration of Independence 1776	____________
____________	The end of the Civil War 1865	____________

6.10.02.04A create and solve word problems related to skills learned in this module

STRATEGY 1:

Separate the students into two groups. Ask the students in each group to create a problem related to the following:

Group A

the amount of time spent on various subjects when completing daily homework assignments

Group B

the number of weeks we have been in school

Ask the students from Group A to have their recorder read the problem they created.

e.g.,

> Yesterday, Kevin had homework in two subjects -- mathematics and language arts. He spent $\frac{1}{2}$ hour on mathematics and $\frac{1}{4}$ hour on language arts. How many minutes did he spend doing homework?

Ask a student from Group B to come to the chalkboard to solve the problem while the other students work at their seats. Say, "How will we solve this problem?" (We can find the number of minutes in each of the fractional parts of an hour and find their sum in minutes.)

STRATEGY 1: (cont'd)

e.g.,

$\frac{1}{2}$ hour = ____ minutes

1 hour = 60 minutes

$\frac{1}{2} \times 60 = \frac{1}{2} \times \frac{60}{1} = \frac{60}{2} = 30$

$\frac{1}{2}$ hour = 30 minutes

$\frac{1}{4}$ hour = ____ minutes

1 hour = 60 minutes

$\frac{1}{4} \times 60 = \frac{1}{4} \times \frac{60}{1} = \frac{60}{4} = 15$

$\frac{1}{4}$ hour = 15 minutes

Say, "How many minutes did Kevin spend doing his homework? (He spent 30 minutes + 15 minutes or 45 minutes.) How did the steps practiced in Strategy 1, P.O. 6.10.02.01A help us to solve the problem?" (The rule is to multiply when changing from a greater to a smaller unit.)

Ask the students from Group B to have their recorder read the problem they created.

e.g.,

> Thirty-five days have passed since school started this year. How many weeks is this?

Ask a student from group A to come to the chalkboard to solve the problem while the other students work at their desks. Say, "How will you solve this problem?" (We can find the number of weeks in 35 days by dividing 35 days by the number of days in a week.)

e.g.,

35 days = ____ weeks

35 + 7 = 5

35 days = 5 weeks

Say, "How did you use the rule?" (The rule is to divide when you want to change from a smaller unit to a greater unit.)

Repeat this procedure to solve other word problems involving time.

P.O. 6.10.02.01-.02&.04

NAME: ______________________________ **CLASS:** _______ **DATE:** ______________

Follow the directions below.

A. Complete the time relationships.

a. 85 min = _____ hr ____ min

b. 75 sec = _____ min _____ sec

c. 32 hr = _____ day _____hr

d. 24 mo = _____ yr

e. 42 days = _____ weeks

B. Write >, <, or = to make each statement true.

a. 6 weeks ○ 42 days

b. 2 years ○ 36 months

c. 18 months ○ 4 years

d. 180 seconds ○ 3 hours

e. 24 hours ○ 1440 minutes

f 2 min 30 sec ○ 150 seconds

C. Television commercials can be 30 seconds long or 60 seconds long. If there are 6 one-minute commercials and 8 thirty-second commercials during the cartoon program, how much time is spent watching the commercials?

Commercial time = __________ seconds or __________ minutes

D. What part of...

a. a year is 6 months? __________________

b. a year is 9 months? __________________

c. a year is 3 months? __________________

d. a day is 6 hours? ____________________

e. a minute is 45 seconds? ______________

f. an hour is 15 minutes? _______________

P.O. 6.10.02.01-.04

NAME: ______________________________ **CLASS:** _______ **DATE:** __________

Follow the directions.

A. Complete the time relationships.

a. 36 months = ______ years
b. 6 months = ______ year
c. 3 months = ______ year
d. 2 decades = ______ years
e. 1 century = ______ years

B. Can you imagine? Write about your future in the next century (2000 and beyond).

In the year 2000, I will be _____ years _____ months old.
I plan ____
____
____

C. Connie has a vacation home away from New York City. She spends $\frac{3}{4}$ of the year in New York City and $\frac{1}{4}$ of the year at her vacation home.

How many months does she spend in New York City? ______________
How many months does she spend at her vacation home? ____________
How many days does she spend at her vacation home? ______________
How many days does she spend in New York City? _________________

D. Use fractions to complete each statement.

a. 6 months = ____ year
b. 3 months = ____ year
c. 10 months = ____ year
d. 9 months = ____ year

CHALLENGE!

The U.S. Military uses a 24 hour clock in telling time. Two o'clock in the morning is expressed as 0200 hours. Two o'clock in the afternoon is expressed as 1400 hours. How would you express 7:45 in the evening using the 24 hour clock? How would you express 10:20 in the evening?

______________________ ______________________

6.10.03 OPERATIONS WITH CUSTOMARY MEASURES

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

6.10.03.01A add and subtract lengths expressed in customary units;

e.g.,

$$\begin{array}{l} \text{3 yd 2 ft} \\ \underline{\text{+1 yd 2 ft}} \\ \text{4 yd 4 ft = 5 yd 1 ft} \end{array} \qquad \begin{array}{l} \text{5 ft 6 in. = 4 ft 18 in.} \\ \underline{\text{-2 ft 9 in.}}\text{ = }\underline{\text{2 ft 9 in.}} \\ \text{2 ft 9 in.} \end{array}$$

6.10,03.02A add and subtract weights expressed in customary units;

e.g.,

$$\begin{array}{l} \text{4 lb 10 oz} \\ \underline{\text{+5 lb 9 oz}} \\ \text{9 lb 19 oz = 10 lb 3 oz} \end{array} \qquad \begin{array}{l} \text{7 lb 3 oz = 6 lb 19 oz} \\ \underline{\text{-4 lb 8 oz}}\text{ = }\underline{\text{4 lb 8 oz}} \\ \text{2 lb 11 oz} \end{array}$$

*6.10.03.03A add and subtract capacities expressed in customary units;

e.g.,

$$\begin{array}{l} \text{6 gal 3 qt} \\ \underline{\text{+1 gal}\qquad} \\ \text{7 gal 3 qt} \end{array} \qquad \begin{array}{l} \text{2 qt 1 pt} \\ \underline{\text{-}\qquad\text{1 pt}} \\ \text{2 qt} \end{array}$$

6.10.03.04A create and solve word problems related to skills learned in this module.

e.g.,

Ms. Wong bought 5 yd 1 ft of fabric. She used 1 yd 2 ft of the fabric to make a skirt. How much fabric does she have left?

PROBLEM EXPLORATION SKILLS:

- to adopt a different point of view
- to look for hidden assumptions
- to estimate

VOCABULARY	SYMBOL
yard	yd
foot	ft
inch	in.
pound	lb
ounce	oz
gallon	gal
quart	qt
pint	pt

PREVIOUS UNDERSTANDINGS:

- identify equivalent measures of customary units, e.g., 1 yard = 3 feet
- rename customary units, e.g., 2 feet 4 inches = 1 foot 16 inches
- customary units are not renamed by using base ten rules

GETTING STARTED:

Materials:

- yardstick
- ruler
- customary weights
- customary capacity containers

Write the following examples on the chalkboard:

(a) $\begin{array}{r}234\\ \underline{+125}\end{array}$ (b) $\begin{array}{r}114\\ \underline{+653}\end{array}$ (c) $\begin{array}{r}958\\ \underline{+467}\end{array}$ (d) $\begin{array}{r}748\\ \underline{+396}\end{array}$ (e) $\begin{array}{r}586\\ \underline{+895}\end{array}$

Say, "Which of the examples require renaming using base ten rules? (Examples c, d, and e require renaming ones to tens, tens to hundreds, and hundreds to thousands.) Why do examples c, d, and e require renaming?" Whenever the sum of the digits in any column is greater than 9 we must exchange. Write the following examples on the chalkboard:

(f) $\begin{array}{r}645\\ \underline{-123}\end{array}$ (g) $\begin{array}{r}936\\ \underline{-729}\end{array}$ (h) $\begin{array}{r}453\\ \underline{-168}\end{array}$

Say, "Which of these examples require renaming before we can subtract? Explain." (Examples g and h require exchange of hundreds to tens and tens to ones. We always rename one ten to ten ones or one hundred to ten tens when we are subtracting in the base ten number system.)

GETTING STARTED: (cont'd)

Display the various customary tools of linear measurement, the weights, and the containers. Ask a student to come to the chalkboard and, after being given a ruler, a weight, or a container, write both the name of the measuring tool and an equivalent.

e.g.,

1 yard = 3 feet
1 foot = 12 inches
1 gallon = 4 quarts
1 quart = $\frac{1}{4}$ gallon
8 ounces = $\frac{1}{2}$ pound
2 pints = 1 quart

Repeat this procedure until all the materials have been listed with an equivalent measure. Say, "How do we regroup when we rename in arithmetic? (We regroup by tens and powers of ten.) Why must we know equivalent measures?" (It is important to know the equivalent measures because we do not rename using base ten rules when we add/subtract units of customary measure.)

6.10.03

6.10.03.01A add and subtract lengths expressed in customary units

STRATEGY 1:

Write the following on the chalkboard:

e.g.,

$$\text{(a)}\ \begin{array}{r}2\ \text{yd}\ 1\ \text{ft}\\ \underline{+3\ \text{yd}\ 1\ \text{ft}}\end{array}\qquad \text{(b)}\ \begin{array}{r}4\ \text{yd}\ 2\ \text{ft}\\ \underline{+3\ \text{yd}\ 2\ \text{ft}}\end{array}\qquad \text{(c)}\ \begin{array}{r}5\ \text{ft}\ \ 8\ \text{in.}\\ \underline{+2\ \text{ft}\ \ 6\ \text{in.}}\end{array}$$

Say, "What is the requirement for adding customary units of measure?" (Addition can only be performed with measurements having the same units.) Ask a student to complete the examples at the chalkboard while the other students work at their desks.

e.g.,

$$\text{(a)}\ \begin{array}{r}2\ \text{yd}\ 1\ \text{ft}\\ \underline{+3\ \text{yd}\ 1\ \text{ft}}\\ 5\ \text{yd}\ 2\ \text{ft}\end{array}\qquad \text{(b)}\ \begin{array}{r}4\ \text{yd}\ 2\ \text{ft}\\ \underline{+3\ \text{yd}\ 2\ \text{ft}}\\ 7\ \text{yd}\ 4\ \text{ft}\end{array}\qquad \text{(c)}\ \begin{array}{r}5\ \text{ft}\ \ 8\ \text{in.}\\ \underline{+2\ \text{ft}\ \ 6\ \text{in.}}\\ 7\ \text{ft}\ 14\ \text{in.}\end{array}$$

Say, "Will it be necessary to rename any of the answers? Explain." (Yes, examples b and c should be renamed. The smaller units of measure in the answers can be renamed as larger units of measure.) How can we rename 7 yd 4 ft?" (The smaller measurement in the answer, 4 ft, could be renamed as 1 yd and 1 ft. We then combine 1 yd and 7 yd for a total of 8 yd 1 ft.)

e.g.,

$$\text{(b)}\ \begin{array}{rl}4\ \text{yd}\ 2\ \text{ft} & \\ \underline{+3\ \text{yd}\ 2\ \text{ft}} & \\ 7\ \text{yd}\ 4\ \text{ft} & \\ 8\ \text{yd}\ 1\ \text{ft} & \text{(Renamed 7 yd 4 ft)}\end{array}$$

Say, "What prior knowledge is needed to solve the example? (3 ft = 1 yd) How does this procedure differ from the base ten system we use in arithmetic computation?" (We are combining equivalent units of measure rather than exchanging as in the place value system.) Ask a student to complete example c at the chalkboard.

e.g.,

$$\text{(c)}\ \begin{array}{r}5\ \text{ft}\ \ 8\ \text{in.}\\ \underline{+2\ \text{ft}\ \ 6\ \text{in.}}\\ 7\ \text{ft}\ 14\ \text{in.}\\ 8\ \text{ft}\ \ 2\ \text{in.}\end{array}$$

Say, "What is the final answer? (8 ft 2 in.) Which prior knowledge was needed to solve the problem?" (12 in. = 1 ft) Repeat this procedure by solving other examples and identifying the prior knowledge needed.

STRATEGY 2:

Write the following examples on the chalkboard:

$$\text{(a)}\quad \begin{array}{r} 9\text{ yd }3\text{ ft}\\ \hline -2\text{ yd }2\text{ ft} \end{array} \qquad \text{(b)}\quad \begin{array}{r} 8\text{ ft }11\text{ in.}\\ \hline -2\text{ yd }\ \ 3\text{ ft} \end{array}$$

Say, "Which units of measure are being subtracted in example a? (Yards are subtracted from yards and feet are subtracted from feet.) Which units are we asked to subtract in example b? (We are asked to subtract feet from inches and yards from feet.) Can subtraction be performed in example b? Explain." (No, the units of measure are not the same.)

Display the following examples:

$$\text{(c)}\quad \begin{array}{r} 9\text{ ft }\ 6\text{ in.}\\ \hline -5\text{ ft }10\text{ in.} \end{array} \qquad \text{(d)}\quad \begin{array}{r} 6\text{ ft }7\text{ in.}\\ \hline -2\text{ ft }9\text{ in.} \end{array} \qquad \text{(e)}\quad \begin{array}{r} 8\text{ ft }7\text{ in.}\\ \hline -2\text{ ft }3\text{ in.} \end{array}$$

Say, "Which of the examples require renaming in order to subtract? Explain." (Examples c and d require renaming because 10 in. is larger than 6 in. and 9 in. is larger than 7 in.) Ask a student to complete examples c and d at the chalkboard while the others write in their notebooks.

e.g.,

$$\text{(c)}\quad \begin{array}{r} 9\text{ ft }\ 6\text{ in.}\\ -5\text{ ft }10\text{ in.}\\ \hline 3\text{ ft }\ 8\text{ in.} \end{array}$$

THINK: Rename 9 ft 6 in. as 8 ft 18 in.

$$\text{(d)}\quad \begin{array}{r} 6\text{ ft }\ 7\text{ in.}\\ -2\text{ ft }\ 9\text{ in.}\\ \hline 3\text{ ft }10\text{ in.} \end{array}$$

THINK: Rename 6 ft 7 in. as 5 ft 19 in.

Say, "What prior knowledge is required to complete each example?" (1 ft = 12 in.)

Repeat this procedure using examples containing other units of customary linear measurement.

6.10.03.02A **add and subtract weights expressed in customary units**

STRATEGY 1:

Write the following on the chalkboard:

STRATEGY 1: (cont'd)

e.g.,

(a) 4 lb 6 oz	(b) 2 lb 8 oz	(c) 3 lb 2 oz
+2 lb 12 oz	+6 lb 7 oz	+4 lb 1 oz

Say, "When adding customary units of measure, what must you look for first? (You must check to see that the units of measure are the same.) Let's estimate the answer before we perform the actual computation. What prior knowledge is required in order to estimate the sums? Explain. (16 oz = 1 lb and 8 oz = $\frac{1}{2}$ lb. If the number of ounces is 7 or less, we round down. If the number of ounces is 8 or more, we round up.) This is similar to the procedure we developed for rounding numbers. We had to make a convention for dealing with cases where we were rounding numbers such as 7.5, that were exactly halfway between two whole numbers. In cases like that, there was no nearest whole. Therefore, we agreed we would round up. Since 8 ounces is exactly half a pound, we are using the same thinking here."

Ask the students to round the amounts, estimate the sum for each example and solve.

e.g.,

			Estimate	Actual
(a)	4 lb 6 oz	rounds to	4 lb	4 lb 6 oz
	+2 lb 12 oz	rounds to	+3 lb	+2 lb 12 oz
			7 lb	6 lb 18 oz

			Estimate	Actual
(b)	2 lb 8 oz	rounds to	3 lb	3 lb 2 oz
	+4 lb 1 oz	rounds to	+4 lb	+4 lb 3 oz
			7 lb	7 lb 3 oz

Say, Will it be necessary to rename any of the answers?" (Yes, the actual solutions for examples a and b must be renamed.) Demonstrate.

e.g.,

(a) 4 lb 6 oz
+2 lb 12 oz **THINK:** There are 16 oz in 1 lb,
6 lb 18 oz therefore I rename 18 oz as 1 lb
7 lb 2 oz 2 oz.

STRATEGY 1: (cont'd)

(b) $\begin{array}{r} 2\text{ lb }\ \ 8\text{ oz} \\ \underline{+6\text{ lb }\ \ 9\text{ oz}} \\ 8\text{ lb }17\text{ oz} \\ 9\text{ lb }\ \ 1\text{ oz} \end{array}$

THINK: There are 16 oz in 1 lb, therefore I rename 17 oz as 1 lb 1 oz.

Say, "How did the estimates compare to the actual answers?" (The estimates established the reasonableness of the actual sums.)

STRATEGY 2:

Adapt P.O. 6.10.03.01A, Strategy 2, for computing with pounds and ounces.

6.10.03.03A add and subtract capacities expressed in customary units

STRATEGY 1:

Adapt P.O. 6.10.03.02A, Strategy 1, for computing with gallons and quarts.

STRATEGY 2:

Say, "What is common to operations with customary measures? (Only measurements with the same units can be added and subtracted.) How does estimating help us? (We can check the reasonableness of an answer.) When is it necessary to rename answers?" (It may be necessary to rename answers when smaller measurements in the answer can be renamed as larger measurements. In subtraction, it may be necessary to rename the minuend before you begin to subtract, and finally, renaming in operations with customary measurements is not the same as exchange in the computation used in the base ten system.) Write the following on the chalkboard:

(a) $\begin{array}{r} 7\text{ gal }2\text{ qt} \\ \underline{-3\text{ gal }3\text{ qt}} \end{array}$ (b) $\begin{array}{r} 9\text{ gal }3\text{ qt} \\ \underline{-2\text{ gal }1\text{ qt}} \end{array}$ (c) $\begin{array}{r} 5\text{ qt} \\ \underline{-2\text{ qt }1\text{ pt}} \end{array}$

Say, "In which examples will it be necessary to rename? (It is necessary to rename in examples a and c.) Ask a student to come to

STRATEGY 2: (cont'd)

the chalkboard to complete the examples while the other students work at their seats.

e.g.,

			Estimate	Actual
(a)	7 gal 2 qt	rounds to	8 gal	7 gal 2 qt
	-3 gal 3 qt	rounds to	-4 gal	-3 gal 3 qt
			4 gal	3 gal 3 qt

THINK: 2 < 3, therefore I must rename 7 gal 2 qt as 6 gal 6 qt.

			Estimate	Actual
(c)	5 qt 0 pt	rounds to	5 qt	5 qt 0 pt
	-2 qt 1 pt	rounds to	-3 qt	-2 qt 1 pt
			2 qt	2 qt 1 pt

THINK: 0 pt < 1 pt, therefore I must rename 5 qt as 4 qt 2 pt.

Repeat this procedure using other examples. Ask students to use the procedures common to all operations with customary measures to help them in their work.

6.10.03.04A **create and solve word problems related to skills learned in this module**

STRATEGY 1:

Materials:

- a set of 3" x 5" cards with customary measures written on each card expressed in two units

  e.g., 8 ft 6 in. 7 lb 12 oz 6 gal 2 qt 2 yd 1 ft

Separate the students into small groups and provide each student in the group with a 3" x 5" card. Say, "We are going to create and solve problems related to operations with customary measures. What can we do to create an appropriate algorithm?" (We must find a matching partner or another student who has a card with corresponding units of

STRATEGY 1: (cont'd)

measure.) Allow students to leave the group, if necessary, to find a partner.

e.g.,

Student Partners

3 lb 12 oz   6 lb 7 oz

Say, "Once you have found a partner with corresponding units of measure, write two word problems, one which requires addition and another which requires subtraction."

e.g.,

(a) Mr. Williams bought a bag of potatoes which weighed 6 lb 7 oz and a bag of onions which weighed 3 lb 12 oz. What is their total weight?

(b) Ms. Stevens had a 6 lb 7 oz chunk of modeling clay. She used 3 lb 12 oz for a class activity. How much clay is available for the next class activity?

Ask a student to come to the chalkboard to complete the problems while the other students work at their seats.

e.g.,

```
(a)  6 lb  7 oz     (b)  6 lb  7 oz
    +3 lb 12 oz         -3 lb 12 oz
     9 lb 19 oz          2 lb 11 oz
    10 lb  3 oz
```

Say, "How much did both bags of Mr. Williams' potatoes weigh? (10 lb 3 oz) How much clay is left for sculpting?" (2 lb 11 oz)

Repeat this procedure by creating and solving other word problems related to operations with customary measures.

P.O. 6.10.03.01&.04

NAME: ____________________ **CLASS:** ______ **DATE:** ____________

Estimate each sum or difference. Then find the actual sum or difference.

ⓐ
3 ft 11 in.
+5 ft 10 in.
my estimate
+ ________

ⓑ
8 yd 1 ft
−3 yd 2 ft
my estimate
− ________

ⓒ
5 ft 9 in.
+7 ft 8 in.
my estimate
+ ________

ⓓ
6 ft 8 in.
−2 ft 11 in.
my estimate
− ________

Study the cards and then write one addition and one subtraction problem for each match. One problem has been started for you.

8 ft 2 in. (a)

12 yd 1 ft (b)

6 ft 9 in. (c)

10 yd 2 ft (d)

For my first problem I am using the measurements on cards a **and** c **:**

Julio can run and jump 8 ft 2 in. His brother Hector can run and jump 6 ft 9 in.

For my second problem I am using the measurements on cards ☐ and ☐ :

For my third problem I am using the measurements on cards ☐ and ☐ :

For my fourth problem I am using the measurements on cards ☐ and ☐ :

P.O. 6.10.03.02-.04

NAME: ______________________ CLASS: ________ DATE: ____________

Add or subtract to solve the examples below.

a. 5 lb 10 oz
+3 lb 9 oz

b. 7 lb 3 oz
-2 lb 14 oz

c. 10 lb 2 oz
- 3 lb 15 oz

d. 8 lb 15 oz
+7 lb 14 oz

e. 6 gal 3 qt
-2 gal 1 qt

f. 8 qt
-2 qt 1 pt

g. 3 qt 1 pt
+1 qt 1 pt

h. 16 gal 2 qt
+ 7 gal 3 qt

Write and solve a word problem using the information given.

a. Jack will be having many guests at his house for Thanksgiving Day. He is baking a 19 lb 14 oz turkey. His friend Dana is having fewer guests and a smaller turkey which weighs 8 lb 12 oz.

b. On Saturday, Mr. DeSoto used $12\frac{1}{2}$ gallons of gasoline. The next day he used $15\frac{1}{4}$ gallons.

c. At the school picnic we served 15 gallons 3 quarts of orange drink and 10 gallons 2 quarts of lemonade.

SHARPENING YOUR SKILLS

Units 10, 11, 12

Name: ____________________ **Class:** ____________ **Date:** ________

1. Complete the following:

   a. 1 T = __________ lb

   b. $\frac{1}{4}$ T = __________ lb

   c. $\frac{1}{2}$ T = __________ lb

2. Complete the following:

   a. $\frac{1}{3}$ ft = __________ in.

   b. $\frac{3}{4}$ ft = __________ in.

   c. $\frac{1}{2}$ yd = __________ in.

   d. $\frac{1}{3}$ yd = __________ in.

3. 90 minutes = __________

   ⓐ 1 hr 30 sec ⓒ 2 hr 30 min

   ⓑ 1 hr 30 min ⓓ Not Here

4. 27 hours = __________

   ⓔ 2 day 7 hr ⓖ 1 day 3 hr

   ⓕ 1 day 30 min ⓗ Not Here

5. 75 seconds = __________

   ⓐ 1 hr 15 min ⓒ 1 min 5 sec

   ⓑ 2 min 15 sec ⓓ Not Here

6. 4 decades = __________ years

7. 3 centuries = __________ years

8. 1 century = __________ decades

9.

$$\begin{array}{r} 3 \text{ yd } 2 \text{ ft} \\ \underline{+1 \text{ yd } 1 \text{ ft}} \end{array}$$

   ⓔ 5 yd ⓖ 4 yd 1 ft

   ⓕ 2 yd 1 ft ⓗ Not Here

10.

$$\begin{array}{r} 5 \text{ gal} \\ \underline{-2 \text{ gal } 1 \text{ qt}} \end{array}$$

11. Today's date is ______________.

    Two decades from today will be

    ______________________________.

12. Boris bought $\frac{1}{2}$ lb of peanuts and $\frac{1}{4}$ lb of cashews. How many ounces of nuts did he buy?

    ____

6.14.04 REVIEW: EQUIVALENT FRACTIONS

(1 lesson/ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

6.14.04.01 use the equal multiplication property to generate equivalent fractions;

e.g.,

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8} \quad \text{therefore} \quad \frac{3}{4} = \frac{6}{8}$$

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12} \quad \text{therefore} \quad \frac{3}{4} = \frac{9}{12}$$

(This corresponds to cutting each piece into the same number of equal parts.)

6.14.04.02 rename fractions as simpler equivalent fractions in lower terms by dividing the numerator and the denominator by the same number;

e.g.,

$$\frac{6 \div 3}{9 \div 3} = \frac{2}{3} \quad \text{therefore} \quad \frac{6}{9} = \frac{2}{3}$$

$$\frac{4 \div 2}{8 \div 2} = \frac{2}{4} \quad \text{therefore} \quad \frac{4}{8} = \frac{2}{4}$$

(This corresponds to bundling together fractional pieces into sets of equal size.)

6.14.04.03 express fractions in simplest form (lowest terms).

e.g.,

$$\frac{6}{8} = \frac{3}{4} \qquad \frac{8}{16} = \frac{1}{2} \qquad \frac{30}{40} = \frac{3}{4}$$

PROBLEM EXPLORATION SKILLS:

- to create an analogy
- to adopt a different point of view

VOCABULARY

equivalent	denominator
numerator	simplest form (lowest terms)
greatest common factor (GCF)	

PREVIOUS UNDERSTANDINGS:

- concept of fraction
- equivalent fractions

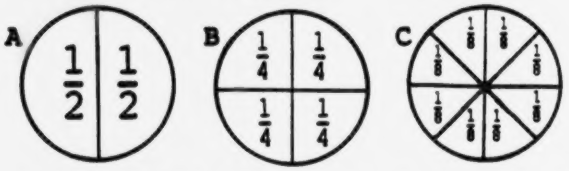
GETTING STARTED:

Materials:

- paper circles, some marked in halves, some in quarters and some in eighths
- scissors

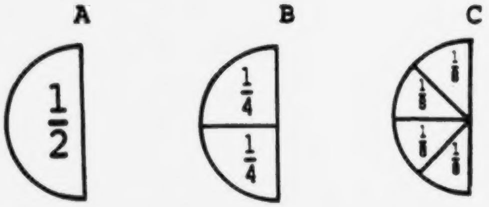
Prepare and distribute 3 paper circles and a pair of scissors to each student or group of students. One circle should be marked in half, one in quarters, and one in eighths.

e.g.,



Ask the students to fold all their circles in half. Say, "Look at each circle. What does circle A show? (It shows a circle marked in 2 equal parts.) Cut along the fold with your scissors. Now look at circle B. What does it show? (It shows a circle marked in 4 equal parts.) Cut along the fold." Continue this procedure with circle C.

e.g.,



Say, "How is each part of circle A represented as a fraction? ($\frac{1}{2}$) ... Circle B ($\frac{2}{4}$)... Circle C ($\frac{4}{8}$). Place your $\frac{1}{2}$ circle on top of your $\frac{2}{4}$ piece. What do you notice? (They are the same size.) What do we call the fractions $\frac{1}{2}$ and $\frac{2}{4}$?" (We call them equivalent fractions.)

GETTING STARTED: (cont'd)

Write this information on the chalkboard and ask the students to write it in their notebooks.

e.g.,

Equivalent Fractions

$$\frac{1}{2} = \frac{2}{4}$$

Repeat this procedure by asking students to compare the $\frac{1}{2}$ and $\frac{4}{8}$ pieces and the $\frac{2}{4}$ and $\frac{4}{8}$ pieces.

Say, "What do you notice about the relationship between the numerator and the denominator when the fraction is equivalent to $\frac{1}{2}$? (The numerator is half the denominator.) Can you express that another way?" (The denominator is twice the numerator.) Ask the students to use this relationship to name other fractions equivalent to $\frac{1}{2}$.

e.g.,

$$\frac{1}{2} = \frac{\square}{12} \qquad \frac{1}{2} = \frac{\square}{20}$$

$$\frac{1}{2} = \frac{8}{\square} \qquad \frac{1}{2} = \frac{50}{\square}$$

6.14.04

6.14.04.01 use the equal multiplication property to generate equivalent fractions

STRATEGY 1:

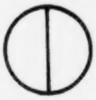
Materials:

- paper circles or rectangles

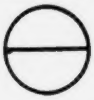
Distribute paper circles or paper rectangles to each student or group of students.

Say, "Fold one circle in half." Demonstrate.

e.g.,



or



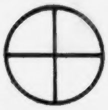
Say, "How many equal parts do we see? (2 equal parts) How can we name each part? ($\frac{1}{2}$) How many halves do we have in all? (two halves) What two names can we give to this circle? (1 whole or 2 halves) What do we know about 1 whole and 2 halves?" (They are equivalent.) Write this on the chalkboard and ask the students to write it in their notebooks.

e.g.,

$$1 \text{ whole} = \frac{2}{2}$$

Say, "Fold each half of the circle in half again."

e.g.,



Say, "How many equal parts do we have now? (We have 4 equal parts.) What can we name each part? (We can name each part $\frac{1}{4}$.) How many fourths do we have in all?" (4) Write this on the chalkboard and ask the students to write it in their notebooks.

e.g.,

$$1 \text{ whole} = \frac{4}{4}$$

STRATEGY 1: (cont'd)

Say, "How many fourths are in $\frac{1}{2}$?" (There are $\frac{2}{4}$ in $\frac{1}{2}$.) Fold the circle into eight equal parts and repeat this procedure. Say, "What is the relationship between $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$? (They are equivalent.) What pattern do you see developing? (When we multiply the numerator and denominator by the same number, we get an equivalent fraction.) How can we rename $\frac{1}{3}$ as an equivalent fraction with a denominator of 6?" (We can multiply the numerator and denominator by 2.) Ask a student to demonstrate this on the chalkboard.

e.g.,

$$\frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

Say, "What happens when we multiply the numerator and the denominator of a fraction by the same number? (We get a fraction equivalent to the one with which we started.) Multiplying each part of the fraction in this way corresponds to subdividing each piece of the original fraction into the same number of equal parts."

Repeat this procedure to generate other equivalent fractions.

e.g.,

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12} \qquad \frac{4 \times 2}{5 \times 2} = \frac{8}{10}$$

STRATEGY 2:

Prepare and display a set of 4 equivalent fractions.

e.g.,

$$\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}$$

Say, "Why are these called equivalent fractions? (They all represent the same quantity.) How can we show that these fractions are equivalent?" (We can show they are equivalent by using multiplication. Whenever we multiply the numerator and the denominator by the same number, we will get an equivalent fraction.) Demonstrate this on the chalkboard.

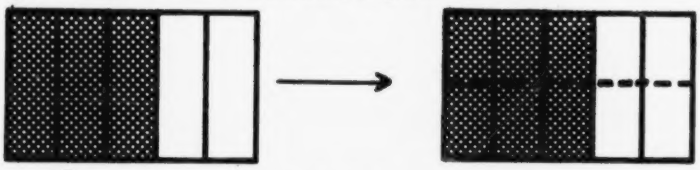
e.g.,

$$\frac{3 \times 2}{5 \times 2} = \frac{6}{10}$$

Say, "Are $\frac{3}{5}$ and $\frac{6}{10}$ equivalent? (yes) What did we do to $\frac{3}{5}$ to rename it as $\frac{6}{10}$? (We multiplied the numerator and the denominator by 2.)

STRATEGY 2: (cont'd)

This corresponds to cutting or folding each of the fifths in half. We get twice as many pieces; ten where we originally had 5." To clarify what is happening, draw the following picture:

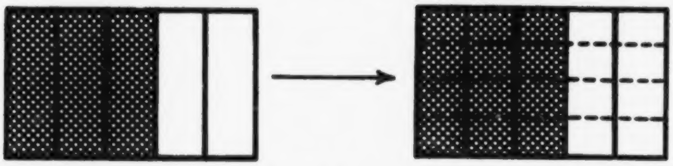


Write the fraction $\frac{3}{5}$ on the chalkboard. Say, "How can we rename fifths to twentieths? (We must multiply 5 by 4 to show that each fifth is being split into 4 equal parts.) What must we do to the 3?" (We must also multiply it by 4.) Ask a student to demonstrate this on the chalkboard.

e.g.,

$$\frac{3 \times 4}{5 \times 4} = \frac{12}{20}$$

To clarify what is happening, draw the following picture:



Write several examples on the chalkboard and ask students to work in their notebooks to find the missing numerators or denominators.

e.g.,

$$\frac{2}{3} = \frac{\Box}{9} \qquad \frac{4}{7} = \frac{16}{\Box} \qquad \frac{4}{5} = \frac{\Box}{35}$$

Say, "In the first example, how did we rename thirds to ninths? (We subdivided each third into three equal parts. To show this, we multiplied the denominator by 3, now we have 9 equal parts.) What must we do to the numerator?" (We must multiply the numerator by 3.) Ask a student to demonstrate on the chalkboard as the others work at their desks.

e.g.,

$$\frac{2 \times \Box}{3 \times 3} = \frac{\Box}{9} \text{ therefore } \frac{2 \times 3}{3 \times 3} = \frac{6}{9}$$

Repeat this procedure for each example.

6.14.04

6.14.04.02 **rename fractions as simpler equivalent fractions in lower terms by dividing the numerator and the denominator by the same number**

STRATEGY 1:

Write several sets of equivalent fractions on the chalkboard. Show the multiplication used to find each equivalent.

e.g.,

$$\frac{3 \times 4}{8 \times 4} = \frac{12}{32} \qquad \frac{4 \times 6}{5 \times 6} = \frac{24}{30} \qquad \frac{3 \times 7}{4 \times 7} = \frac{21}{28}$$

$$\frac{3}{8} = \frac{12}{32} \qquad \frac{4}{5} = \frac{24}{30} \qquad \frac{3}{4} = \frac{21}{28}$$

Ask the students to copy each example into their notebooks. Say, "The numerator and the denominator of a fraction are called its terms. What can we say about $\frac{3}{8}$ and $\frac{12}{32}$? (They are equivalent.) We say that $\frac{12}{32}$ is written in higher terms than $\frac{3}{8}$ because the numbers in the numerator and the denominator are greater. How did we rename $\frac{3}{8}$ to $\frac{12}{32}$? (We multiplied both the numerator and denominator by 4.) Sometimes it is useful to write a fraction in lower terms. A fraction in lower terms retains its value, but the whole is divided into fewer parts." Write the following on the chalkboard: $\frac{1}{2} = \frac{19}{38}$. Say, "Which fraction is easier for us to visualize? ($\frac{1}{2}$) If we used multiplication to generate equivalent fractions in higher terms, how can we generate equivalent fractions in lower terms?" (We could divide the numerator and the denominator by the same number.) Write the following on the chalkboard:

$$\frac{12 \div \square}{32 \div \square} = \frac{\square}{\square}$$

Say, "What numbers can 12 and 32 be divided by evenly?" (2 and 4) Demonstrate.

e.g.,

$$\frac{12 \div 2}{32 \div 2} = \frac{6}{16} \qquad \frac{12 \div 4}{32 \div 4} = \frac{3}{8}$$

STRATEGY 1: (cont'd)

Say, "Both $\frac{6}{16}$ and $\frac{3}{8}$ are equivalent to $\frac{12}{32}$ but are in lower terms." Write the fraction $\frac{24}{30}$ on the chalkboard. Using this procedure, ask the students to find several equivalent fractions for $\frac{24}{30}$ in lower terms. Ask a small group of students to work on the chalkboard as the others write in their notebooks.

e.g.,

$$\frac{24 \div 2}{30 \div 2} = \frac{12}{15} \qquad \frac{24 \div 3}{30 \div 3} = \frac{8}{10} \qquad \frac{24 \div 6}{30 \div 6} = \frac{4}{5}$$

Repeat this procedure to express other fractions in lower, or simpler, terms.

6.14.04.03 express fractions in simplest form (lowest terms)

STRATEGY 1:

Write the following on the chalkboard: $\frac{3}{8}$, $\frac{16}{32}$. Say, "How can we rename $\frac{3}{8}$ as an equivalent fraction in higher terms? (We can multiply the numerator and the denominator by the same number.) Can we multiply by any number? Explain." (Yes, as long as the same number is used to multiply both the numerator and the denominator.) Ask a student to choose a multiplier and work at the chalkboard.

e.g.,

$$\frac{3 \times 5}{8 \times 5} = \frac{15}{40}$$

Say, "Let's rename $\frac{16}{32}$ in lower terms. How do we reduce a fraction to lower terms? (We divide the numerator and the denominator by the same number.) Can we use any number? Explain. (No, because the divisor must be a factor of both the numerator and denominator.) What number can we use to reduce $\frac{16}{32}$?" (We can divide by 2.) Ask a student to use the 2 to reduce $\frac{16}{32}$ to lower terms on the chalkboard while the other students work in their notebooks.

e.g.,

$$\frac{16 \div 2}{32 \div 2} = \frac{8}{16}$$

STRATEGY 1: (cont'd)

Say, "How can $\frac{8}{16}$ be further reduced to lower terms? (We can divide the numerator and the denominator by 2.)" Demonstrate.

e.g.,

$$\frac{8 \div 2}{16 \div 2} = \frac{4}{8}$$

Say, "Can we continue to divide $\frac{4}{8}$ to further reduce its terms? Explain." (Yes, we can divide the numerator and the denominator by 2 again.) Demonstrate.

e.g.,

$$\frac{4 \div 2}{8 \div 2} = \frac{2}{4}$$

Continue this procedure to reduce $\frac{2}{4}$ to $\frac{1}{2}$.

Say, "Have we reduced $\frac{16}{32}$ to lowest terms? (yes) How can we reduce a fraction to lowest terms more efficiently? (We can divide by the greatest common factor of both the numerator and denominator.) How can we find the greatest common factor (GCF) for 16 and 32?" (First, we can list all the factors of 16 and 32.) Ask a student to demonstrate this on the chalkboard.

e.g.,

Factors of 16: 1,2,4,8,16
Factors of 32: 1,2,4,8,16,32

Say, "Which factors appear in both lists? (1,2,4,8,16). What do we call these factors? (common factors) Which is the greatest common factor?" (16) Ask a student to come to the chalkboard and reduce, or simplify, by dividing the numerator and the denominator by 16.

e.g.,

$$\frac{16 \div 16}{32 \div 16} = \frac{1}{2}$$

Say, "Can we further reduce, or simplify, $\frac{1}{2}$? Explain. (No, because only 1 can be used as a divisor and this will not further reduce the fraction.) When we can no longer reduce a fraction, we say it is in lowest terms. How did the GCF help us to reduce $\frac{16}{32}$ to lowest terms?" (Using the GCF is faster and the opportunities for making an error are diminished.)

Repeat this procedure with other fractions. Ask students to find the equivalent fraction which is written in lowest terms.

P.O. 6.14.04.01-.02

NAME: ______________________ **CLASS:** ________ **DATE:** ____________

Follow the directions below.

1. Find equivalent fractions in higher terms.

$\frac{3 \times 4}{8 \times 4} = \boxed{}$ $\frac{3}{8} = \boxed{}$

$\frac{1 \times 6}{9 \times 6} = \boxed{}$ $\frac{1}{9} = \boxed{}$

$\frac{7 \times 3}{10 \times 3} = \boxed{}$ $\frac{7}{10} = \boxed{}$

$\frac{5 \times 7}{6 \times 7} = \boxed{}$ $\frac{5}{6} = \boxed{}$

$\frac{1 \times 8}{2 \times 8} = \boxed{}$ $\frac{1}{2} = \boxed{}$

2. Write an equivalent fraction in higher terms.

$\frac{5 \times \boxed{}}{9 \times \boxed{}} = \boxed{}$

$\frac{4 \times \boxed{}}{7 \times \boxed{}} = \boxed{}$

$\frac{2 \times \boxed{}}{12 \times \boxed{}} = \boxed{}$

$\frac{11 \times \boxed{}}{12 \times \boxed{}} = \boxed{}$

$\frac{1 \times \boxed{}}{3 \times \boxed{}} = \boxed{}$

3. Find the missing numerator or denominator to make each pair of fractions equivalent.

$\frac{3}{4} = \frac{\boxed{}}{8}$

$\frac{7}{9} = \frac{\boxed{}}{54}$

$\frac{5}{8} = \frac{15}{\boxed{}}$

$\frac{7}{12} = \frac{35}{\boxed{}}$

$\frac{2}{3} = \frac{\boxed{}}{21}$

4. Find equivalent fractions in lower terms.

$\frac{3 \div 3}{9 \div 3} = \boxed{}$ $\frac{3}{9} = \boxed{}$

$\frac{8 \div 4}{32 \div 4} = \boxed{}$ $\frac{8}{32} = \boxed{}$

$\frac{12 \div 2}{30 \div 2} = \boxed{}$ $\frac{12}{30} = \boxed{}$

$\frac{18 \div 6}{18 \div 6} = \boxed{}$ $\frac{18}{18} = \boxed{}$

5. Write an equivalent fraction in lower terms.

$\frac{14 \div \boxed{}}{16 \div \boxed{}} = \boxed{}$

$\frac{8 \div \boxed{}}{20 \div \boxed{}} = \boxed{}$

$\frac{12 \div \boxed{}}{32 \div \boxed{}} = \boxed{}$

$\frac{25 \div \boxed{}}{45 \div \boxed{}} = \boxed{}$

6. Find the missing numerator or denominator to make each pair of fractions equivalent.

$\frac{10}{16} = \frac{\boxed{}}{8}$

$\frac{9}{27} = \frac{3}{\boxed{}}$

$\frac{16}{30} = \frac{8}{\boxed{}}$

$\frac{8}{24} = \frac{\boxed{}}{6}$

P.O. 6.14.04.03

NAME: ______________________ **CLASS:** ________ **DATE:** ____________

Follow the directions below.

1. Circle the fractions that are expressed in lowest terms.

$\frac{8}{11}$ $\frac{9}{21}$ $\frac{3}{5}$ $\frac{3}{9}$

$\frac{4}{7}$ $\frac{3}{15}$ $\frac{9}{10}$

$\frac{3}{16}$ $\frac{7}{20}$ $\frac{15}{28}$ $\frac{5}{8}$

$\frac{2}{9}$ $\frac{6}{27}$

2. In each group, find the fraction that is expressed in lowest terms. Write it on the line.

$\frac{2}{6}, \frac{3}{9}, \frac{1}{3}, \frac{4}{12}$ ______________

$\frac{6}{10}, \frac{3}{5}, \frac{12}{20}, \frac{15}{20}$ ______________

$\frac{2}{7}, \frac{6}{21}, \frac{12}{42}, \frac{4}{14}$ ______________

3. Express each fraction in simplest form.

$\frac{4}{10} =$ ☐ $\frac{9}{36} =$ ☐ $\frac{8}{16} =$ ☐ $\frac{9}{24} =$ ☐

$\frac{6}{18} =$ ☐ $\frac{10}{20} =$ ☐ $\frac{12}{24} =$ ☐ $\frac{10}{24} =$ ☐

$\frac{4}{32} =$ ☐ $\frac{5}{25} =$ ☐ $\frac{12}{16} =$ ☐ $\frac{4}{12} =$ ☐

Do you remember?

Identify the prior knowledge needed and solve.

Jackie invited several friends for dinner. Her mother prepared a meat loaf which included: 2 lb 12 oz ground beef, $1\frac{1}{2}$ lb ground veal, 1 lb ground pork. How many pounds and ounces of meat were used to prepare the meat loaf?

Prior knowledge needed ______________________________

Answer: ________ pounds ________ ounces

Unit 14 **6.14.05 REVIEW: LEAST COMMON MULTIPLE (LCM)**

(1 lesson/ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

*6.14.05.01 find the least common multiple (LCM) of two or more numbers;

e.g.,

Find the LCM of 8 and 12.

The set of multiples of 8
$= \{8,16,24,32,40,48,56...\}$

The set of multiples of 12
$= \{12,24,36,48,60...\}$

The set of common multiples of 8 and 12
is $\{24,48...\}$
(This is the intersection of the two sets listed above.)

The LCM of 8 and 12 = 24.
This is the smallest number which is a multiple of 8 and also a multiple of 12.

6.14.05.02 rename two or more fractions as fractions with common denominators.

e.g., $\frac{5}{6}$, $\frac{3}{4}$

To find the lowest common denominator for sixths and fourths, find the LCM of 6 and 4. 12 is the LCM.

$$\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

PROBLEM EXPLORATION SKILLS:

- to adopt a different point of view
- to generalize conclusions clearly

VOCABULARY

LCM
least common multiple
common denominator

PREVIOUS UNDERSTANDINGS:

- multiples of a number
- common multiples
- equivalent fractions

GETTING STARTED:

Display several equivalent fractions on the chalkboard.

e.g.,

$$\frac{3}{4}, \frac{9}{12}, \frac{18}{24}, \frac{27}{36}$$

Say, "The fractions shown on the chalkboard are equivalent fractions. How can we prove that $\frac{3}{4}$ is equivalent to $\frac{18}{24}$? (We could change $\frac{3}{4}$ to 24ths, or we could reduce $\frac{18}{24}$.) How would you change $\frac{3}{4}$ to 24ths? (Multiply the numerator and the denominator in $\frac{3}{4}$ by the same number, 6, in order to prove that $\frac{3}{4}$ is equivalent to $\frac{18}{24}$.) What allows us to do this? (the equal multiplication property) Ask a student to demonstrate this information on the chalkboard.

e.g.,

$$\frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$

Say, "What is the relationship between the fractions $\frac{3}{4}$ and $\frac{18}{24}$? (They are equivalent.) Since the new numerator and the new denominator are larger numbers, we say that this equivalent fraction is written in higher terms." Ask several students to find other fractions that are equivalent to $\frac{3}{4}$.

Repeat this procedure using other equivalent fractions.

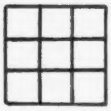
6.14.05

6.14.05.01 **find the least common multiple (LCM) of two or more numbers**

STRATEGY 1:

On the chalkboard draw a 3 x 3 grid and a chart on which students can list the multiples of two numbers.

e.g.,



Multiples of

6	9

Say, "We will divide the students into two teams, team A, and team B. One student from each team will list the first ten multiples of each number on the chalkboard chart. What are multiples? (A multiple of a number is the product of that number and another number.) What is the first multiple of 6? (The first multiple of 6 is 6, because that is the product of 1 x 6.) Some numbers will appear on both lists. What do we call these numbers?" (We call them common multiples of 6 and 9 because they are common to both lists.) After the two players list the first ten multiples of both numbers, they will circle the common multiples and identify the lowest number circled.

e.g.,

Multiples of

6	9
6	9
12	⑱
⑱	27
24	㊱
30	45
㊱	54
42	65
48	72
54	81
60	90

STRATEGY 1: (cont'd)

Say, "What is the lowest number circled? (18) What is the special name given to this common multiple?" (It is called the least common multiple or LCM.) On the chalkboard write: Least Common Multiple (LCM) of 6 and 9 = 18. Say, "The member of the team who finds the LCM first can write his/her team's letter, A or B, on the grid. As in Tic-Tac-Toe, when a team gets three of its letters in any row, that team wins."

e.g.,

A		
	A	
		A

Ask two students to begin the game by listing the multiples of 8 and 5. Repeat this procedure using the multiples of other numbers.

STRATEGY 2:

Display a multiples chart using any two 1-digit numbers.

e.g., **Multiples Chart**

Multiples of 3										
Multiples of 5										

Say, "What is a multiple of a number? (A multiple is a product we get when we multiply that number by another number.) How can we make a multiples chart? (We can list several multiples of each number consecutively.) What is the first multiple of 3?" (The first multiple of 3 is the product of 1 x 3, or 3.)

Ask a student to complete the chart for threes while another student completes the chart for fives. The other students may work in their notebooks.

e.g., **Multiples Chart**

Multiples of 3	3	6	9	12	15	18	21	24	27	30
Multiples of 5	5	10	15	20	25	30	35	40	45	50

Say, "Are there any numbers which appear on both lists of multiples? (Yes, 15 and 30 are on both lists of multiples.) What do we call numbers which appear on both lists?" (They are called common multiples.)

STRATEGY 2: (cont'd)

Ask the students to circle the common multiples.

e.g.,

Multiples Chart

Multiples of 3	3	6	9	12	15	18	21	24	27	30
Multiples of 5	5	10	15	20	25	30	35	40	45	50

Say, "What is the definition of the least common multiple, or LCM? (The LCM is the lowest number of the set of common multiples.) What is the LCM of 3 and 5?" (15)

Repeat this procedure to find the LCM of other numbers. The same activity can be done using 3 or more numbers.

e.g.,

Multiples Chart

Multiples of 2	2	4	6	8	10	12	14	16	18	20
Multiples of 3	3	6	9	12	15	18	21	24	27	30
Multiples of 4	4	8	12	16	20	24	28	32	36	40

LCM of 2, 3, and 4 = 12

6.14.05.02 **rename two or more fractions as fractions with common denominators**

STRATEGY 1:

Display the following word problem:

> Sally ran $\frac{3}{4}$ mile, Betsy ran $\frac{2}{3}$ mile, and Ann ran $\frac{1}{2}$ mile. Who ran the longest distance? Who ran the shortest distance?

Say, "What are we asked to find? (We are asked to find out who ran the longest distance and who ran the shortest distance.) What contributes to the difficulty of this problem? (All the fractions have different denominators.) How can we compare fractions with unlike denominators? (We must first rename the fractions as equivalents with common denominators.) How can we find a denominator which is common to all three fractions?" (We can make a multiples chart and find the LCM of the denominators 2, 3, and 4. The LCM will

STRATEGY 1: (cont'd)

be the new denominator of the three fractions. Then we will have to rename the numerators so that each new fraction is equivalent to its original fraction.) Ask a student to show this work on the chalkboard as the other students work in their notebooks.

e.g.,

Multiples Chart

Multiples of 2	2	4	6	8	10	12	14	16	18	20
Multiples of 3	3	6	9	12	15	18	21	24	27	30
Multiples of 4	4	8	12	16	20	24	28	32	36	40

Say, "What is the LCM of 2, 3, and 4? (12) Now we can find equivalent fractions using 12 as the denominator." Ask a student to show this work at the chalkboard as the other students work in their notebooks.

e.g.,

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12} \qquad \frac{1 \times 6}{2 \times 6} = \frac{6}{12} \qquad \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

Say, "How can we compare the fractions?" (Since the fractions now have like denominators, we can compare the numerators.) What are the solutions to the word problem?" (Sally ran the longest distance, $\frac{9}{12}$ mile, and Ann ran the shortest distance, $\frac{6}{12}$ mile.)

Repeat this procedure using similar word problems.

P.O. 6.14.05.01-.02

NAME: ______________________________ **CLASS:** _______ **DATE:** ______________

Circle the common multiples of each set. Write the LCM.

1. 3, 5

3 5 6 8 10 12

15 17 30 40 45 90

The LCM of 3 and 5 is ______.

2. 4, 12

4 6 8 9 12 15

18 20 24 30 36 48

The LCM of 4 and 12 is ______.

Find the LCM of each group of numbers.

3. 2, 4, 5

Multiples of 2: ____________________

Multiples of 4: ____________________

Multiples of 5: ____________________

The LCM of 2, 4, and 5 is ___________.

4. 4, 8, 10

Multiples of 4: ____________________

Multiples of 8: ____________________

Multiples of 10: ____________________

The LCM of 4, 8, and 10 is ___________.

Rename each set of fractions as equivalent fractions with common denominators. Write them from least to greatest.

5. $\frac{5}{8}, \frac{3}{7}$

________ < ________

6. $\frac{2}{3}, \frac{3}{5}$

________ < ________

7. $\frac{2}{3}, \frac{5}{6}, \frac{7}{12}$

_______ < _______ < _______

8. $\frac{2}{3}, \frac{3}{4}, \frac{1}{2}$

_______ < _______ < _______

Unit 15

6.06.01 COMPARING FRACTIONS

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

6.06.01.01A use the common denominator method to compare two fractions using the correct relation symbol (>, <, =);
e.g.,

$\frac{3}{5} \bigcirc \frac{7}{9}$

$\frac{3 \times 9}{5 \times 9} = \frac{27}{45}$

$\frac{7 \times 5}{9 \times 5} = \frac{35}{45}$

$\frac{27}{45} < \frac{35}{45}$ therefore $\frac{3}{5} < \frac{7}{9}$

6.06.01.02A arrange a set of fractions from the least to the greatest;
e.g.,

$\frac{3}{4}, \frac{2}{5}, \frac{7}{10}$

$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$

$\frac{2 \times 4}{5 \times 4} = \frac{8}{20}$

$\frac{7 \times 2}{10 \times 2} = \frac{14}{20}$

$\frac{8}{20} < \frac{14}{20} < \frac{15}{20}$ therefore $\frac{2}{5} < \frac{7}{10} < \frac{3}{4}$

6.06.01.03A create and solve word problems related to skills learned in this module.

e.g.,

As part of her exercise program, Doreen walked $\frac{3}{4}$ mile on Monday, $\frac{7}{10}$ mile on Wednesday, and $\frac{4}{5}$ mile on Friday. On which day did she walk the greatest distance?... the least distance?

PROBLEM EXPLORATION SKILLS:

- to create an analogy
- to adopt a different point of view

6.06.01

VOCABULARY

common denominator
equivalent

PREVIOUS UNDERSTANDINGS:

- equivalent fractions

GETTING STARTED:

Materials:

- numeral cards, each with a digit from 1 through 50

Display a large equal symbol on the chalkboard. Ask two students to stand on the left side of the equal sign. Give one student a numeral card with the digit 4 and give the other student the numeral card with the digit 5. Ask a third student to stand on the right side of the equal sign with the card having the numeral 10.

Say, "When there are numbers on both sides of an equal sign, what does it mean? (It means that the amounts on each side have the same value.) Let's multiply the numbers that are on the same side of the equal sign. How much are 4 fives? (4 fives are 20.) Are the two sides equal? (No, there is 20 on one side and 10 on the other.) We must find a number by which to multiply 10 to make the values on both sides equal. What number will accomplish this?" (2) Demonstrate this on the chalkboard as the students write it into their notebooks. e.g.,

$$\begin{aligned} 4 \times 5 &= 10 \times 2 \\ 20 &= 20 \end{aligned}$$

Repeat this activity using numeral cards 6 and 3 on one side and 2 on the other. Say, "Are the values on each side of the equal symbol worth the same amount? (no) How can we make this equation true?" (Since 6 x 3 has a value of 18, we can multiply 9 by 2, thus giving the right side of the equation a value of 18 as well.) After the students compute 9 as the missing factor, write the equality on the chalkboard and ask the students to write it into their notebooks. e.g.,

$$\begin{aligned} 6 \times 3 &= 2 \times 9 \\ 18 &= 18 \end{aligned}$$

Repeat this activity using other digit cards.

6.06.01

6.06.01.01A **use the common denominator method to compare two fractions using the correct relation symbol**

STRATEGY 1:

Display two pairs of fractions, one with common denominators and the other with different denominators.

e.g.,

$$\frac{7}{8} \quad \frac{5}{8} \qquad\qquad \frac{4}{7} \quad \frac{2}{3}$$

Say, "Look at the first pair of fractions. What do you notice about the denominators? (The denominators are the same.) What should we look at in order to compare the size of the two fractions? (We should look at the numerators.) What do these numerators tell us?" (Since 7 is greater than 5, $\frac{7}{8}$ is greater than $\frac{5}{8}$.) Write this relationship on the chalkboard and ask the students to write it into their notebooks.

e.g.,

$$\frac{7}{8} > \frac{5}{8}$$

Say, "It is easy to compare fractions which have the same denominator. One way of comparing fractions with different denominators is to rename them with a common denominator. How can we rename the second pair of fractions?" (We can use the LCM of the two denominators to rename both fractions.) Ask a student to show this on the chalkboard while the other students write it in their notebooks.

e.g.,

Multiples of 3	3	6	9	12	15	18	21	24
Multiples of 7	7	14	21	28				

Say, "What is the LCM of 3 and 7?" (21) Ask a student to circle the number 21 in each set of multiples.

e.g.,

Multiples of 3	3	6	9	12	15	18	㉑	24
Multiples of 7	7	14	㉑	28				

Say, "Using the equal multiplication property, rename each fraction using 21 as the denominator." Demonstrate.

e.g.,

$$\frac{4 \times 3}{7 \times 3} = \frac{12}{21} \qquad\qquad \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$$

Say, "How does $\frac{12}{21}$ compare to $\frac{14}{21}$?" ($\frac{12}{21}$ is less than $\frac{14}{21}$ or $\frac{14}{21}$ is greater than $\frac{12}{21}$.) Ask a student to write this comparison using the correct relation symbol on the chalkboard.

STRATEGY 1: (cont'd)

e.g.,

$\frac{12}{21} < \frac{14}{21}$ or $\frac{14}{21} > \frac{12}{21}$ Therefore $\frac{4}{7} < \frac{2}{3}$ or $\frac{2}{3} > \frac{4}{7}$

$\frac{12}{21}$ is less than $\frac{14}{21}$ $\frac{14}{21}$ is greater than $\frac{12}{21}$

Repeat this procedure using other examples.

6.06.01.02A arrange a set of fractions from the least to the greatest

STRATEGY 1:

Say, "Joe, Tara, and Blake were each reading copies of the same book. Joe has read $\frac{2}{3}$ of the book, Tara has read $\frac{3}{4}$ of the book, and Blake has read $\frac{5}{8}$ of the book. Who read the greatest part of the book? Who read the least part of the book?" Ask a student to write the three fractions on the chalkboard as the other students write them in their notebooks.

e.g., $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{8}$

Say, "What can we do to make our task of comparing these fractions easier? (We can rename all the fractions with a common denominator and then compare the numerators.) How do we find the common denominator?" (We can make a multiples chart of the denominators to find the LCM.)

Ask a student to make a multiples chart on the chalkboard and circle the LCM as the other students do the same in their notebooks.

e.g.,

Multiples of 3	3	6	9	12	15	18	21	㉔
Multiples of 4	4	8	12	16	20	㉔	28	32
Multiples of 8	8	16	㉔	32	40	48	56	64

LCM = 24

Say, "Write the new fractions in order from least to greatest in your notebooks." Ask one student to write them on the chalkboard.

STRATEGY 1: (cont'd)

e.g., $\frac{15}{24} < \frac{16}{24} < \frac{18}{24}$

Ask a student to compare the original fractions on the chalkboard as the other students work in their notebooks.

e.g., $\frac{5}{8} < \frac{2}{3} < \frac{3}{4}$

Say, "Who read the most? (Tara) Who read the least?" (Blake)

Repeat this procedure using other fractions.

STRATEGY 2:

Prepare and display a word problem on the chalkboard or chart paper.

e.g.,

> Linda, Mike, and Nelson had a milk drinking contest. The chart below shows what part of a quart each of them drank.

Milk Drinking Contest

Linda	$\frac{3}{4}$ quart
Mike	$\frac{2}{3}$ quart
Nelson	$\frac{5}{6}$ quart

Say, "Who drank the least amount of milk? ... the most? What are we asked to find? (We are asked to compare the sizes of various fractions from the least to the greatest.) What is the difficulty in ordering these fractions? (They have different denominators). How can we solve this problem? (We can rename all three fractions as equivalent fractions with common denominators.) How can we find the common denominator?" (We can construct a multiples chart to find the LCM.) Ask a student to make the chart on the chalkboard while the others work at their desks.

e.g.,

Multiples of 3	3	6	9	12	15	18	21	24	27	30
Multiples of 4	4	8	12	16	20	24	28	32	36	40
Multiples of 6	6	12	18	24	30	36	42	48	54	60

LCM = 12

STRATEGY 2: (cont'd)

Ask the students to write equivalent fractions using 12 as the common denominator.

e.g.,

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12} \qquad \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \qquad \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

$$\frac{3}{4} = \frac{9}{12} \qquad \frac{2}{3} = \frac{8}{12} \qquad \frac{5}{6} = \frac{10}{12}$$

Ask a student to order the fractions from least to greatest.

e.g.,

$$\frac{8}{12} < \frac{9}{12} < \frac{10}{12} \quad \text{therefore} \quad \frac{2}{3} < \frac{3}{4} < \frac{5}{6}$$

Say, "Using the order of fractions, explain the solution to the problem." (Mike drank the least, Linda is in the middle, and Nelson drank the most.)

Repeat this procedure with similar problems.

6.06.01.03A create and solve word problems related to skills learned in this module

STRATEGY 1:

Display the following word problem:

> A frog jumping contest was held in Toad County. Three frogs participated in the finals. Frog A jumped $\frac{3}{4}$ yd, Frog B jumped $\frac{5}{6}$ yd, and Frog C jumped $\frac{7}{12}$ yd. How did each frog place in the contest?

Ask a student to read the problem aloud. Say, "What are we asked to find? (how the three frogs placed in the contest) What will we have to do in order to decide how each frog placed in the contest? (We have to compare $\frac{3}{4}$ yd, $\frac{5}{6}$ yd, and $\frac{7}{12}$ yd.) We want to compare the distance each frog jumped. What must we do in order to make this comparison? (We must change the fractions to equivalent fractions having a common denominator.) What common denominator will we use? Explain. (The LCM of 4, 6, and 12 is 12.) What must we do in order to change each fraction to an equivalent fraction with 12 as its denominator? (We must determine what number times the denominator will equal 12 and then multiply the numerator by that same number.) What number times 4 equals 12? (3) ... times 6 equals 12? (2) ...times 12 equals 12?" (1) Ask a student to compare the fractions

STRATEGY 1: (cont'd)

using this method on the chalkboard as the other students work in their notebooks.

e.g.,

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12} \qquad \frac{5 \times 2}{6 \times 2} = \frac{10}{12} \qquad \frac{7 \times 1}{12 \times 1} = \frac{7}{12}$$

$$\frac{7}{12} < \frac{9}{12} < \frac{10}{12} \quad \text{therefore} \quad \frac{7}{12} < \frac{3}{4} < \frac{5}{6}$$

Say, "How did each frog place in the contest? (Frog B placed first, Frog A placed second, and Frog C placed third.) Is there another way to solve this problem?" (Yes, convert all measurements to inches.) Ask a student to demonstrate this information at the chalkboard as the other students work in their notebooks.

e.g.,

$\frac{3}{4}$ yd = ____ in. $\frac{5}{6}$ yd = ____ in. $\frac{7}{12}$ yd = ____ in.

$\frac{3}{4} \times \frac{36}{1} = 27$ in. $\frac{5}{6} \times \frac{36}{1} = 30$ in. $\frac{7}{12} \times \frac{36}{1} = 21$ in.

Say, "How many inches are there in $\frac{3}{4}$ yd? (27) ... in $\frac{5}{6}$ yd? (30) ... in $\frac{7}{12}$ yd? (21) What can we conclude from this information?" (It still shows that $\frac{5}{6}$ yd is greater than $\frac{3}{4}$ yd which is greater than $\frac{7}{12}$ yd.)

Repeat this procedure to solve other word problems some of which are created by the students.

P.O. 6.06.01.01

NAME: ______________________ **CLASS:** _______ **DATE:** ______________

Find the LCM for each pair of fractions. Change them to equivalent fractions with a common denominator. Write <, >, or = in each circle.

1. $\frac{2}{5}, \frac{4}{9}$

LCM = $\boxed{45}$

$\frac{2}{5} = \frac{18}{\boxed{45}}$ $\frac{4}{9} = \frac{20}{\boxed{45}}$

$\frac{2}{5} \textcircled{<} \frac{4}{9}$

2. $\frac{5}{8}, \frac{3}{4}$

LCM = $\boxed{}$

$\frac{5}{8} = \boxed{}$ $\frac{3}{4} = \boxed{}$

$\frac{5}{8} \bigcirc \frac{3}{4}$

3. $\frac{12}{14}, \frac{18}{21}$

LCM = $\boxed{}$

$\frac{12}{14} = \boxed{}$ $\frac{18}{21} = \boxed{}$

$\frac{12}{14} \bigcirc \frac{18}{21}$

4. $\frac{4}{10}, \frac{3}{8}$

LCM = $\boxed{}$

$\frac{4}{10} = \boxed{}$ $\frac{3}{8} = \boxed{}$

$\frac{4}{10} \bigcirc \frac{3}{8}$

5. $\frac{5}{6}, \frac{3}{4}$

LCM = $\boxed{}$

$\frac{5}{6} = \boxed{}$ $\frac{3}{4} = \boxed{}$

$\frac{5}{6} \bigcirc \frac{3}{4}$

6. $\frac{3}{7}, \frac{3}{5}$

LCM = $\boxed{}$

$\frac{3}{7} = \boxed{}$ $\frac{3}{5} = \boxed{}$

$\frac{3}{7} \bigcirc \frac{3}{5}$

7. $\frac{6}{11}, \frac{1}{2}$

LCM = $\boxed{}$

$\frac{6}{11} = \boxed{}$ $\frac{1}{2} = \boxed{}$

$\frac{6}{11} \bigcirc \frac{1}{2}$

8. $\frac{7}{15}, \frac{9}{20}$

LCM = $\boxed{}$

$\frac{7}{15} = \boxed{}$ $\frac{9}{20} = \boxed{}$

$\frac{7}{15} \bigcirc \frac{9}{20}$

Do you remember?

Express the following base five numerals as base ten equivalents.

142_{five} = __________ten
2024_{five} = __________ten
313_{five} = __________ten
1440_{five} = __________ten

P.O. 6.06.01.02-.03

NAME: ____________ **CLASS:** ______ **DATE:** ____________

Write the fractions from least to greatest.

1. $\frac{6}{11}, \frac{1}{2}$

________ < ________

2. $\frac{5}{8}, \frac{7}{10}, \frac{2}{5}$

________ < ________ < ________

3. $\frac{2}{3}, \frac{1}{2}, \frac{5}{6}$

________ < ________ < ________

4. $\frac{2}{3}, \frac{5}{18}, \frac{3}{4}$

________ < ________ < ________

5. $\frac{3}{4}, \frac{7}{12}, \frac{5}{8}$

________ < ________ < ________

6. $\frac{1}{8}, \frac{3}{16}, \frac{5}{32}$

________ < ________ < ________

Solve the following word problems:

7. Ted, Jill, and Sally discovered that they were reading the same book for a book report. Ted has read $\frac{3}{4}$ of the book, Jill has read $\frac{4}{5}$ of the book, and Sally has read $\frac{7}{10}$ of the book. Who has read the least number of pages? Who has read the greatest number of pages?

____

____

8. The three sixth grade classes in the Lander's Elementary School sold five dozen boxes of cookies among them. Class 6A sold $\frac{2}{5}$ of the boxes, class 6B sold $\frac{4}{15}$ of the boxes, and class 6C sold $\frac{1}{3}$ of the boxes. Which class sold the least number of boxes of cookies? Which class sold the greatest number of boxes of cookies?

____

____

9. Create and solve a word problem which involves comparing fractions.

____

____

____

____

____

SHARPENING YOUR SKILLS

Units 13, 14, 15

Name: ____________ **Class:** ____________ **Date:** ________

1. Complete each pair of equivalent fractions.

   a. $\frac{1}{2} = \frac{\square}{10}$

   b. $\frac{3}{5} = \frac{9}{\square}$

   c. $\frac{4}{9} = \frac{16}{\square}$

   d. $\frac{2}{7} = \frac{\square}{49}$

2. Express each fraction below in simplest form (lowest terms).

   a. $\frac{3}{9}$ = ______ d. $\frac{6}{8}$ = ______

   b. $\frac{4}{8}$ = ______ e. $\frac{10}{30}$ = ______

   c. $\frac{5}{25}$ = ______ f. $\frac{6}{16}$ = ______

3. The LCM of 4 and 10 is ________.

   ⓐ 2 ⓒ 20

   ⓑ 4 ⓓ Not Here

4. The LCM of 3 and 7 is ________.

   ⓔ 3 ⓖ 42

   ⓕ 21 ⓗ Not Here

5. Express the fractions below as equivalent fractions with the lowest common denominator.

   $\frac{1}{2}$ = ________

   $\frac{2}{5}$ = ________

6. Fill in the oval with the correct answer, yes or no.

   a. Does $\frac{2}{5} = \frac{4}{10}$? (yes) (no)

   b. Does $\frac{4}{7} = \frac{6}{9}$? (yes) (no)

   c. Does $\frac{3}{8} = \frac{1}{3}$? (yes) (no)

   d. Does $\frac{1}{4} = \frac{10}{40}$? (yes) (no)

   e. Does $\frac{6}{8} = \frac{9}{12}$? (yes) (no)

7. Write >, <, or = to make each number sentence true.

   a. $\frac{2}{3}$ ○ $\frac{4}{5}$

   b. $\frac{3}{8}$ ○ $\frac{1}{4}$

   c. $\frac{5}{20}$ ○ $\frac{2}{8}$

8. Tanya bought $\frac{2}{3}$ pound of peanuts, $\frac{3}{8}$ pound of cashews, and $\frac{1}{2}$ pound of pistachio nuts. Which bag of nuts is the heaviest?

   ____

Unit 16

6.07.01 DECIMALS - TEN THOUSANDTHS

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

6.07.01.01A identify the place value of decimal places through ten-thousandths; e.g.,

23.7146 =

tens	ones	tenths	hundredths	thousandths	ten-thousandths
2	3	7	1	4	6

The digit 6 is in the ten-thousandths place. The total value of the digit 6 is 6 ten-thousandths.

6.07.01.02A write tenths, hundredths, thousandths, and ten-thousandths as decimals; e.g.,

$\frac{6}{10} = 0.6$ $\frac{6}{1000} = 0.006$

$\frac{6}{100} = 0.06$ $\frac{6}{10,000} = 0.0006$

*6.07.01.03A compare decimals using the symbols >, <, or =; e.g.,

$0.47 > 0.048$ $1.2345 < 1.357$ $0.4 = 0.400$

6.07.01.04A order a set of decimals from least to greatest. e.g.,

0.53, 0.2991, 0.674

$0.2991 < 0.53 < 0.674$

Note: An appropriate strategy is to express all decimals with a common denominator; e.g., to compare 0.5 and 0.17, express both as hundredths: 0.50 > 0.17.

PROBLEM EXPLORATION SKILLS:

- to adopt a different point of view
- to consider a related but simpler problem

VOCABULARY

decimal point
decimal number
tenths
hundredths
thousandths
ten-thousandths

PREVIOUS UNDERSTANDINGS:

- identify, write, and compare place value of decimal numbers through thousandths.

GETTING STARTED:

Materials:
- place value chart through thousandths

Display the place value chart.
e.g.,

Tens	Ones	Tenths	Hundredths	Thousandths

Write a word problem about the digits in a decimal number (thousandths) on the chalkboard.
e.g.,

> What's my number? In my tens place there is a 2. The digit in my hundredths place is 3 times my tens. The digit in my thousandths column is the sum of my tens digit and my hundredths digit. My ones digit and my tenths digit are the same and their sum also equals my thousandths digit.

Ask a student to read the problem aloud. Say, "What must we find? (We must find the five digits of the number.) Where should we start?" (Since we are given the tens digit, we should start there.)

Continue this procedure until all five digits are discovered. Say, "What is the number?" (The number is twenty-four and four hundred sixty-eight thousandths.) Ask a student to write the standard form of the number on the chalkboard, e.g., 24.468.

Repeat this procedure with similar problems.

6.07.01

6.07.01.01A identify the place value of decimal places through ten-thousandths

STRATEGY 1:

> **Materials:**
> - place value chart

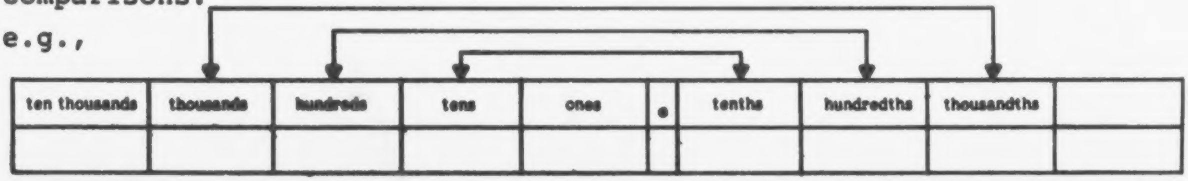
Display a 9-column place value chart labeled from the ten thousands place through the thousandths place, leaving the ten-thousandths place unlabeled.

e.g.,

ten thousands	thousands	hundreds	tens	ones	•	tenths	hundredths	thousandths	

Say, "We know that, with the exception of the ones place, the columns on both sides of the decimal point (whole numbers on the left and decimal fractions on the right) have names which are quite similar sounding. Because of this, we must be extremely careful when speaking or listening to their names. What columns are to the left and the right of ones place?" (the tens column on the left and the tenths column on the right) A digit in the tens place has ten times the value it would have in the ones place. The same digit in the tenths place would have one tenth its value in the ones place." Repeat this questioning for hundreds and hundredths place and for thousands and thousandths places. Demonstrate by drawing arrows stressing these comparisons.

e.g.,



Say, "Look at the first and last columns on the chart. To the left of thousands is ten thousands place. What must the column be to the right of thousandths?" (ten-thousandths place) Label it and ask the students to write this completed chart in their notebooks. Draw an arrow from ten thousands to ten-thousandths place as we did for the others.

STRATEGY 1: (cont'd)

arrow from ten thousands to ten-thousandths place as we did for the others.

Write the numeral 53,896.1274 on the chart and ask students to identify the place value of each digit. Say, "What digits are in the tens place and the tenths place? (The 9 is in the tens place and the 1 is in the tenths place.) What are their values? (The 9 represents ninety, or 9 tens, and the 1 means one tenth.) What are the values of the digits in the hundreds place and the hundredths place? (The 8 in hundreds place means 800 while the 2 in hundredths place is $\frac{2}{100}$.) What are the values of the digits in the thousands and the thousandths place? (The 3 means 3000 and the 7 means $\frac{7}{1000}$.) What does the 5 represent? (5 ten thousands or 50,000) What is the value of the digit in ten-thousandths place? (The 4 must be 4 ten-thousandths or $\frac{4}{10,000}$.) How would we read this entire number? (It is read fifty-three thousand, eight hundred ninety-six and one thousand, two hundred seventy-four ten-thousandths.) What does the word and represent?" (The decimal point or the end of the part of the numeral which is one or larger.)

Display and label the following decimal fractions:

A	B	C	D
0.0402	0.3060	0.0007	0.0048

Say, "How are these numbers different from the numbers we first worked with? (The only digit to the left of the decimal point is zero. They have no whole number parts.) In sample A, what is the value of the non-zero digits? (The 4 is $\frac{4}{100}$ and the 2 is $\frac{2}{10,000}$). When we read a decimal which contains a digit in ten-thousandths place, we must express this place in its name. How can we read this decimal number?" (402 ten-thousandths) If necessary, demonstrate that $\frac{4}{100} + \frac{2}{10,000} = \frac{402}{10,000}$.

Repeat this procedure for the other samples. For sample B, elicit that we can express 0.3060 as 3060 ten-thousandths or we can express it as 306 thousandths by eliminating the zero in ten-thousandths place.

6.07.01

6.07.01.02A write tenths, hundredths, thousandths, and ten-thousandths as decimals

STRATEGY 1:

Materials:
- place value chart

Display a place value chart from ones place through ten-thousandths place.

e.g.,

ones	•	tenths	hundredths	thousandths	ten-thousandths
	■				

Say, "Any common fraction which contains a denominator which is a power of ten such as 10, 100, 1000 or 10,000 can also be written as a decimal fraction. I will ask you to write numerals on the place value chart. I will give you a common fraction and you must read it and write its equivalent decimal on the chart in its appropriate column." Write the decimal $\frac{8}{10}$ on the chalkboard to the left of the chart. Say, "How do we read this fraction? (8 tenths) How should we write it on the place value chart?" (We should write an 8 in the tenths column and a zero in the ones column.) Ask a student to write the decimal numeral on the chart.

e.g.,

	ones	•	tenths	hundredths	thousandths	ten-thousandths
$\frac{8}{10}$ =	0	■	8			

Write the equivalence $\frac{8}{10} = 0.8$. Say, "Placing a zero in the ones column when the number is smaller than one whole alerts the reader that the number has a decimal point and helps him avoid misreading it." Ask the students to write it into their notebooks. Write the fraction $\frac{38}{100}$ on the chalkboard. Say, "How do we read and write this fraction?" (This is the fraction $\frac{38}{100}$. To write it on the chart we write the 8 in the hundredths column and the 3 in the tenths column.) Ask a student to demonstrate this on the chart.

STRATEGY 1: (cont'd)

e.g.,

$\frac{38}{100}$ =

ones	•	tenths	hundredths	thousandths	ten-thousandths
0	■	3	8		

Write the equation $\frac{38}{100}$ = 0.38 on the chalkboard and ask the students to write it in their notebooks. Say, "How would we write $\frac{7}{10,000}$ as a decimal?" (We would write a 7 in the ten-thousandths column and zeros in all the columns left of it up to and including the ones place.) Ask a student to write this on the chart. Say, "How do we know where to write the digits in decimal form? (The last digit always goes in the column which names the decimal.) If we write $\frac{5}{100}$, the 5 goes in the hundredths column. If we write $\frac{562}{1000}$, the 2 goes in the thousandths column, the 6 in the hundredths column, the 5 in the tenths column. If we write $\frac{306}{10,000}$, the 6 goes in the ten-thousandths column, the 0 in the thousandths column, and the 3 in the hundredths place. Another 0 is needed as a place holder in the tenths place. Also, if there is no whole number, we indicate this by writing a zero in the ones column." Record this information and ask the students to write it in their notebooks.

e.g., $\frac{306}{10,000}$ = 0.0306

Repeat this procedure using other examples.

6.07.01.03A compare decimals using the symbols >, <, or =

STRATEGY 1:

Materials:
- place value chart

Display the place value chart. Write two numerals on the chalkboard.

e.g.,

0.47 0.048

Write the symbols >, <, = on the chalkboard. Say, "We can use the place value chart and these symbols to compare decimal numerals. First, we will write the two numerals on the chart." Ask a student to

STRATEGY 1: (cont'd)

write the numerals on the chart while the other students write them in their notebooks.

e.g.,

ones	•	tenths	hundredths	thousandths	ten-thousandths
0		4	7		
0		0	4	8	

Say, "When we compare numerals, we may look to see which is greater or which is less. We start on the left and look for the greater or lesser digit one column at a time. Let's try it with this pair of numerals. What do we see in the ones place? (In each numeral the digit is zero.) The next column is tenths. What do we see there? (The top numeral has 4 tenths, the bottom numeral has 0 tenths.) What does this tell us? (The top numeral, 0.47, is greater.) Look at the order in which the numerals are written. Which symbol would we use to compare 0.47 and 0.048 if 0.47 were written on the left?" (We would use the 'is greater than' symbol.) Ask a student to demonstrate on the chalkboard while the other students write it in their notebooks.

e.g., 0.47 > 0.048 47 hundredths is greater than 48 thousandths

Reverse the order of the digits and ask another student to write the correct inequality.

e.g., 0.048 < 0.47 48 thousandths is less than 47 hundredths

Say, "Some people prefer to use a slightly different approach to this task. They begin by writing zeros in the empty place or places at the right of the numerals with fewer decimal places, until both decimals are the same length. Now they have given the two numerals like denominators. Here's how it would look with the pair we just compared." Write the following on the chalkboard:

0.47 = 0.470
0.048 = 0.048

Say, "This makes it obvious that the first number, 470 thousandths, is greater than the second, 48 thousandths. You may use either method to compare decimals."

Repeat this procedure with other pairs of numbers.

STRATEGY 1: (cont'd)

Display a series of number sentences with the correct relation symbol missing.
e.g.,

0.4   0.16   0.4   0.400   0.71   1.7

Say, "What is missing from each number sentence? (The relation symbol in the middle is missing.) Look at the first two numbers. Are they equal? (No, they have different values.) What is this type of sentence called? (It is an inequality.) Which symbol should we use for the first sentence? Explain." (Both numbers have a 0 in the ones place. The 0.4 has a greater digit in the tenths place, so we would say 0.4 is greater than 0.16.) Ask a student to fill in the correct symbol on the chalkboard while the others write it in their notebooks.
e.g.,

0.4 > 0.16   4 tenths is greater than 16 hundredths

Say, "What symbol should we use to compare the second set of numerals? Explain." (They both have 0 in the ones place and 4 in the tenths place. 0.4 has no other digits. 0.400 has more digits but they are both zeros. So the two numbers are equal in value.) Ask a student to show the equality on the chalkboard while the others work in their notebooks.
e.g.,

0.4 = 0.400   4 tenths is equal to 400 thousandths

Repeat this procedure with the third example as well as with other number sentences, where a relation symbol (<, >, =) is necessary to compare the decimals.

6.07.01.04A order a set of decimals from least to greatest

STRATEGY 1:

Materials:
- place value chart

Display a place value chart through ten-thousandths and the following problem:

STRATEGY 1: (cont'd)

A botanist studied the growth rate of three plants over a period of one week. Plant A grew 0.53 in., plant B grew 0.2991 in., and plant C grew 0.674 in.

Write the three numerals on the chalkboard.
e.g.,

Plant A	Plant B	Plant C
0.53 in.	0.2991 in.	0.674 in.

Say, "How can we find which plant grew the most?...the least?" (We can use the place value chart to help us compare the numbers. Then we can write them in size order. We will then know which amount is the greatest and which is the least.) Ask a student to write the numbers on the place value chart.
e.g.,

ones	•	tenths	hundredths	thousandths	ten-thousandths
0	•	5	3		
0	•	2	9	9	1
0	•	6	7	4	

Say, "When we compare decimal numerals, we line them up so that the decimal points are directly under each other. This puts digits with the same place value in the same column, making them easier to compare. We want to order these numerals from least to greatest. Where do we start comparing? (We start all the way on the left with the place which has the greatest value. All three numbers have a zero in the ones column.) Next we look at the tenths. What do you see? (The first numeral has a digit 5, the next a 2, and the last a 6 in the tenths place.) Which of these digits is least? (The 2 is least. This means that 0.2991 is the least of the three numerals in value.) Which digit is next in value? (the 5) This means that numeral 0.53 is next in value. Which numeral is greatest?" (0.674 is greatest in value.) Ask a student to write the numbers in that order on the chalkboard;
e.g.,

0.2991   0.53   0.674

STRATEGY 1: (cont'd)

Say, "Does it matter that 0.2991 has the greatest number of digits? (No, the numeral with the highest value digit in the left-most place is the numeral with the greatest total value. What symbol will we use between the numbers to compare them?" (Since 0.2991 is less than 0.53, we will write an 'is less than' symbol between 0.2991 and 0.53. Since 0.53 is less than 0.674, we will write an 'is less than' symbol between these two numbers.)

Ask a student to write these symbols on the chalkboard while the others write them in their notebooks.

e.g.,

$$0.2991 < 0.53 < 0.674$$

Say, "Which plant, A, B, or C, grew the most? (plant C) Which grew the least?" (plant B)

Repeat this procedure with other numbers.

Caution the students that whenever more than two numerals are being compared in a single sentence they must be arranged in size order (either ascending or descending). This is true for all kinds of numerals, not just decimals. Sentences such as $7 > 3 < 5$ and $5 > 2 < 9$ are useless, because there is no way to tell how the outside numerals compare.

Write three decimals on the chalkboard and ask the students to write them into their notebooks.

e.g.,

0.7 0.1939 0.194

Say, "We want to order these numerals from least to greatest. Often, when comparing fractions, we find it useful to rewrite them so that they have the same denominator. What would be the least common denominator for these three numerals? (ten-thousandths) Converting a decimal numeral to an equivalent with a greater power-of-ten denominator is easy. All we need to do is show the zeros in the places to the right of the given numeral. What do we know about writing zeros in the decimal places to the right of a decimal numeral?" (We can show as many zeros as we want in the places to the right of a decimal. This does not change the value of the numeral.)

Ask a student to write as many zeros as necessary at the right of each decimal to make them all of equal length.

e.g.,

0.7000 0.1939 0.1940

Say, "Now that they all extend to the same place, the ten-thousandths place, they are easy to compare." Ask a student to order these numerals from least to greatest at the chalkboard while the others do so in their notebooks.

e.g.,

0.1939 0.1940 0.7000

Ask another student to remove the extra zeros so we return to the original numerals. Say, "What symbols shall we write?" (We will write an 'is less than' symbol between each pair.) Ask a student to fill in the 'is less than' symbol.

e.g.,

$0.1939 < 0.194 < 0.7$

Say, "Does it matter how many digits the decimal has?" (No, because we can always write enough zeros to the right to make them all end at the same place, without changing their values.)

Repeat this procedure with other sets of decimals.

P.O. 6.07.01.01-.02

NAME: ____________________ **CLASS:** ________ **DATE:** ____________

Follow the directions below.

1. Write the place value of each underlined digit.

0.7482 hundredths
36.5109 ____________
3.6750 ____________
17.0863 ____________
4.7811 ____________
26.0005 ____________
2.3596 ____________
90.1234 ____________

2. Write the value of each underlined digit, both as a common fraction and as a decimal fraction equivalent.

6.1029 $\frac{2}{1000} = 0.002$
1.2345 ____________
15.0870 ____________
0.8901 ____________
0.7583 ____________
0.0009 ____________
3.5467 ____________
11.0236 ____________

3. What's my number?
I have a 6 in my tens column.
My thousandths digit is 4 less than my tens.
My ten-thousandths digit equals my tens plus my thousandths.
My ones and hundredths digits are the same and have a sum of 10.
If you subtract my ones digit from my hundredths digit, the difference is my tenths digit.

____ ____. ____ ____ ____ ____

Do you remember?

Reduce to lowest terms:

$\frac{6}{12} = \frac{1}{2}$ $\frac{4}{12} = \frac{\square}{\square}$ $\frac{9}{12} = \frac{\square}{\square}$ $\frac{2}{12} = \frac{\square}{\square}$

$\frac{8}{12} = \frac{\square}{\square}$ $\frac{3}{12} = \frac{\square}{\square}$ $\frac{10}{12} = \frac{\square}{\square}$ $\frac{12}{12} = \frac{\square}{\square}$

P.O. 6.07.01.02

NAME: ____________ **CLASS:** ______ **DATE:** ____________

Follow the directions below.

1. Write the decimal number for each.

five tenths ______
nine hundredths ______
seven thousandths ______
three ten-thousandths ______
fifteen ten-thousandths ______
thirty-five hundredths ______
eleven ten-thousandths ______
ninety-nine thousandths ______

2. Rewrite each fraction as a decimal.

$\frac{5}{100}$ = ______
$\frac{7}{10}$ = ______
$\frac{6}{1000}$ = ______
$\frac{125}{10,000}$ = ______
$\frac{6}{10,000}$ = ______
$\frac{25}{100}$ = ______
$\frac{95}{1000}$ = ______
$\frac{6}{100}$ = ______
$\frac{3102}{10,000}$ = ______
$\frac{6}{10}$ = ______
$\frac{99}{100}$ = ______
$\frac{32}{10,000}$ = ______

3. Rewrite each mixed numeral as a decimal number.

$3\frac{7}{10}$ = ______
$16\frac{9}{100}$ = ______
$33\frac{17}{1000}$ = ______
$2\frac{42}{10,000}$ = ______
$5\frac{4}{1000}$ = ______
$4\frac{15}{100}$ = ______
$48\frac{89}{10,000}$ = ______
$1\frac{9}{10,000}$ = ______
$10\frac{1}{10,000}$ = ______
$\frac{12}{100}$ = ______
$2\frac{11}{1000}$ = ______
$5\frac{8}{10}$ = ______

CHALLENGE!

Write the missing decimal number using the following clues:

A. The tenths is twice the tens.
B. The thousandths is three less than the tenths.
C. The hundredths is the sum of the tens and tenths.
D. The ten-thousandths and ones place are each the difference of the tenths and thousandths.

2 ____ . ____ ____ ____ ____

P.O. 6.07.01.03-.04

NAME: ____________________ **CLASS:** ______ **DATE:** ______________

Follow the directions below.

1. Fill in the proper symbol (>, <, or =) in each circle below to make correct number sentences.

1.4375 ○ 1.4199

6.001 ○ 5.9999

0.3924 ○ 0.393

1.0427 ○ 1.0428

2.3 ○ 2.3000

5.01 ○ 5.10

2.2 ○ 02.21

1.65 ○ 1.3

12.4 ○ 12.400

0.7 ○ 0.75

1.776 ○ 17.76

3.020 ○ 3.02

2. Write a decimal number on each line that will make the sentence true.

0.75 > __________

0.02 < __________

__________ = 3.60

2.0005 > __________

3.154 < __________

1.073 = __________

6.2517 > __________

6.2517 < __________

__________ < 6.02

5.62 > __________

3. Order each set of numbers from least to greatest.

3.6, 3.085, 2.9987 __________ < __________ < __________

0.8, 0.709, 0.7832 __________ < __________ < __________

2.75, 8.7097, 8.699 __________ < __________ < __________

0.4, 0.4001, 0.981 __________ < __________ < __________

6.07.02 ROUND DECIMALS

(1 lesson)

PERFORMANCE OBJECTIVES

The student will be able to...

6.07.02.01A round decimals to the nearest tenth; e.g.,

0.26 rounds to 0.3
365.318 rounds to 365.3

6.07.02.02A round decimals to the nearest hundredth. e.g.,

0.428 rounds to 0.43
23.992 rounds to 23.99
4.997 rounds to 5.00

PROBLEM EXPLORATION SKILLS:

- to generalize
- to recognize a convention

VOCABULARY

estimate
round

PREVIOUS UNDERSTANDINGS:

- rounding whole numbers
- reading and ordering decimals through ten-thousandths place

---

GETTING STARTED:

Materials:
- chart shown below

Display the following chart:

Numeral	Nearest Ten	Nearest Hundred	Nearest Thousand
6709			
5398			
2655			

Say, "Which digit are we concerned with when rounding to the nearest ten? (The digit we are most concerned with is in the tens column.) How do we decide whether to round up or down?" (We look at the column to its right, the ones. If the digit is 4 or less, the tens digit remains the same and the ones digit becomes 0. If the ones digit is 5 or more, we round to the higher ten and the ones digit becomes 0.) Show an example of this on the chalkboard.

e.g.,
36 rounded to the nearest ten is 40
73 rounded to the nearest ten is 70

Say, "How do we round to the nearest hundred?... nearest thousand?" (We look at the digit to the right of the place to which we are rounding and round accordingly.) Ask the students to complete the chart.

e.g.,

Numeral	Nearest Ten	Nearest Hundred	Nearest Thousand
6709	6710	6700	7000
5398	5400	5400	5000
2655	2660	2700	3000

Repeat this procedure using other numerals.

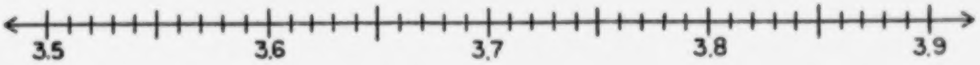
6.07.02.01A round decimals to the nearest tenth

STRATEGY 1:

Materials:
- number line

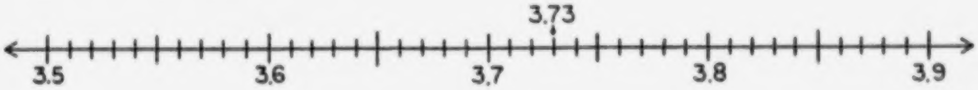
Prepare and display a number line showing numbers divided into tenths. Say, "Here is a number line. The labeled points show tenths, from 3.5 to 3.9".

e.g.,



Say, "How much does each marked division represent on this number line?" (Since there are ten divisions to each tenth, each represents a tenth of a tenth, or one hundredth.) Write the numeral 3.73 on the chalkboard. Say, "Where would we find the numeral 3.73 on the number line?" (It would be between 3.7 and 3.8.) Ask a student to find the number 3.73 on the number line;

e.g.,



Say, "When we round 3.73 to the nearest tenth, we must choose between 3.7 and 3.8. To which of those tenths is 3.73 closer?" (It's closer to 3.7.) Write this fact on the chalkboard.

e.g.,

3.73 rounded to the nearest tenth is 3.7

Write 3.5627 on the chalkboard. Say, "Which two tenths does 3.5627 come between? (It is between 3.5 and 3.6.) Which tenth is it closer to on the number line?" (It is closer to 3.6.)

Ask a student to write this fact on the chalkboard.

e.g.,

3.5627 rounded to the nearest tenth is 3.6

Repeat this procedure with several other examples.

STRATEGY 2:

Write the following on the chalkboard:

Number	Rounded to the Nearest Tenth
35.16	
4.3852	
0.428	
0.9275	

Say, "When we learned to round whole numbers to the nearest ten, where did we have to look in order to find out whether to round up or down? (We had to look one column to the right, at the ones place.) What did we do then? (If the ones digit was 1,2,3, or 4, we left the tens digit alone and changed the ones digit to zero; that is, we rounded down. If the ones digit was 5,6,7,8, or 9, we rounded up to the next higher ten and wrote a zero in the ones place.) We always round the same way. If we want to round to a particular place, we look at the digit to the right. If we want to round a decimal to the nearest whole number, where should we look? (We should look at the digit in the tenths place.) If we want to round a decimal to the nearest tenth, where should we look? (We should look at the digit in the hundredths place.) Does it matter how many digits there are in the number?" (No, we are only concerned with the place to which we are rounding and the digit to its right.) Demonstrate this on the chalkboard.

e.g.,

0.3872 rounded to the nearest tenth is 0.4

Say, "The digit in the hundredths place is more than 4 so we round up, making the digit in the tenths place 4 instead of 3. When we round whole numbers, everything to the right of the rounding column becomes a zero. We must do that with decimal numbers too, although we don't need to show the zeros. We just omit the places to the right of the place we are rounding to."

Ask the students to complete the chart displayed on the chalkboard.

STRATEGY 2: (cont'd)

e.g.,

Number	Rounded to the Nearest Tenth
35.16	35.2
4.3852	4.4
0.428	0.4
0.9275	0.9

6.07.02.02A round decimals to the nearest hundredth

STRATEGY 1:

Adapt Strategy 2, P.O. 6.07.02.01A.

STRATEGY 2:

Prepare and display the following word problem on the chalkboard:

The fraction $\frac{5}{8}$ written as a decimal is 0.625.
What is this decimal rounded to the nearest hundredth?

Say, "What are we asked to do? (Round 0.625 to the nearest hundredth.) How can we solve the problem? (We can write the entire decimal and then look at the digit to the right of the hundredths place, that is, the digit in the thousandths column.) What digit is there? (There is a 5 in the thousandths column.) What should we do next?" (With a 5 in the thousandths place, we round up, changing the hundredths digit from 2 to 3.) Ask a student to write the solution on the chalkboard as the others write it in their notebooks.

e.g.,

0.625 rounded to the nearest hundredth is 0.63

Repeat this procedure with similar problems.

e.g.,

"What is 0.6666 rounded to the nearest hundredth?" (0.67)

P.O. 6.07.02.01-.02

NAME: ____________________ CLASS: ________ DATE: ____________

A. Round each decimal to the nearest tenth.

5.650 ________
2.87 ________
16.095 ________
11.749 ________
3.138 ________
0.066 ________

3.7751 ________
8.9108 ________
6.285 ________
3.71 ________
2.88 ________
3.69 ________

6.07 ________
1.108 ________
5.37 ________
5.42 ________
2.04 ________
1.98 ________

B. Round each decimal to the nearest hundredth.

2.569 ________
3.714 ________
1.395 ________
6.272 ________
8.065 ________
9.991 ________

0.250 ________
0.259 ________
6.586 ________
8.9825 ________
4.3076 ________
16.1508 ________

14.7179 ________
0.201 ________
1.666 ________
2.135 ________
3.531 ________
7.7777 ________

C. Write a numeral that, when rounded to the nearest tenth or hundredth, would be the given number. The first one is done for you.

4.669 rounded to the nearest hundredth is 4.67
__________ rounded to the nearest hundredth is 3.75
__________ rounded to the nearest hundredth is 4.59
__________ rounded to the nearest tenth is 2.1
__________ rounded to the nearest tenth is 8.0
__________ rounded to the nearest tenth is 16.8

6.14.06 REVIEW: METRIC MEASUREMENT

(1 lesson/ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

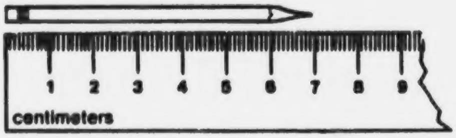
6.14.06.01 recognize and explain the meaning of metric prefixes and their abbreviations;

e.g.,

milli - one thousandth
centi - one hundredth
kilo - one thousand
mm - millimeter
cm - centimeter
m - meter
km - kilometer

6.14.06.02 estimate and measure lengths using millimeters, centimeters, and meters;

e.g.,



The pencil measures 7 centimeters.

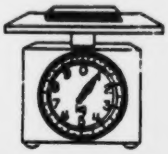
6.14.06.03 estimate and measure capacity using milliliters, deciliters, and liters;

e.g.,

The glass holds 450 mL of liquid.

6.14.06.04 estimate and measure weight using grams and kilograms;

e.g.,



The book weighs 1 kg.

6.14.06.05 create and solve word problems related to skills learned in this module.

e.g.,

A man took a long hike, walking 8000 meters. How many kilometers did he walk?

PROBLEM EXPLORATION SKILLS:

- to use analogy
- to generalize
- to keep good records

VOCABULARY

milli
centi
kilo
deci

PREVIOUS UNDERSTANDINGS:

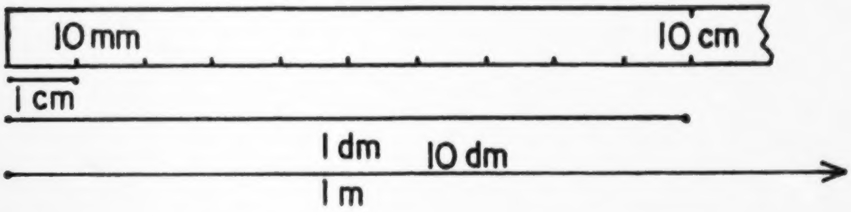
- measuring with metric units

GETTING STARTED:

Materials:
- meter stick

Say, "What is metrics? (It is a system of measurement.) Why must we learn the metric system? (The United States uses the customary or English system for most measurements, but scientists and most other countries perform measurements using metric units.) How can the metric system be used? (to measure length, capacity, and weight/mass) Which basic units of measure does the metric system use? (The meter is used to measure length, the liter to measure capacity, and the gram to measure weight/mass.) Are there other units of measure in the metric system? Explain. (Yes, since it is a decimal system, we can increase or decrease a basic unit by multiplying or dividing by ten and its powers.) Describe the relationships among meters, decimeters, centimeters, and millimeters." (A meter is made up of ten decimeters. A decimeter is made up of ten centimeters. A centimeter is made up of ten millimeters.) Ask a student to demonstrate this using a meter stick.

e.g.,



Say, "How do the units compare?" (Each unit is ten times larger than the next smaller unit.) Ask students to record this information in their notebooks.

e.g.,

10 mm = 1 cm
10 cm = 1 dm
10 dm = 1 m

6.14.06

6.14.06.01 recognize and explain the meaning of metric prefixes and their abbreviations

STRATEGY 1:

Materials:
- place value chart shown below

Display the following place value chart:

Unit							
Symbol/Abbreviation							
	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
	1000	100	10	1	0.1	0.01	0.001

Say, "Look at the place value chart on the chalkboard. How are the columns related? (As we move from right to left each number is multiplied by ten.) What happens as we move from left to right? (The number to the right is one-tenth the previous number.) How does our place value system compare to the metric system? (They are both base ten systems.) How can we complete the headings on the chart? (We can use prefixes and the root word to recognize and explain the different units of metric measurement.) Which unit will we use as our basic unit for linear measurement in the metric system?" (The meter is the basic unit for linear measurement.) Ask a student to come to the chalkboard to write the word meter and its abbreviation in the appropriate column.

e.g.,

Unit				Meter			
Symbol/Abbreviation				m			
	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
	1000	100	10	1	0.1	0.01	0.001

Continue by asking a student to come to the chalkboard and fill in the columns to the left of the base word, meter.

e.g.,

Unit	Kilometer	Hectometer	Dekameter	Meter			
Symbol/Abbreviation	km	hm	dam	m			
	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
	1000	100	10	1	0.1	0.01	0.001

STRATEGY 1: (cont'd)

Say, "How will each of these words help us to recognize and explain the metric prefixes and their abbreviations?" (As we move to the left we know the number is ten times greater than the previous column/place.) How will the prefixes help us? (The prefix deka means 10, hecto means 100 and kilo means 1000.) Do you recognize their derivation or roots?" (Yes, they are Greek prefixes and they extend the base unit, the meter.) Ask a student to come to the chalkboard to complete the columns to the right of the base word, meter.

e.g.,

Unit	Kilometer	Hectometer	Dekameter	Meter	Decimeter	Centimeter	Millimeter
Symbol/Abbreviation	km	hm	dam	m			
	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
	1000	100	10	1	0.1	0.01	0.001

Say, "How does each of these words help us to recognize and explain the metric prefixes and their abbreviations?" (As we move to the right of the base word, meter, the number named is $\frac{1}{10}$ of the previous place.) How do the prefixes help us? (The prefix deci means $\frac{1}{10}$, centi means $\frac{1}{100}$, and milli means $\frac{1}{1000}$.) Do you recognize their derivation or roots?" (Yes, they are Latin prefixes and they are subdivisions of the base unit, meter.) Complete the chart by writing the symbols or abbreviations dm, cm, and mm.

e.g.,

Unit	Kilometer	Hectometer	Dekameter	Meter	Decimeter	Centimeter	Millimeter
Symbol/Abbreviation	km	hm	dam	m	dm	cm	mm
	Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
	1000	100	10	1	0.1	0.01	0.001

Repeat this procedure using units of weight and capacity.

6.14.06

6.14.06.02 estimate and measure lengths using millimeters, centimeters, and meters

STRATEGY 1:

Materials:

- rulers calibrated in millimeters and centimeters
- meter sticks

Display the following chart:

Object/Item To Be Measured	Estimate	Actual Measurement	Difference

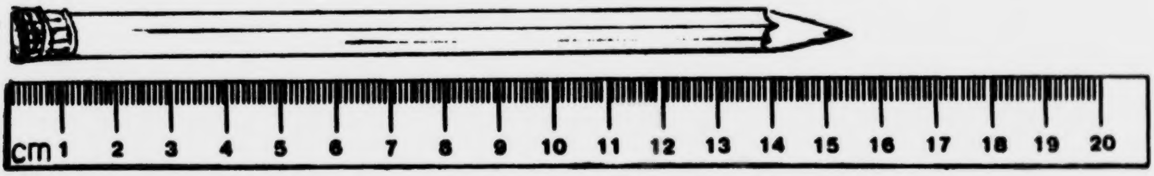
Say, "How can we improve our measurement skills?" (We must practice to become expert at measurement.) Display a pencil. Say, "Which unit of measure can be used to measure a pencil? (centimeters or millimeters) Which unit is more appropriate? (centimeters) What is a reasonable estimate of the length of the pencil?" (18 cm) Record this estimate on the chart.

e.g.,

Object/Item To Be Measured	Estimate	Actual Measurement	Difference
pencil	18 cm		

Ask a student to measure the actual length of the pencil.

e.g.,



Say, "Which unit markings are shown on the ruler? (The longer markings represent centimeters and the shorter markings represent millimeters.) What is the length of the pencil? (Is is slightly more than 15 centimeters in length.) Let's round our measurement to 15 cm." Ask the students to use the chart to record the length and the difference between the estimate and the actual measurement. The other students will write the information in their notebooks.

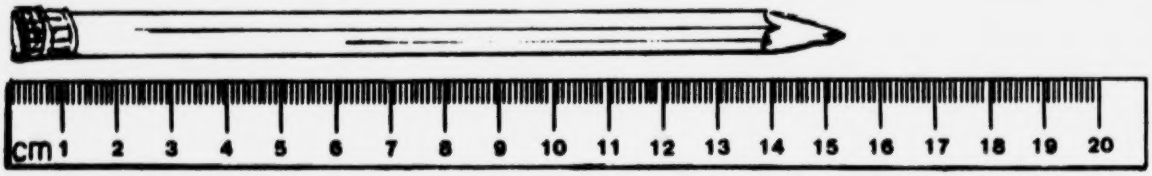
STRATEGY 1: (cont'd)

e.g.,

Object/Item To Be Measured	Estimate	Actual Measurement	Difference
pencil	18 cm	15 cm	3 cm

Say, "Could we have made a more accurate measurement?" (Yes, since the length of the pencil is between 15 and 16 centimeters, we could have selected a more precise unit of measurement, the millimeter.) Ask a student to come to the chalkboard to find the length of the pencil in millimeters.

e.g.,



Say, "What is the more accurate measurement? (154 millimeters)

Repeat this procedure by asking students to estimate, measure, record and find the difference of items/objects around the classroom.

Note: Since all measurement is approximate, we are measuring (rounding) to the nearest specified unit.

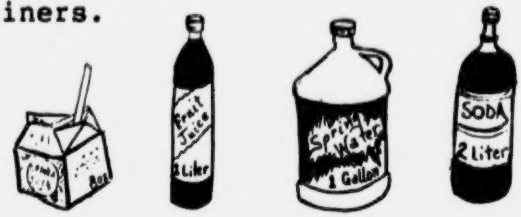
6.14.06.03 estimate and measure capacity using milliliters, deciliters, and liters

STRATEGY 1:

Materials:
- containers and cartons

Display various containers which provide information as to the capacity of the containers.

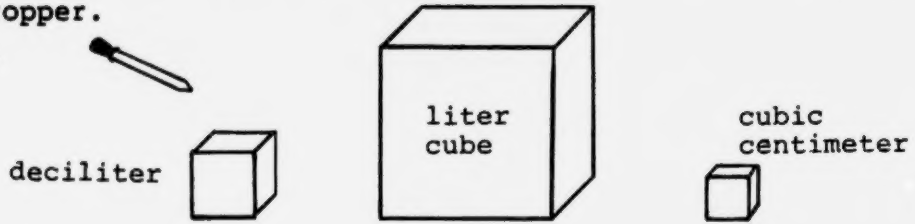
e.g.,



STRATEGY 1: (cont'd)

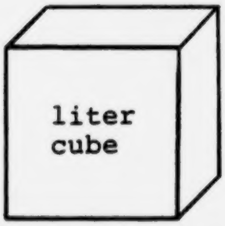
Ask a student to read the capacity on each label. Say, "What does the capacity tell us about the container?" (It tells us how much liquid a container can hold.) Write the following on the chalkboard: milli, deci, kilo. Say, "When have we used these prefixes? (We have used them in linear metric measurement.) Today we are going to use these prefixes for measuring the capacity of containers." Display a cubic centimeter, a liter cube (a metric unit of volume equal to a cubic decimeter), a container whose capacity is a deciliter (100 mL), and an eyedropper.

e.g.,



Say, "What is the standard metric unit of length? (the meter) What is the standard unit of capacity?" (the liter) Display the liter cube (cubic decimeter).

e.g.,



Say, "The liter cube has a capacity of one liter. If we could fill the cubic centimeter with liquid and pour it into the liter cube, we would need 1000 pourings to fill the liter cube. Therefore, the cubic centimeter holds $\frac{1}{1000}$ of a liter. Which metric prefix means $\frac{1}{1000}$? (milli) Then we can say that a cubic centimeter has a capacity of one milliliter. What is the symbol for a liter? (The symbol for a liter is 'L') Why do we use a capital L?" (The lower case might be mistaken for the number 1.) Write the symbols/abbreviations for a milliliter, a deciliter, and a liter.

e.g.,

milliliter - mL deciliter - dL liter - L

STRATEGY 1: (cont'd)

Display a two-liter soda container.

e.g.,

Fill the 2-liter container with colored liquid. Select the deciliter (dL) container and say, "What is another way to express the capacity of this container? (100 milliliters) How do the metric prefixes help us? (The prefix in the metric system gives us the clue. It tells the same number value no matter which unit is used.) Using metric prefixes, can you explain the relationship between the milliliter and the deciliter? (Yes, the prefix milli in the word milliliter means one-thousandth. Therefore, a milliliter is one-thousandth of the basic unit, the liter. The prefix deci in the word deciliter means one-tenth, therefore, a deciliter is one-tenth of the basic unit, the liter.) How is measuring capacity similar to measuring length?" (The prefixes are helpful for understanding a specific unit as it relates to the basic unit and it is important to select an appropriate unit of measure.) Write the following chart on the chalkboard:

Container	Unit Selected	Estimate	Actual Amount	Difference

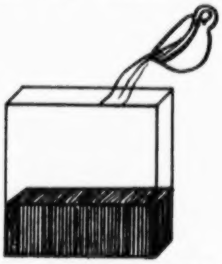
Display the various cups and containers. Ask a student to select the 1-cup container. Say, "Let's find the capacity of this container. Which unit of measure will we use? (milliliters) What is a reasonable estimate of the cup's capacity?" (300 milliliters) Record the unit selected (mL) and the estimate (300 mL) on the chart.

e.g.,

Container	Unit Selected	Estimate	Actual Amount	Difference
	milliliters	300 mL		

STRATEGY 1: (cont'd)

Say, "How can we find the capacity of the cup?" (Fill the cup with colored liquid and pour it into the liter cube.) Demonstrate.
e.g.,



Say, "What's the actual capacity?" (350 milliliters) Record the actual capacity and difference on the chart.
e.g.,

Container	Unit Selected	Estimate	Actual Amount	Difference
	milliliters	300 mL	350 mL	50 mL

Repeat this procedure using other containers.

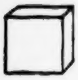
Note: Determine the students' ability to recognize that the shape of a container has no bearing on its capacity by using different containers which have the same capacity.

6.14.06.04 estimate and measure weight using grams and kilograms

STRATEGY 1:

Materials:
- metric scale and/or a balance scale
- metric weights

Say, "What is the basic unit of weight/mass in the metric system?" (the gram) Display the following models of a gram:



cubic centimeter

paper clip

1 g

sweetener packet

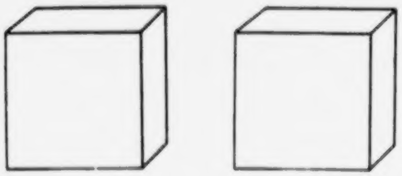
1 g

Note: A cubic cm holds 1 mL of water and 1 mL of water weighs 1 g.

STRATEGY 1: (cont'd)

Say, "How can we use information about metric prefixes to help us understand the kilogram?" (The prefix kilo means a thousand. Therefore, a kilogram is equal to one thousand grams.) Display metric weights and models of a kilogram.

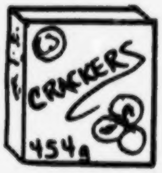
e.g.,

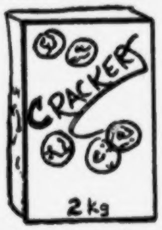


Note: The cubic decimeter holds 1000 cubic centimeters. It also holds 1 liter of water, which has a mass of 1 kilogram.

Display various items labeled in metric weight.

e.g.,





Ask a student to read each label aloud and record the information on the chalkboard.

e.g.,

Object/Item	Weight
Crackers	454 grams or 454 g
Crackers (Family Size)	2 kilograms or 2 kg

Write the following on the chalkboard:

Object/Item To Be Weighed	Unit Selected	Estimate	Actual Amount	Difference

Display objects to be weighed, e.g., book, stapler, crayon box. Ask a student to select an object to be weighed. Say, "Which is an appropriate unit of measure? (gram) What is a reasonable estimate of its weight?" (Answers may vary) Record the information on the chart.

STRATEGY 1: (cont'd)

e.g.,

Object/Item To Be Weighed	Unit Selected	Estimate	Actual Amount	Difference
crayon box	gram	350 g		

Ask a student to use the metric scale and/or balance scale to weigh the object, record its weight, and find the difference between the actual weight and the estimated weight.

e.g.,

Object/Item To Be Weighed	Unit Selected	Estimate	Actual Amount	Difference
crayon box	gram	350 g	324 g	26 g

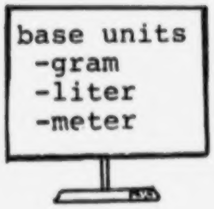
Repeat this procedure by selecting the appropriate metric unit, estimating, weighing, finding the difference, and recording the data for each item selected.

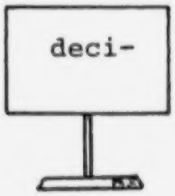
6.14.06.05 create and solve word problems related to skills learned in this module

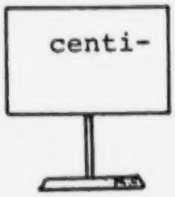
STRATEGY 1:

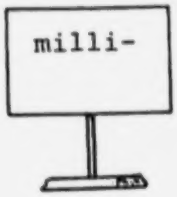
Materials
- signs
- measurement instruments

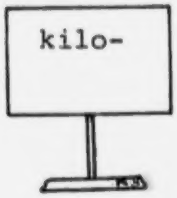
Prepare and display the following signs:







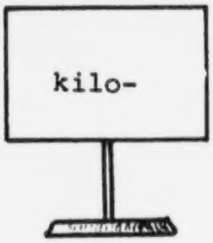




STRATEGY 1: (cont'd)

Separate the students into five groups. Say, "Today we are going to create a measurement game similar to 'JEOPARDY'. Each group is to write a problem with its solution using the sign at your table as the basis for the question.

e.g.,



Question: How does the prefix kilo help us to determine the length of the specific unit, kilometer?

Solution: The prefix kilo tells us that the length is a thousand times more than the base unit, meter. Therefore, a kilometer is a thousand meters.

Ask students to include questions which require the use of measuring instruments such as rulers, metric scales, balance scales, and liter cubes. Say, "Which topics may be used for developing questions and solutions?" (linear measurement, mass/weight, and capacity) After each group has written a question and a solution, rotate the signs.

Repeat this procedure by asking students to solve the class questions.

P.O. 6.14.06.01-.02&.05

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

Complete the charts below.

OBJECTS/ITEMS MEASURED IN CLASSROOM	UNIT	ESTIMATED LENGTH	ACTUAL LENGTH	DIFFERENCE
1. length of the desk				
2. pencil				
3.				
4.				
5.				

OBJECTS/DISTANCES MEASURED IN THE SCHOOL	UNIT	ESTIMATED LENGTH	ACTUAL LENGTH	DIFFERENCE
1. length of the yard				
2.				
3.				
4.				
5.				

OBJECTS/ITEMS/PERSONS MEASURED AT HOME	UNIT	ESTIMATED LENGTH	ACTUAL LENGTH	DIFFERENCE
1. kitchen table				
2. height of refrigerator				
3.				
4.				
5.				

Complete and solve the following word problem:

Keith is 162 cm tall. His brother William is 182 cm.

____

____

____

____

P.O. 6.14.06.02&.05

NAME: ______________________________ **CLASS:** _______ **DATE:** ____________________

Estimate and measure length in meters, centimeters, and millimeters.

Small Objects/Items	Estimate		Actual Length		Difference	
	cm	mm	cm	mm	cm	mm
1. a pen						
2. a pencil						
3. any book						
4.						
5.						
6.						
7.						
8.						
9.						
10.						

Measure the distance between:	Estimate in Meters	Actual Length in Meters	Difference
1. two rooms in a school			
2. two street signs			
3.			
4.			
5.			

CHALLENGE!

Students in class 6-301 wanted to measure the length and width of the auditorium but had no meter stick. They had a metric ruler and adding machine tape. They decided to construct their own metric tapes.

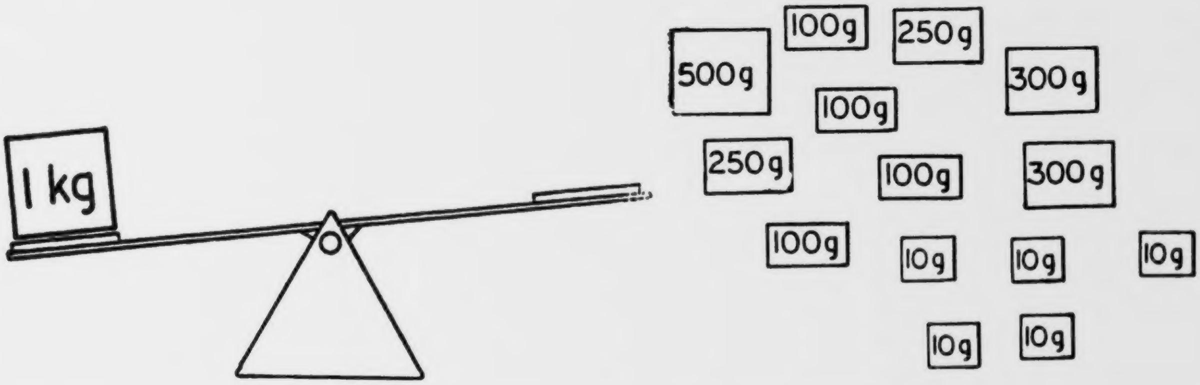
1) Construct a meter stick using the metric ruler and the adding machine tape.
2) Measure the length of the school's auditorium.

P.O. 6.14.06.03-.05

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

Follow the directions below.

I. Use the fewest number of weights to balance the beam. Circle the weights you would use.



II. **Solve.**

1. Jason hit two home runs. The first traveled a distance of 98 meters. The second landed in the bleacher section, 109 meters from home plate. How much farther did the second home run travel than the first? ______________________

2. What is the equivalent length in centimeters of a desk which is $2\frac{1}{2}$ m long? ______________________

3. If a 1-liter bottle contains 565 mL of soda, how much liquid is missing from the container? ______________________

4. David's mathematics textbook weighs $\frac{1}{4}$ kg. His science book weighs $3\frac{1}{2}$ kg. What is their combined weight in grams?

   ____

5. Create and solve your own problem.

   ____

   ____

   ____

   ____

   ____

SHARPENING YOUR SKILLS

Units 16, 17, 18

Name: ____________________ **Class:** ____________ **Date:** _________

1. The value of the 6 in 4.167 is __________.

ⓐ 60 ⓒ 0.06
ⓑ 6 ⓓ Not Here

2. The value of the 3 in 51.243 is __________.

ⓔ 3 thousandths
ⓕ 3 tenths
ⓖ 3 ten-thousandths
ⓗ Not Here

3. Express each fraction below as a decimal.

a. $\frac{4}{10}$ = ____ d. $\frac{3421}{10,000}$ = ____
b. $\frac{34}{100}$ = ____ e. $\frac{9}{100}$ = ____
c. $\frac{15}{1000}$ = ____ f. $\frac{246}{10,000}$ = ____

4. Write >, <, or = to make each number sentence true.

a. 0.36 ○ 0.036
b. 0.456 ○ 0.4
c. 3.84 ○ 0.3845
d. 2.51 ○ 25.1
e. 0.38 ○ 0.3800

5. Which group of decimals is in order from least to greatest?

ⓐ 0.53, 0.581, 0.6
ⓑ 1.35, 1.035, 1.53
ⓒ 0.664, 0.0656, 0.466
ⓓ Not Here

6. 0.379 rounded to the nearest tenth is __________.

ⓔ 0.3 ⓖ 1.0
ⓕ 0.4 ⓗ Not Here

7. 4.228 rounded to the nearest tenth is __________.

ⓐ 4.23 ⓒ 4.0
ⓑ 4.3 ⓓ Not Here

8. 3.677 rounded to the nearest hundredth is __________.

ⓔ 3.67 ⓖ 3.77
ⓕ 3.68 ⓗ Not Here

9. 27.008 rounded to the nearest hundredth is __________.

ⓐ 27.08 ⓒ 27.1
ⓑ 27.01 ⓓ Not Here

10. Complete the following:

a. $\frac{1}{10}$ meter = 1 ______
b. $\frac{1}{1000}$ meter = 1 ______
c. 1000 grams = 1 ______
d. 1000 milliliters = 1 ______

Unit 19 6.10.04 COMPARISONS WITH METRIC MEASURES

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

6.10.04.01A state equivalents for given measures of length;
e.g.,
2.3 m = 230 cm
150 mm = 15 cm

6.10.04.02A state equivalents for given measures of weight;
e.g.,
44 g = 0.044 kg
2.3 kg = 2300 g

*6.10.04.03A state equivalents for given measures of capacity;
e.g.,
5.1 L = 51 dL
1250 mL = 1.25 L

6.10.04.04A draw approximate comparisons between metric units and customary units;
e.g.,
A yard is a little less than a meter (about 10% less).
A quart is a little less than a liter.
A pound is a little less than half a kilogram.
A mile is about 1.6 kilometers.

6.10.04.05A create and solve word problems related to skills learned in this module.
e.g.,
A piece of lumber is 2 meters long. If a 78 cm section is cut from it, how many meters long will the piece of lumber be?

PROBLEM EXPLORATION SKILLS:

- to adopt a different point of view
- to look for hidden assumptions

VOCABULARY

milli
centi
deci
kilo
converting
meter
liter
gram

SYMBOLS

mm
cm
dm
m
mg
kg
g
L
mL
dL

PREVIOUS UNDERSTANDINGS:

- subdivisions of a metric base unit
- multiplying and dividing decimals by 10, 100, and 1000

GETTING STARTED:

Materials:

- instruments for metric measurement, e.g., rulers, scales and balances, liter cubes (containers)

Write the following examples on the chalkboard:
e.g.,

100 x 24 = ☐   100 x 35 = ☐

1000 x 24 = ☐   1000 x 35 = ☐

Say, "What do we do when multiplying whole numbers by tens? ... by hundreds? ... by thousands?" (Multiply the non-zero digits and then write one, two, or three zeros in the ones, tens, and hundreds places.) Ask a student to do the mental computation work at the chalkboard while the other students work at their desks.
e.g.,

100 x 24 = 2400   100 x 35 = 3500

1000 x 24 = 24,000   1000 x 35 = 35,000

Write the following examples on the chalkboard:

9.24 x 10 = ☐   9.24 x 100 = ☐   9.24 x 1000 = ☐

GETTING STARTED: (cont'd)

Say, "Are there similar procedures for multiplying a decimal by 10, by 100, or by 1000?" (Yes, move the decimal point one, two, or three places to the right respectively.) Ask a student to come to the chalkboard to do the work as the other students complete the work at their seats.

e.g.,

9.24 x 10 = 92.40 or 92.4
9.24 x 100 = 924.0 or 924
9.24 x 1000 = 9240.0 or 9240

Say, "Why does this procedure work?" (When the decimal point is moved in this way, each place value becomes ten times as great for each place the decimal point has been shifted to the right.)

Write the following examples on the chalkboard:

3.8 ÷ 10 = ☐
3.8 ÷ 100 = ☐
3.8 ÷ 1000 = ☐

Say, "Is there a rule or pattern for dividing a decimal by 10, 100, or 1000? (Yes, move the decimal point one, two, or three places to the left respectively.) How can this be explained another way?" (To divide a decimal by a power of ten, move the decimal point one place to the left for each zero in the power of ten.) Ask a student to come to the chalkboard to complete the work as the other students work at their desks.

e.g.,

3.8 ÷ 10 = 0.38
3.8 ÷ 100 = 0.038
3.8 ÷ 1000 = 0.0038

6.10.04.01A state equivalents for given measures of length

STRATEGY 1:

Display the following chart:

Object/Item/Distance To Be Measured	Estimate	Measurement					Difference
		mm	cm	dm	m	km	

Say, "Today, we are going to play a metric game." The rules are as follows:

1. A player selects an object/item/distance to be measured.
2. After selecting an appropriate unit, the player makes an estimate of its length in that same unit of measure.
3. A player scores a point for each column completed.
4. A player loses a point for an estimate which is greater/less than the actual measure by 5 or more units.
5. Other team players may gain points for their team by providing an appropriate equivalent for their teammate.

Say, "Let's play the metric game together. What is a reasonable estimate for the length of a business envelope?" (25 cm) Record this information on the chart.

Object/Item/Distance To Be Measured	Estimate	Measurement					Difference
		mm	cm	dm	m	km	
business envelope	25 cm						

Ask a student to find the actual length of the business envelope in centimeters. Say, "How long is the business envelope?" (24 cm) Continue writing the information on the chart.

STRATEGY 1: (cont'd)

e.g.,

Object/Item/Distance To Be Measured	Estimate	Measurement					Difference
		mm	cm	dm	m	km	
business envelope	25 cm		24				

Say, "What other metric units would people be likely to choose to describe the length of this envelope? (millimeters and decimeters) How many millimeters are equivalent to 24 cm? Explain. (Since 1 cm equals 10 mm, we can multiply 24 cm by 10 which equals 240 mm.) What is the equivalent of 24 cm in decimeters?" (Since 10 cm equals 1 decimeter, we can divide 24 cm by ten to get 2.4 dm.) Demonstrate.

e.g.,

24 cm x 10 = 240 mm 24 cm ÷ 10 = 2.4 dm

Say, "What is the difference between the estimate and the actual measurement?" (1 cm) Write this information in the appropriate columns on the chart.

e.g.,

Object/Item/Distance To Be Measured	Estimate	Measurement					Difference
		mm	cm	dm	m	km	
business envelope	25 cm	240	24	2.4			1 cm

Say, "How many columns have we completed? (5) What is the total number of points scored?" (5)

Repeat this procedure using other objects, items, or distances to be measured.

6.10.04.02A state equivalents for given measures of weight

STRATEGY 1:

Materials:
- charts shown below

Display the following:

Mr. Lee delivers cold cuts to grocery stores around the city. The following orders were received by him for tomorrow's deliveries:

S&A Provisions	
Quantity	Description
1.5 kg	Bologna
2.6 kg	Salami

P&A Provisions	
Quantity	Description
800 g	Ham
450 g	Bologna

Roadmark	
Quantity	Description
4.2 kg	Ham
2.9 kg	Swiss Cheese
4.8 kg	American Cheese

Joe's Superette	
Quantity	Description
3.5 kg	Swiss Cheese
4.1 kg	Salami

Tony's Bodega	
Quantity	Description
750 g	Ham
1600 g	Swiss Cheese
900 g	Bologna

Say, "Since each order must be given in both kilograms and grams, let's help Mr. Lee change the units given to their equivalent metric units."

STRATEGY 1: (cont'd)

Ask the students to gather data from the charts and record this information in the following manner:

Mr. Lee's Equivalency Table										
	Bologna		Ham		Salami		American Cheese		Swiss Cheese	
Store	kg	g	kg	g	kg	g	kg	g	kg	g
S&A	1.5				2.6					
P&A		450		800						
Superette					4.1				3.5	
Bodega		900		750						1600
Roadmark			4.2				4.8		2.9	

Say, "Now we can complete the chart by converting each unit of weight into an equivalent unit. Which metric units of weight are to be considered? (grams and kilograms) What conversions must be made to complete the chart? (We must convert from grams to kilograms or from kilograms to grams.) Which is the larger unit? (Kilograms is a larger unit than grams.) What have we previously learned about converting units, or stating equivalents, from one unit to another?" (When we change from a larger unit to a smaller unit, we multiply. When we change from a smaller unit to a larger unit, we divide.) Ask a student to come to the chalkboard and explain the operations required to state an equivalent for each given measure.

e.g.,

	Swiss Cheese		
Store	kg	g	Explanation
Bodega		1600	smaller to larger (÷)
Roadmark	2.9		larger to smaller (x)

STRATEGY 1: (cont'd)

Say, "What is the operation for finding the equivalent for Tony's Bodega's order of Swiss Cheese? (Since there are 1000 grams in a kilogram, we can divide 1600 g by 1000.) What is a simple method for performing this operation?" (The decimal point can be moved three places to the left.) Demonstrate.

e.g.,

1600 g + 1000 = 1.600 or 1.6 kg

Say, "How can we find the equivalent for Roadmark's order? (Multiply 2.9 kg by 1000 since a kilogram is 1000 grams.) Since this requires multiplication, the decimal point in the product can be moved three places to the right."

e.g.,

2.9 kg x 1000 = 2900 g

Ask a student to complete the Swiss Cheese Order chart while the other students complete it in their notebooks. Continue this procedure to complete the entire chart.

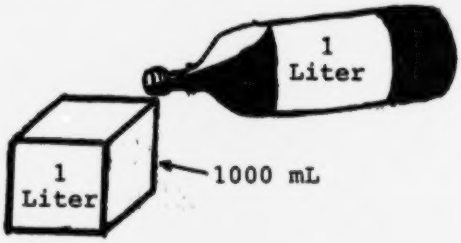
6.10.04.03A state equivalents for given measures of capacity

STRATEGY 1:

Materials:
- a liter cube (10 cm x 10 cm x 10 cm)
- 1-liter soda bottle

Display the liter cube. Say, "What are the properties of this cube? (It is a liter cube, which can be described as 1000 cubic centimeters.) What is its capacity?" (Its capacity is 1000 milliliters of liquid.) Pour the contents of a one-liter soda container into the liter cube.

e.g.,



STRATEGY 1: (cont'd)

Say, "Today we will express equivalents for various metric units of capacity." Write the following problems on the chalkboard:

(a) 354 mL = ____ L (b) 3.54 dL = ____ mL

Say, "How do we find the equivalent in example (a)? (Since a liter is 1000 milliliters, divide 354 mL by 1000.) What is the 'shortcut' method?" (Move the decimal point three places to the left.)

e.g., 354.0 mL + 1000 = 0.354 L

Say, "How can we find the equivalent in example (b)? (Multiply 3.54 dL by 100, since a deciliter is the same as 100 milliliters.) What is the 'shortcut' method?" (Move the decimal point two places to the right.)

e.g., 3.54 dL x 100 = 354 mL

Repeat this procedure by pouring other containers into the model and expressing them as equivalents.

6.10.04.04A draw approximate comparisons between metric units and customary units

STRATEGY 1:

Materials:
- metric and customary cards

Say, "Does the size, weight, or capacity of an object change if we use the metric or the customary system of measurement. (No, the objects remain the same.) What is the difference?" (The system of measurement and the units are different. The number of units of the different types in the given object are different.) Display the sets of metric and customary cards (linear, weight/mass, capacity).

e.g.,

Customary Cards		Metric Cards	
1 mi	1 lb	0.9 m	30 cm
1 ft	1 qt	2.5 cm	0.92 L
1 yd	1 in.	0.4 kg	1.6 km

STRATEGY 1: (cont'd)

Divide the class into two teams, metric and customary. Ask a player from each team to sort the cards for each team into 3 separate groups, linear, weight/mass, and capacity.

e.g.,

CUSTOMARY TEAM

Deck #1 Linear	Deck #2 Weight/Mass	Deck #3 Capacity
1 yd	1 lb	1 qt
1 in.		
1 ft		
1 mi		

METRIC TEAM

Deck #1 Linear	Deck #2 Weight/Mass	Deck #3 Capacity
0.9 m	0.4 kg	0.92 L
2.5 cm		
30 cm		
1.6 km		

Place each card face down. Ask a student from the Customary Team to select a card from Deck #1, e.g., 1 mi. Ask a student from the Metric Team to estimate the equivalent metric measure. Say, "What is an equivalent metric measure for miles? (kilometers) How can the length of a mile be compared to the length of a kilometer?" (A mile is a little more than one and a half kilometers.) Ask a student to select and display the appropriate card from the Metric Deck #1, e.g., 1.6 km.

Repeat this procedure by alternating the teams making the initial selection.

6.10.04.05A create and solve word problems related to skills learned in this module

STRATEGY 1:

Display the following chart:

e.g.,

Class 6-301 Hiking Trip	
Day	Distance Hiked
Monday	3.6 km
Tuesday	4.2 km
Wednesday	925 m
Thursday	875 m

STRATEGY 1: (cont'd)

Say, "What information can be gleaned from the chart? (We can find the distance Class 6-301 hiked on each of four separate days.) What kinds of questions and/or problems can be solved by using the data on the chart? (We could find how much farther Class 6-301 traveled on Tuesday than on Monday.) What are some other questions which can be explored? (Responses will vary, e.g., We can learn whether there is a developing pattern. The possibility for predicting the hiking distance on Friday may be explored. We can also discover whether graphing the data would serve a useful purpose.) What additional condition exists if we are to find the total distance hiked during the four days? (We must decide if our answer is to be expressed in meters or in kilometers.) Which measurement is more appropriate? (kilometers) What is the correct operation for performing the conversion from meters to kilometers? Explain. (When smaller units are converted to larger units, we divide.) By which number will we divide? (Since there are 1000 meters in a kilometer, we can divide by 1000.) Is there a simple method by which we can divide by 1000?" (Yes, move the decimal point 3 places to the left.)

e.g.,

$$925.0\text{ m} \div 1000 = 0.925\text{ km}$$

$$875.0\text{ m} \div 1000 = 0.875\text{ km}$$

Say, "What was the purpose of finding the equivalents?" (In order to find the total distance hiked over the four day period, each distance must be expressed in the same unit of measure.) Demonstrate.

e.g.,

$$\begin{array}{lcr} 3.6\text{ km} & \longrightarrow & 3.6\text{ km} \\ 4.2\text{ km} & \longrightarrow & 4.2\text{ km} \\ 925\text{ m} & \longrightarrow & 0.925\text{ km} \\ 875\text{ m} & \longrightarrow & +\ \underline{0.875\text{ km}} \\ \text{Total Distance} & & 9.600\text{ km} \end{array}$$

Say, "How far did Class 6-301 hike? (They hiked 9.6 km) How could

STRATEGY 1: (cont'd)

the total distance hiked be expressed another way?" (By multiplying 9.6 by 1000, we could express the answer in meters.)
e.g.,

$$9.6 \text{ km} \times 1000 = 9600 \text{ m}$$

Repeat this procedure using other units of measure of length, weight/mass, and capacity.

P.O. 6.10.04.01&.05

NAME: ______________________________ **CLASS:** _______ **DATE:** ______________

Complete the following.

I. a) 5000 m = ______ km
b) 60 mm = ______ cm
c) 286 cm = ______ m
d) 5250 mm = ______ m
e) 35 mm = ______ cm
f) 24 m = ______ cm
g) 162 cm = ______ m
h) 1000 cm = ______ m
i) 1.25 m = ______ cm
j) 5.2 km = ______ m

II. Estimate, measure, and record the length of eight different items/objects/distances. Express the measurement at least two ways. Include your height and that of one other member of your family.

Item/Object Distance	Estimate	mm	cm	dm	m	km
1. pen	18 cm	145	14.5			
2. Mother's height						
3.						
4.						
5.						
6.						
7.						
8.						

III. Which unit (km, m, dm, cm, or mm) would you use to best express the lengths or distances of each of the following?

1. trip from N.Y.C. to L.A. ____
2. distance of 1 city block ____
3. length of a paper clip ____
4. height of a 6th grader ____
5. length of a school bus ____
6. width of your pinky finger ____
7. thickness of a dime ____
8. length of your classroom ____
9. height from desk to floor ____
10. length of a looseleaf book ____

P.O. 6.10.04.02&.05

NAME: ______________________ **CLASS:** ________ **DATE:** ____________

Complete the following.

a) 54 g = ______ kg

b) 2.6 kg = ______ g

c) 7.8 g = ______ mg

d) 4 kg = ______ g

e) 846 kg = ______ g

f) 130 mg = ______ kg

g) 0.924 kg = ______ g

h) 1.9 kg = ______ g

i) 813 mg = ______ g

j) 907 g = ______ mg

II. Look at labels on cans, bottles, and boxes which display metric units of measure. List the items and the weights shown on their labels and circle them. Express the equivalent weights in the other two columns on the chart.

Container	Circle the weight shown on the label		
	kg	g	mg
1. Krispy Cereal Box	0.284	[✓ 284]	284,000
2.			
3.			
4.			
5.			
6.			
7.			
8.			

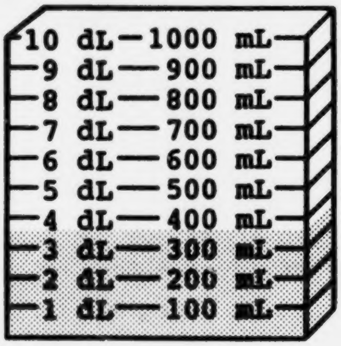
III. Use the information in the chart above to create and solve your own problem.

P.O. 6.10.04.03

NAME: ______________________ **CLASS:** ________ **DATE:** ____________

Find the capacity on six different labels. "Pour" or shade in the approximate amount on the liter cubes for questions A to D. Express the amount three different ways. The first one is completed for you. For questions E and F, write the approximate amount that is shown in each cube in 3 equivalent measures.

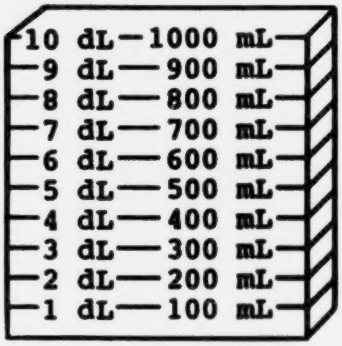
A.


10 dL—1000 mL
9 dL—900 mL
8 dL—800 mL
7 dL—700 mL
6 dL—600 mL
5 dL—500 mL
4 dL—400 mL
3 dL—300 mL
2 dL—200 mL
1 dL—100 mL

a 0.354 L can of soda

3.54 dL .354 L 354 mL

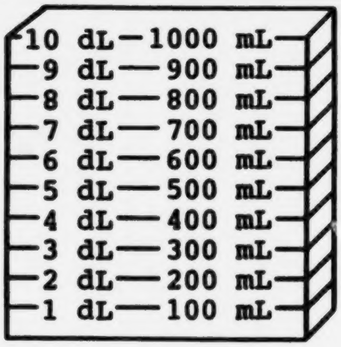
B.


10 dL—1000 mL
9 dL—900 mL
8 dL—800 mL
7 dL—700 mL
6 dL—600 mL
5 dL—500 mL
4 dL—400 mL
3 dL—300 mL
2 dL—200 mL
1 dL—100 mL

one liter of milk

____ dL ____ L ____ mL

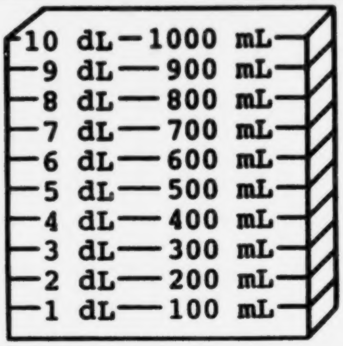
C.


10 dL—1000 mL
9 dL—900 mL
8 dL—800 mL
7 dL—700 mL
6 dL—600 mL
5 dL—500 mL
4 dL—400 mL
3 dL—300 mL
2 dL—200 mL
1 dL—100 mL

one-half liter of oil

____ dL ____ L ____ mL

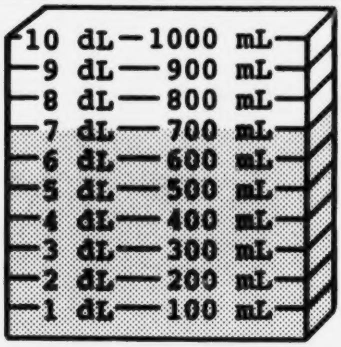
D.


10 dL—1000 mL
9 dL—900 mL
8 dL—800 mL
7 dL—700 mL
6 dL—600 mL
5 dL—500 mL
4 dL—400 mL
3 dL—300 mL
2 dL—200 mL
1 dL—100 mL

3.75 dl of water

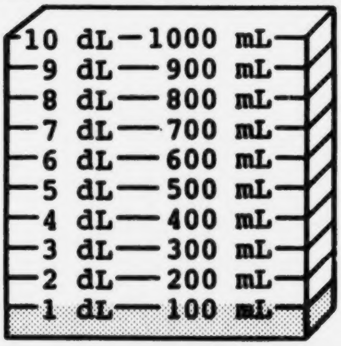
____ dL ____ L ____ mL

E.


10 dL—1000 mL
9 dL—900 mL
8 dL—800 mL
7 dL—700 mL
6 dL—600 mL
5 dL—500 mL
4 dL—400 mL
3 dL—300 mL
2 dL—200 mL
1 dL—100 mL

____ dL ____ L ____ mL

F.


10 dL—1000 mL
9 dL—900 mL
8 dL—800 mL
7 dL—700 mL
6 dL—600 mL
5 dL—500 mL
4 dL—400 mL
3 dL—300 mL
2 dL—200 mL
1 dL—100 mL

____ dL ____ L ____ mL

P.O. 6.10.04.04-.05

NAME: ____________________ **CLASS:** ________ **DATE:** ____________

I. Which is the approximate equivalent of each measure? Circle the best answer.

1. A(n) _______ is a little less than 3 centimeters.
   (A) inch (B) foot (C) yard

2. A _______ is a little more than 2 pounds.
   (A) gram (B) kilogram (C) milligram

3. A _______ is a little less than 2 kilometers.
   (A) yard (B) mile (C) 10 miles

4. A _______ is about the same as a vitamin tablet.
   (A) kilogram (B) milligram (C) gram

5. A _______ is a little more than a yard.
   (A) meter (B) kilometer (C) centimeter

6. A _______ is about 2 quarts.
   (A) liter (B) gallon (C) 2 liters

7. _______ is about the length of your arm.
   (A) 60 centimeters (B) 60 feet (C) 2 meters

8. A 500 gallon fuel truck holds about _______.
   (A) 500 quarts (B) 2000 liters (C) 200 liters

9. The height of a 10-story building is about _______.
   (A) 300 meters (B) 30 meters (C) 3 meters

II. Solve each problem

1. Jill has a 1-meter length of ribbon. If she cuts off 30 centimeters, how many centimeters are left? ____________________

2. A 2-liter bottle of water fell over. If 65 milliliters spilled out, how many liters of water were still in the bottle? ______________

3. Pedro weighs 68 kilograms. How many grams is this? ____________

4. The distance from Michael's house to school is 1250 meters. How many kilometers is this? ______________________________

5. A baseball bat is 98 centimeters long.
   (a) What part of a meter is it? ______________________
   (b) How many millimeters long is it? ______________________

Unit 20 6.14.07 REVIEW: MULTIPLICATION OF WHOLE NUMBERS

(1 lesson/ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

6.14.07.01 estimate products by rounding factors to the left-most place;

e.g.,

```
 325 rounds to    300
x 39 rounds to     40
               12,000 is the estimated product

 264 rounds to    300
x139 rounds to    100
               30,000 is the estimated product
```

6.14.07.02 multiply a 3-digit number by a 2- or 3-digit number;

e.g.,

```
   325         264
  x 39        x139
 2 925       2 376
 9 750       7 920
12,675      26 400
            36,696
```

6.14.07.03 create and solve word problems related to skills learned in this module.

e.g.,

During one month, Flora worked a total of 105 hours at Big Bob's Burger Barn. If she is paid $3.70 per hour, how much did she earn that month?

PROBLEM EXPLORATION SKILLS:

- to estimate
- to consider a related but simpler problem
- to state the goals of a problem clearly

VOCABULARY

factors
multiplier
multiplicand
partial products
product

PREVIOUS UNDERSTANDINGS:

- rounding numbers
- identifying multiples of tens and hundreds

GETTING STARTED:

Display the following problem:

> Donald's Diner used 39 dozen eggs to make breakfast for its customers this morning. How many eggs were used in all?

Ask a student to read the problem aloud. Say, "Which operation would best solve this problem? (multiplication) How many eggs are there in a dozen? (12) To find out how many eggs Donald's Diner used, what factors do we multiply?" (39 and 12)

Ask the students to solve the multiplication problem in their notebooks while two students solve it on the chalkboard. Reverse the order of the factors for each problem on the chalkboard, labeling them A and B.

e.g.,

$$\text{(A)}\quad \begin{array}{r} 39 \\ \underline{\times 12} \\ 78 \\ \underline{390} \\ 468 \end{array} \qquad\qquad \text{(B)}\quad \begin{array}{r} 12 \\ \underline{\times 39} \\ 108 \\ \underline{360} \\ 468 \end{array}$$

Say, "How many eggs were used for breakfast? (468 eggs were used.) What is the product? (468) What are the partial products in example A? (78 and 390) In sample B? (108 and 360) What are other names for the factors in a multiplication problem? (multiplier and multiplicand) What is the multiplier in A? (12) ...in B? (39) In which problem, A or B, is 12 the multiplicand? (B) Which property have we demonstrated by showing that 12 x 39 = 39 x 12?" (the commutative property for multiplication)

6.14.07.01 **estimate products by rounding factors to the left-most place**

STRATEGY 1:

On the chalkboard write: $\begin{array}{r}475\\ \underline{\times\ \ 31}\end{array}$. Say, "If Mr. Brown sold 475 quarts of milk in his store every day for 31 days, we can find exactly how many quarts he sold. But it is not always necessary to know an exact answer. Sometimes, an answer reasonably close to the real one is sufficient. The process of finding a value close to the real answer is called estimating. We can estimate the number of quarts of milk Mr. Brown sold by rounding the factors 475 and 31 to their left-most places and then multiplying. What is the leftmost place in 475? (hundreds place) What is 475 rounded to the nearest hundred? (500) What is the left-most place in 31? (tens place) What is 31 rounded to the nearest ten?" (30) Write this on the chalkboard.

e.g.,

$$\begin{array}{rcr}475 & \text{rounds to} & 500\\ \underline{\times\ 31} & \text{rounds to} & \underline{\times\ 30}\end{array}$$

Say, "One method in rounding factors (numerals) is to think in terms of multiples. A 3-digit factor is a numeral in hundreds, so the rounded factor is a multiple of one hundred. To which multiple of one hundred is 475 closest? (500) A 2-digit factor is a numeral in tens, so the factor is a multiple of ten. To which multiple of ten is 31 closest? (30) These understandings help us to estimate the product quickly without finding exact partial products. When multiplying multiples of tens and hundreds, simply count the zeros in the rounded factors and record them. Then multiply the remaining digits and record this result at the left-most place. This gives us the estimated product."

e.g.,

STEP 1. Count and record zeros.

$$\begin{array}{rl}500 & \text{rounded factor}\\ \underline{\times\ 30} & \text{rounded factor}\\ 000 & \end{array}$$

STEP 2. Multiply the remaining digits and record.

$$\begin{array}{rl}500 & \\ \underline{\times\ 30} & \\ 15000 & \text{is the estimated product}\end{array}$$

STRATEGY 1: (cont'd)

Demonstrate the efficiency of this method. Ask a student to multiply 500 x 30 using the traditional method of multiplying by first finding partial products. Compare the answer with that of the short-cut method.

e.g.,

$$\begin{array}{r} 500 \\ \underline{\times\ 30} \\ 000 \\ \underline{15\ 000} \\ 15{,}000 \end{array}$$

Say, "Let's estimate the product of two 3-digit factors, 332 and 189. Remember to round the factors before multiplying." Ask a student to show the computation at the chalkboard while the other students work in their notebooks.

e.g.,

$$\begin{array}{rlrcr} 332 & \text{rounds to } 300 & 300 & \text{or} & 300 \\ \underline{\times 189} & \text{rounds to } 200 & \underline{\times 200} & & \underline{\times 200} \\ & & 000 & & 60{,}000 \\ & & 0\ 000 & & \\ & & \underline{60\ 000} & & \\ & & 60{,}000 & & \end{array}$$

Say, "To which place are both factors rounded? (hundreds) Which factor is the multiplicand? (332) To what did it round? (300) Which is the multiplier? (189) To what did it round? (200) Using the partial product or the short-cut method, what is the estimated product?" (60,000)

Repeat this procedure for other multiplication examples using 3-digit multiplicands and 2-digit and 3-digit multipliers.

6.14.07.02 multiply a 3-digit number by a 2- or 3-digit number

STRATEGY 1:

Materials:
- place value chart to hundred thousands

Display the place value chart.

STRATEGY 1: (cont'd)

e.g.,

hundred thousands	ten thousands	thousands	hundreds	tens	ones

Say, "Let's use a place value chart to solve a multiplication problem. Where shall we place the digits of a 3-digit multiplicand?" (in the hundreds, tens, and ones places) Write the multiplicand 253 on the place value chart.

e.g.,

hundred thousands	ten thousands	thousands	hundreds	tens	ones
			2	5	3

Say, "Where shall we place the multiplier, 13?" (In the tens and ones places under 253.) Write 13 under 253, the multiplication symbol, and the line to complete the multiplication algorithm.

e.g.,

hundred thousands	ten thousands	thousands	hundreds	tens	ones
			2	5	3
			x	1	3

Ask a student to find the first partial product and write it on the chart. Say, "What is the first partial product? (759) How can we compute the second partial product? (We multiply 253 by the digit 1 in the tens place.) In which place do we begin to record our product? (tens place) What number must we use as a place holder in the ones place?" (zero) Ask a student to complete the computation for the second partial product on the place value chart. Say, "Why is the second partial product 2530 and not 253? (Because 253 was actually multiplied by 10, not by 1, the digit 2 in this partial product is in the thousands place and its place value is thousands, not hundreds.) How do we find the final product?" (Add the partial products.)

STRATEGY 1: (cont'd)

e.g.,

hundred thousands	ten thousands	thousands	hundreds	tens	ones	
			2	5	3	
			x	1	3	
			7	5	9	(3 x 253)
		+ 2	5	3	0	(10 x 253)
		3	2	8	9	

Say, "What is the product of the factors 253 and 13?" (3289)

Repeat this procedure by multiplying other 3-digit multiplicands by 2-digit and 3-digit multipliers.

STRATEGY 2:

Write the following word problem on the chalkboard:

> For lunch today, 564 oranges were delivered to each of 28 public schools. How many oranges were delivered?

Ask a student to read the problem aloud. Say, "What are we being asked to find? (We are asked to find the total number of oranges delivered to 28 schools.) What operation can we use to solve the problem? (multiplication or addition) Which operation is more efficient and reduces the number of possible errors? (multiplication) How can we estimate the number of oranges delivered? (We can estimate the product.) How do we estimate? (Round the factors to their left-most places and multiply.) What are the factors?" (564 and 28) Demonstrate.

e.g.,

564 rounds to 600
x 28 rounds to x 30
18,000 estimated product

Say, "Approximately how many oranges were delivered? (18,000) How can we find the actual number of oranges delivered?" (We can find the product of 564 and 28.) Ask students to multiply 564 by the 8 ones in 28 to find the first partial product. Demonstrate.

STRATEGY 2: (cont'd)

e.g.,

$$\begin{array}{rl} 564 & \\ \underline{\times \quad 28} & \\ 4\,512 & (8 \times 564) \end{array}$$

Say, "How can we find the second partial product?" (We can multiply 564 by the 20 in 28.)

e.g.,

$$\begin{array}{rl} 564 & \\ \underline{\times \quad 28} & \\ 4\,512 & (8 \times 564) \\ 11\,280 & (20 \times 564) \end{array}$$

Say, "How can we find the final product?" (Add the two partial products.) Ask a student to show this at the chalkboard.

e.g.,

$$\begin{array}{rl} 564 & \\ \underline{\times \quad 28} & \\ 4\,512 & (8 \times 564) \\ \underline{11\,280} & (20 \times 564) \\ 15{,}792 & \end{array}$$

Say, "What is the exact number of oranges? (15,792) Is our estimate of 18,000 reasonably close to this amount?" (yes)

Repeat this procedure by asking students to create and solve other problems using 3-digit multiplicands and 2- or 3-digit multipliers.

6.14.07.03 create and solve word problems related to skills learned in this module

STRATEGY 1:

Display the following word problem:

> Fred's computer store received 40 cases of floppy disks. If each case contained 185 disks, how many disks did Fred receive?

Say, "What are we asked to find? (the number of disks Fred received) What information is needed to solve this problem? (the number of disks in each case, 185, and the total number of cases received, 40) Which operation will we use? Explain." (Since each case contains the same number of disks, we can multiply by 40.) Elicit that multiplication is a faster and more efficient way to do repeated

STRATEGY 1: (cont'd)

addition. Ask a student to write the example needed to solve the problem on the chalkboard.

e.g.,

$$\begin{array}{r} 185 \\ \underline{\times\ 40} \end{array}$$

Say, "Estimate by rounding each factor to its left-most place and then multiply."

e.g.,

$$\begin{array}{rlr} 185 & \text{rounds to} & 200 \\ \underline{\times\ 40} & \text{rounds to} & \underline{\times\ 40} \\ & & 8000 \end{array}$$

Say, "Use the partial product method to find the actual cost of the disks." Demonstrate.

e.g.,

$$\begin{array}{rl} 185 & \\ \underline{\times\ 40} & \\ 000 & (0\ \times 185) \\ \underline{7400} & (40 \times 185) \\ 7400 & \end{array}$$

Say, "Is the actual product, 7400, reasonably close to the estimated product of 8000? (yes) Would the product change if the factors were reversed?" (no) Ask a student to demonstrate the commutative property of multiplication on the chalkboard, e.g., 185 x 40 = 40 x 185.

Repeat this procedure to solve other word problems, some of which are created by the students, to multiply other 3-digit numbers by 2- or 3-digit numbers.

STRATEGY 2:

Materials:
- chart shown below

Display the following chart:

STRATEGY 2: (cont'd)

Price List for Nails		
NAILS	QUANTITY PER BOX	PRICE PER BOX
Small Box	35	$ .59
Medium Box	75	$ .85
Large Box	125	$ 1.15

Say, "What does our chart show? (The prices for boxes of nails.) In which column would you look to find the number of nails in each box? (Quantity Per Box) What does the 'Price Per Box' column tell us? (the price of each box of nails) What is the price of a large box? ($1.15) What is the quantity of nails contained in a medium-sized box?" (75)

Write the following problem on the chalkboard:

How many nails are there in 123 small boxes?

Ask a student to record the example needed to solve the problem on the chalkboard as the other students write in their notebooks.

e.g.,

$$\begin{array}{r} 123 \\ \underline{\text{x } 35} \end{array}$$

Say, "Let's estimate the product and then find the exact number of nails in 123 small boxes." Demonstrate.

e.g.,

$$\begin{array}{rlr} 123 & \text{rounds to} & 100 \\ \underline{\text{x } 35} & \text{rounds to} & \underline{\text{x } 40} \\ & & 4000 \end{array} \qquad \begin{array}{rl} 123 & \\ \underline{\text{x } 35} & \\ 615 & \\ \underline{3690} & \\ 4305 & \text{nails} \end{array}$$

Say, "How do the products compare?" (The estimated product of 4000 is very close to the actual product of 4305.)

Ask the students to create and solve other word problems using the information on the chart.

P.O. 6.14.07.01-.02

NAME: ______________________ CLASS: ________ DATE: ____________

1. **Match Them Up**
Draw a line from each number on the left to its rounded number on the right.

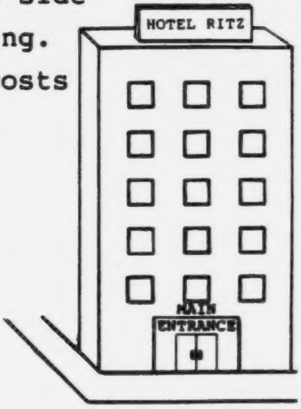
34
450
67
55
893
13
749

70
900
30
60
700
10
500

2. For each multiplication problem, circle the example which will provide the best estimate of the actual product.

A.	350 x 24	400 x 30	400 x 20	300 x 20
B.	242 x388	300 x400	200 x300	200 x400
C.	687 x 19	700 x 20	700 x 10	600 x 20

3. The Hotel Ritz is fixing all the windows on the main entrance side of the building. Each window costs $9.45 to fix.



A. Estimate the cost of fixing the windows. ____________

B. What is the actual cost of fixing the windows? ____________

4. Fill in the missing digits for this problem.

$$\begin{array}{r} 1\;2\;3 \\ \underline{\times 4\;5\;6} \\ 7\;3\;8 \\ 6\;1\;\square\;\square \\ \underline{4\;9\;\square\;\square\;\square} \end{array}$$

A. Find the product. ____________

For items B-D, circle the correct answer.

B. 123 is the ________.
multiplier multiplicand

C. 738 is called the ________.
partial product product

D. This problem has ________ partial products.
1 2 3 4

P.O. **6.14.07.03**

NAME: ______________________ **CLASS:** ________ **DATE:** ____________

Use the advertisement to answer the questions below.

Fox's Floor Tile Sale

BUY:	PAY:
50 - 74 tiles	$1.45 per tile
75 - 99 tiles	$1.35 per tile
100 or more tiles	$1.15 per tile

SIZE OF EACH TILE - 1 SQ FT

Quantities less than 50 NOT on sale	$1.89 per tile

TAX INCLUDED

1. Mrs. Johnson is putting floor tiles on her kitchen floor. If she buys 75 tiles at the sale price, how much will she pay? ____________

2. A bathroom floor measures 12 feet in length and 6 feet in width.
   a. How many square feet is the bathroom floor? ________
   b. How many tiles are needed to cover the bathroom floor? ________
   c. How much will each tile cost? ________

3. Mr. Glass needs 122 tiles. How much will he pay? ____________

4 Mrs. Johnson purchased floor tiles for three different rooms in her house. She paid $105.90 for one room and $85.78 for each of the other two rooms. How much did she pay to tile the three rooms? ____________

5. Estimate the cost of buying 45 tiles at the regular price. ____________

6. Mr. Smith has four $20 bills in his wallet. Is this enough money to buy 50 tiles at the sale price? yes _____ no _____
   a. If yes, how much change will he get back? ________
   b. If no, explain. ______________________________________

Unit 21

6.04.01 MULTIPLICATION OF 4-DIGIT NUMBERS BY 3-DIGIT NUMBERS

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

*6.04.01.01A multiply multiples of 1000 by multiples of 100;

e.g.,

$$\begin{array}{r} 5\ 000 \\ \times\ \ \ 100 \\ \hline 500{,}000 \end{array} \qquad \begin{array}{r} 4\ 000 \\ \times\ \ \ 200 \\ \hline 800{,}000 \end{array}$$

6.04.01.02A estimate products by rounding factors to the left-most place;

e.g.,

$$\begin{array}{rlr} 2\ 234 & \text{rounds to} & 2\ 000 \\ \underline{\times\ \ \ 281} & \text{rounds to} & \underline{\ \ \ \ 300} \\ & & 600{,}000 \end{array}$$

600,000 is the estimated product

6.04.01.03A multiply a 4-digit number by a 3-digit number;

e.g.,

$$\begin{array}{r} 2\ 234 \\ \times\ \ \ 281 \\ \hline 2\ 234 \\ 178\ 720 \\ 446\ 800 \\ \hline 627{,}754 \end{array}$$

6.04.01.04A create and solve word problems related to skills learned in this module.

e.g.,

A store owner purchased 325 jackets that cost $19.95 each. What was the total cost for these jackets?

PROBLEM EXPLORATION SKILLS:

- to use estimation to check the reasonableness of an answer

VOCABULARY

multiplier
multiplicand
factors
partial product
product

PREVIOUS UNDERSTANDINGS:

- multiplying 4-digit by 2-digit numbers
- multiplying 3-digit by 2- and 3-digit numbers
- rounding factors to estimate the product
- partial products

GETTING STARTED:

Materials:

- nine small index cards labeled 1 through 9

Display the following on the chalkboard:

$$\begin{array}{r} \boxed{}\boxed{}\boxed{}\boxed{} \\ \underline{\times \quad\quad 3\ \ 2} \end{array}$$

Say, "What can we call 32 in this example?" (Answers may vary, e.g., a factor, a 2-digit number, a multiplier.) Elicit all three answers. Say, "What can we name the factor with the missing digits? (multiplicand) What is the value of its left-most digit?" (thousands) Assign four students a place value from one through thousands. Ask each student to select an index card and record the digit in the appropriate box.

e.g.,

$$\begin{array}{r} \boxed{3}\boxed{8}\boxed{6}\boxed{4} \\ \underline{\times \quad\quad 3\ \ 2} \end{array}$$

Say, "How can we find the estimated product?" (We can round the factors and then multiply.) Ask a student to demonstrate on the chalkboard.

e.g.,

$$\begin{array}{rlr} 3864 & \text{rounds to} & 4000 \\ \underline{\times\ 32} & \text{rounds to} & \underline{\times\ 30} \\ & & 120{,}000 \quad \text{estimated product} \end{array}$$

GETTING STARTED: (cont'd)

Say, "What is the estimated product? (one hundred twenty thousand) How can we find the actual product? (We can compute the partial products and add.) What is the correct procedure?" (First, multiply 3864 by the 2 ones in 32. Then, multiply 3864 by the 3 tens, or 30. Finally, add the partial products.) Ask a student to work at the chalkboard as the other students work at their desks.

e.g.,

$$\begin{array}{rll}
3\,864 & & \\
\underline{\times\ \ \ 32} & & \\
7\,728 & \text{first partial product} & (2 \times 3864) \\
\underline{115\,920} & \text{second partial product} & (30 \times 3864) \\
123{,}648 & \text{product} &
\end{array}$$

Say, "What is the actual product? (123,648) How does the actual product compare to the estimated product?" (The actual product is reasonably close to the estimated product.)

6.04.01

6.04.01.01A multiply multiples of 1000 by multiples of 100

STRATEGY 1:

> **Materials:**
> - 9 index cards, each with a multiple of 100 through 900
> - 9 index cards, each with a multiple of 1000 through 9000

Display the following on the chalkboard:

Set A

$$\begin{array}{rrrr} 6 & 60 & 600 & 6000 \\ \underline{\times 8} & \underline{\times\ 8} & \underline{\times\ \ 8} & \underline{\times\ \ \ 8} \\ 48 & 480 & 4800 & \boxed{} \end{array}$$

Set B

$$\begin{array}{rrrrr} 7 & 7 & 70 & 700 & 7000 \\ \underline{4} & \underline{\times 40} & \underline{\times 40} & \underline{\times\ 40} & \underline{\times\ \ 40} \\ 28 & 280 & 2800 & 28000 & \boxed{} \end{array}$$

Set C

500 x 9 = 4500

500 x 90 = 45,000

500 x 900 = 450,000

500 x ____ = ______

Say, "Within each set of problems the non-zero digits remain the same. What do you notice about the zeros in the factors? (In each example a zero has been added to one of the factors.) How is the product affected? (The product also includes one extra zero.) Using this information, complete the examples in Sets A and B." Ask a student to complete the examples at the chalkboard as the other students write it in their notebooks.

e.g.,

$$\begin{array}{r} 6\ 000 \\ \underline{\times\ \ \ \ \ 8} \\ 48{,}000 \end{array} \qquad \begin{array}{r} 7\ 000 \\ \underline{\times\ \ \ \ 40} \\ 280{,}000 \end{array}$$

Say, "What are the products? (48,000 and 280,000) Following the pattern established in Set C, what is the missing factor? (9000) ...missing product?" (4,500,000) Ask a student to write the missing digits.

e.g.,

500 x 9000 = 4,500,000

Say, "Let's continue to multiply multiples of 1000 by multiples of 100 using this method." Display the two stacks of index cards face down. Ask a student to select one card from each stack, display them to the class, and write them as factors on the chalkboard.

STRATEGY 1: (cont'd)

e.g.,

$$\begin{array}{r} 4000 \\ \underline{\times\ 600} \end{array}$$

Say, "How many digits will the product have? Explain. (The product will contain 7 digits. Five of the digits will be zeros and two digits will be the product of 6 and 4.) How much is 6 times 4?" (24) Ask another student to complete the example at the chalkboard.

e.g.,

$$\begin{array}{r} 4\ 000 \\ \underline{\times\ \ 600} \\ 2{,}400{,}000 \end{array}$$

Say, "What is the product?" (two million, four hundred thousand) Ask other students to select "factors" from the stacks of cards and solve other multiplication algorithms.

6.04.01.02A estimate products by rounding factors to the left-most place

STRATEGY 1:

Materials:
- place value chart
- index cards, each with a large digit from 1 through 9 written on it

Ask a student to randomly select four index cards, e.g., 4, 2, 7, 5. Beginning with the thousands place, record the digits on the place value chart.

e.g.,

thousands	hundreds	tens	ones
4	2	7	5

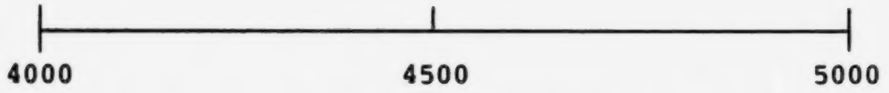
Say, "The four digits picked were used to form a number through the thousands place. This number will be the multiplicand in a multiplication problem." Replace and shuffle the cards. Ask another student to randomly pick a 3-digit multiplier, (e.g., 4, 1, 2). Say, "These three digits form a number through the hundreds place. We call this 'hundreds' number the multiplier." Ask a student to record this number on the place value chart.

STRATEGY 1: (cont'd)

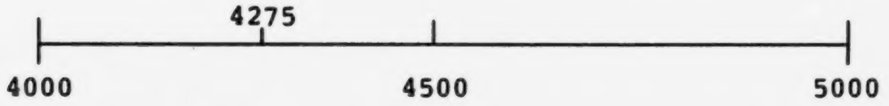
e.g.,

thousands	hundreds	tens	ones
4	2	7	5
x	4	1	2

Say, "How can we estimate the product? (We can round both factors to their left-most places and multiply.) What is the left-most place in the factor 4275?" (thousands) What multiplier of 1000 comes before and after 4275?" (4000 and 5000) Display the following on the chalkboard:



Ask a student to place 4275 in its correct place on the number line.
e.g.,



Say, "What is 4275 rounded to the nearest thousand? (4000) Can you describe how we determine which multiple is nearest a number?" (Yes, any number less than halfway between multiples is always rounded to the lower multiple.)

Repeat this procedure to round 412 to the nearest multiple of 100.

Say, "What is the multiplier, 412, rounded to the left-most place?" (400) Write the following on the chalkboard:

4275 rounds to 4000
x 412 rounds to x 400

Ask students to compute the estimated product using the shortcut method.
e.g.,

4275 rounds to 4 000
x 412 rounds to x 400
1,600,000

STRATEGY 1: (cont'd)

Say, "What is the estimated product of the factors 4275 and 412? (1,600,000) Is this estimate more or less than the actual product? Explain. (It is less because we rounded each factor down. The smaller the factors, the smaller the product.) How will the actual product compare to the estimate?" (The actual product will be greater than 1,600,000.)

Repeat this activity by asking students to estimate products for other 3- and 4-digit factors.

6.04.01.03A multiply a 4-digit number by a 3-digit number

STRATEGY 1:

Adapt Strategy 1 or 2, P.O. 6.14.07.02 (Unit 20).

6.04.01.04A create and solve word problems related to skills learned in this module

STRATEGY 1:

Display the following problem:

> Lindy's airplane travels a distance of 2814 miles each day. If it makes this trip for 281 days, about how many miles will it travel?

Ask a student to read the problem aloud. Say, "What are we asked to find? (the number of miles an airplane traveled) Does this problem ask for an estimated or an exact product? Explain. (We are asked to find an estimated product. The word 'about' tells us to estimate.) How can the problem be solved?" (One way to solve the problem is to round each factor to its left-most place and multiply.) Demonstrate. e.g.,

```
 2814 rounds to    3000
x 281 rounds to   x 300
                900,000 estimated miles
```

STRATEGY 1: (cont'd)

Say, "What is the estimated product? (900,000 miles) Will the actual product be larger or smaller than our estimate? Explain." (Since both factors were rounded up, this estimate is larger than the actual product.) Ask a student to solve the problem using the partial product format as the other students work in their notebooks to determine if the actual product is smaller.

e.g.,

```
     2 814
   x   281
     2 814     (1   x   2814)
   225 120     (80  x   2814)
  +562 800     (200 x   2814)
   790,734 miles
```

Say, "How many miles did the airplane actually travel? (790,734 miles) What is the advantage to estimating the products for large factors?" (Rounding factors allows us to multiply quickly to determine approximately how large our answer will be.)

Repeat this procedure using word problems involving multiplication of 4-digit and 3-digit factors. Some of the word problems should be created by the students.

STRATEGY 2:

Display the following chart:

GAME HUT	
ITEMS SOLD	PRICE PER ITEM
124 dolls 576 games 2000 puzzles	\$23.99 \$12.25 \$ 7.00

Say, "What is the information given on the chart? (the number of games and toys sold at the Game Hut store and their prices) I am going to dictate a word problem. Listen carefully to the question and write the algorithm needed to solve the problem. Use the chart to help you." Dictate the following:

STRATEGY 2: (cont'd)

How much money did the Game Hut earn selling puzzles?

Say, "Which data is required to find the solution? (the number of puzzles sold, 2000, and the price per puzzle, $7.00) Which basic operation is to be performed? (multiplication) Why is multiplication appropriate? (There are 2000 puzzles and each one cost $7.00.) Does it matter which factor is used as the multiplier? Explain." (No, multiplication is commutative.) Ask a student to write the correct example on the chalkboard.

e.g.,

```
    2000                $7.00
 x $7.00      or       x 2000
```

Say, "We could use the partial product method or the short-cut method. Which method should we use?" (This problem should be solved using the short-cut method since both factors, 2000 and $7.00, end with zeros.) Ask a student to complete the example on the chalkboard as the other students work at their desks.

e.g.,

```
       2000
    x $7.00
 $14,000.00
```

Say, "What is the answer to the problem?" (The Game Hut earned $14,000.00 selling puzzles.)

Say, "Listen carefully. Locate the appropriate information and record it in your notebooks." Dictate the following:

How much money was taken in when all the dolls were sold?

Say, "What are the factors needed to solve this problem? (124 and $23.99) How is this problem different from the first problem? (It is more difficult to solve because the factors are not multiples of 1000 and 100.) Which method must be used to find the product. (We must use the partial product method to find the actual answer.) How can we determine whether our actual answer is reasonable?" (We can first

STRATEGY 2: (cont'd)

estimate by rounding products to the left-most place before finding our actual answer. However, since one of the rounded factors will be 100, we can get a better estimate by rounding $23.99 to the nearest dollar.) Ask students to find the estimated and actual products. e.g.,

$$\begin{array}{rlrl} \$23.99 & \text{rounds to} & \$24 & \\ \underline{\times\ \ \ 124} & \text{rounds to} & \underline{\times\ 100} & \\ & & \$2400 & \text{estimated earnings} \end{array}$$

and

$$\begin{array}{r} \$23.99 \\ \underline{\times\ \ \ 124} \\ \$2974.76 \end{array}$$

Repeat this procedure by having students create and solve other word problems using the chart.

P.O. **6.04.01.01-.02**

NAME: ______________________ **CLASS:** ________ **DATE:** ____________

I. Circle the name of the student who estimated the factors by rounding to their left-most places.

5876
x 745

Troy	Kim	Tanya	David
5000 x 700	6000 x 800	5900 x 700	6000 x 700

II. Would you round up or down?

Example:

Round 8272 down to 8000.

A. Round 6439 ______ to ______.

B. Round 354 ______ to ______.

C. Round 2511 ______ to ______.

D. Round 289 ______ to ______.

E. Round 535 ______ to ______.

F. Round 7500 ______ to ______.

G. Round 1499 ______ to ______.

III. Match the factors to the estimated products.

FACTORS	ESTIMATED PRODUCTS
1253 x 296	3,200,000
348 x 6811	1,000,000
7506 x 412	300,000
9289 x 554	2,100,000
249 x 4649	5,400,000

IV. Round factors before you multiply. Shade in the circle of the correctly estimated product on the answer grid at the bottom of the page.

Sample: 4924 5 000
x 311 x 300
1,500,000

ⓐ 150,000 ⓒ 1,600,000
ⓑ 1,500,000 ⓓ 15,000

1. 6735
x 436

ⓐ 2,400,000 ⓒ 3,500,000
ⓑ 280,000 ⓓ 2,800,000

2. 5319
x 762

ⓔ 4,000,000 ⓖ 3,500,000
ⓕ 400,000 ⓗ 4,800,000

3. 2730
x 637

ⓐ 180,000 ⓒ 1,800,000
ⓑ 18,000,000 ⓓ 1,200,000

4. 2499
x 854

ⓔ 2,400,000 ⓖ 1,600,000
ⓕ 2,700,000 ⓗ 1,800,000

ANSWER GRID

Sample:	ⓐ	●	ⓒ	ⓓ
1.	ⓐ	ⓑ	ⓒ	ⓓ
2.	ⓔ	ⓕ	ⓖ	ⓗ
3.	ⓐ	ⓑ	ⓒ	ⓓ
4.	ⓔ	ⓕ	ⓖ	ⓗ

P.O. 6.04.01.03

NAME: ______________________ CLASS: _______ DATE: ______________

I. Find the products. Match your answers with those found in the message. Write the letter of the example above the matching product to complete the message below.

Ⓥ 5 116 × 842

Ⓗ 3 652 × 421

Ⓛ 1 527 × 345

Ⓜ 4 377 × 219

MESSAGE

W	E	______	O	______	E	______	A	T	______
		526,815		4,307,672		958,563			1,537,492

II. Using the examples above, shade in the circle of the correct answer for each question.

1. In example V the multiplier is: ⓐ 5116 ⓑ 842 ⓒ 10,232
2. In example H the multiplicand is: ⓐ 1,537,492 ⓑ 3652 ⓒ 421
3. The second partial product in example L is: ⓐ 526,815 ⓑ 7635 ⓒ 61,080
4. In example M the product is: ⓐ 958,563 ⓑ 219 ⓒ 39,393

Do you remember?

What is the value of each underlined digit?

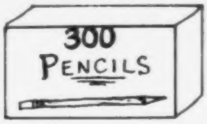
1. 92$\underline{4}$,407,030 ⓐ 4 million ⓑ 4 ones ⓒ 40 thousand
2. 378,3$\underline{2}$5,900 ⓐ 2 thousand ⓑ 20 million ⓒ 20 thousand
3. $\underline{3}$6,500,000,000 ⓐ 3 billion ⓑ 30 million ⓒ 30 billion
4. 5$\underline{6}$1,571,000,000 ⓐ 60 billion ⓑ 60 million ⓒ 600 billion
5. 589,$\underline{8}$90,000,000 ⓐ 80 million ⓑ 8 million ⓒ 8 hundred million
6. $\underline{9}$25,000,000,000 ⓐ 9 billion ⓑ 900 million ⓒ 900 billion

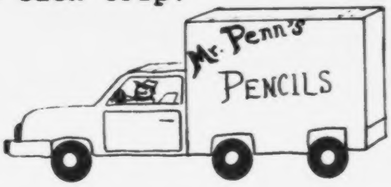
P.O. 6.04.01.04

NAME: ______________________ **CLASS:** ______ **DATE:** ______________

I. Mr. Penn uses a truck to deliver boxes of pencils to the schools. His truck carries 2000 boxes of pencils on each trip.

One box of pencils:





1. How many pencils does Mr. Penn deliver in one trip? __________
2. If Mr. Penn makes 200 trips each year, how many boxes of pencils does he deliver? __________

II.

Baseball Ticket Prices	
Adults	$8.00
Children	$4.75

a. There were 4132 children at the ball park. About how much money was spent for children's tickets? (estimate) __________

b. How much did the children's tickets actually cost?

____

c. Five thousand adult tickets were purchased. How much money was spent for the adults' tickets?

____

III.

Sam's Concession

Food Prices	
Frankfurter	$2.25
Popcorn	.85
Soda	1.00

a. How much did Kim spend for a frankfurter, a soda, and popcorn? __________

b. How much change did she receive if she paid with a $5 bill? __________

c. If 2113 people bought frankfurters at the ball park, how much money was spent for all the frankfurters combined?

____

d. How much money did Mr. Garcia spend for three frankfurters and a soda? __________

SHARPENING YOUR SKILLS

Units 19, 20, 21

Name: ____________________ Class: ____________ Date: ________

I. Find the equivalent metric measurement to solve the coded message.

900 cm - T
8.6 km - T
913.0 m - S
12.5 cm - E
16.0 m - G

4.26 dL - O
342.0 mL - M
3.5 kg - E
627 - S
7.8 g - L

Coded message:

________ ________ ________, ________ ________ ________
0.0078 mg 125 mm 9 m 0.913 km 160dm 426mL

________ ________ ________ ________
0.342 L 3500 g 8600 mL 0.627 mg

II. Estimate and solve. Use the chart to solve the problems below.

Major Rivers in North America	
River	Miles
Alabama	729
Allegheny	610
Delaware	390

	Estimate	Solve
1. The Alabama river is _____ yards long.	________	_____
2. The Delaware river is _____ yards shorter than the Allegheny river.	________	_____
3. The Alabama river and the Allegheny river combined are _____ yards long.	________	_____

III. Solve the following word problem.

CIMS University has a faculty of 5000 professors. If each professor graded 200 assignments each week, what is the total number of assignments graded at the Cims University?

____

IV. Round each factor to the left-most place to find the estimated product.

a. 1420 rounds to ________
x 180 rounds to x ________

____
estimated product

b. 5103 rounds to ________
x 223 rounds to x ________

____
estimated product

6.14.08 REVIEW: DIVISION

(1 lesson/ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

6.14.08.01 divide a 4- or 5-digit dividend by a 2-digit divisor and check;
e.g.,

$$\begin{array}{r} 268 \\ 24\overline{)6433} \\ \underline{-48}\downarrow \\ 163 \\ \underline{-144}\downarrow \\ 193 \\ \underline{-192} \\ 1 \end{array} \quad \text{or} \quad \begin{array}{rr} 268 & \\ 24\overline{)6433} & \\ \underline{-4800} & \underline{(200 \times 24)} \\ 1630 & \\ \underline{-1440} & \underline{(60 \times 24)} \\ 193 & \\ \underline{-192} & \underline{(8 \times 24)} \\ 1 & \end{array} \qquad \begin{array}{r} \underline{\text{check}} \\ 268 \\ \underline{\times 24} \\ 1072 \\ \underline{5360} \\ 6432 \\ \underline{+\quad 1} \\ 6433 \end{array}$$

The answer is $268\frac{1}{24}$.

6.14.08.02 create and solve word problems related to skills learned in this module.
e.g.,

A citrus packing plant shipped 10,125 oranges. If they used 45 crates, and placed the same number of oranges in each crate, how many oranges were in each crate?

PROBLEM EXPLORATION SKILLS:

- to estimate
- to identify subgoals

6.14.08

VOCABULARY	**SYMBOLS**
divisor	÷
dividend	$\overline{)\quad\quad}$
quotient	

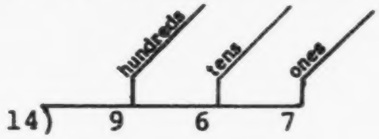
PREVIOUS UNDERSTANDINGS:

- division of a 3-digit dividend by a 1- or 2-digit divisor
- multiplication of a 3-digit number by a 1- or 2-digit number

---

GETTING STARTED:

Write the place value names above the following division example:



Ask the students to write the example and the place value chart in their notebooks. Say, "How many digits are in the dividend? (3) What is the number in the hundreds place? (9) ...in the tens place? (6) ...in the ones place? (7) How many digits are in the divisor? (2) If we have a 2-digit divisor and a 3-digit dividend, how can we estimate the quotient?" (First we should ask ourselves whether or not the quotient is greater than 10. We can multiply the divisor by 10.) Ask a student to solve the example on the chalkboard, e.g., 10 x 14 = 140.

Say, "How does this help you? (Since 140 < 967, our quotient is greater than 10.) How can we continue to estimate our quotient?" (We multiply our divisor by 100.) Ask a student to solve the example at the chalkboard, e.g., 100 x 14 = 1400.

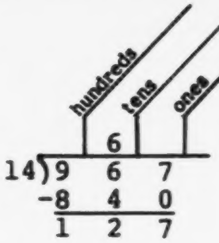
Say, "How does this help us? (Since 10 x 14 = 140 and 100 x 14 = 1400, we can judge whether our quotient is closer to 10 or to 100.) What have we learned about our quotient? (It is greater than 10 but less than 100.) Let's continue to multiply the divisor by multiples of 10." Develop the following information on the chalkboard:

20 x 14 = 280	280 < 967
30 x 14 = 420	420 < 967
40 x 14 = 560	560 < 967
50 x 14 = 700	700 < 967
60 x 14 = 840	840 < 967
70 x 14 = 980	980 > 967

GETTING STARTED: (cont'd)

Say, "What does this information tell us?" (Our quotient will be a number greater than 60 but less than 70.) Ask a student to do the computation at the chalkboard while the other students work in their notebooks.

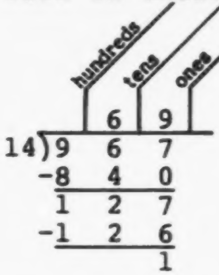
e.g.,


hundreds
tens
ones
6
14)9 6 7
-8 4 0
1 2 7

(60 x 14)

Say, "How much is left to divide? (127) Can we use the information on the chalkboard to divide 127 by 14? (Yes, we already know 10 x 14 = 140 and 127 is close to 140 so we can try 9 groups of 14.) How much is 9 groups of 14?" (9 x 14 = 126 and 126 < 127) Ask a student to continue the computation at the chalkboard while the other students complete the work in their notebooks.

e.g.,

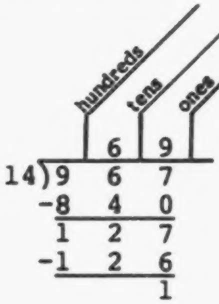

hundreds
tens
ones
6 9
14)9 6 7
-8 4 0
1 2 7
-1 2 6
1

(60 x 14)

(9 x 14)

Say, "Can we subtract any more groups of 14? (no) Can you write the remainder as a fraction? (yes, $\frac{1}{14}$) How can we show this quotient is correct?" (We can check by multiplication and addition.) Ask a student to show the check at the chalkboard as the other students work at their seats.

e.g.,


hundreds
tens
ones
6 9
14)9 6 7
-8 4 0
1 2 7
-1 2 6
1

(60 x 14)

(9 x 14)

check

$$\begin{array}{r} 69 \\ \underline{\times 14} \\ 276 \\ \underline{+690} \\ 966 \\ \underline{+\ \ 1} \\ 967 \end{array}$$

Quotient: $69\frac{1}{14}$

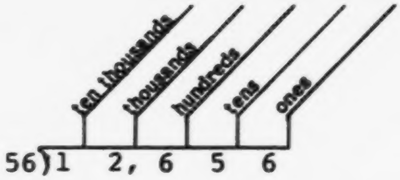
6.14.08

6.14.08.01 **divide a 4- or 5-digit dividend by a 2-digit divisor and check**

STRATEGY 1:

Write the following example on the chalkboard: 56)12,656. Ask a student to write place value names above the digits in the dividend.

e.g.,



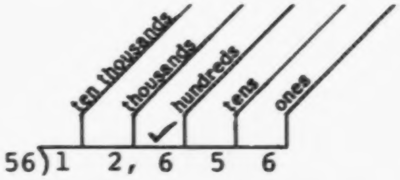
Say, "Let's find the number of digits in the quotient. We can multiply the divisor by 10, 100, and 1000 to determine where to write the first digit of the quotient." Ask a student to show this on the chalkboard as the other students work at ther seats.

e.g.,

10 x 56 = 560   560 < 12,656
100 x 56 = 5600   5600 < 12,656
1000 x 56 = 56,000   56,000 > 12,656

Say, "What can we conclude from this information? (Our quotient will be less than 1000 but greater than 100.) Where will our first digit appear in the place value chart?" (the hundreds place) Ask a student to put a checkmark in the hundreds place.

e.g.,



Say, "We can multiply the divisor by multiples of 100 to find the number that will appear in the hundreds place." Ask a student to record this information on the chalkboard as the other students work at their desks.

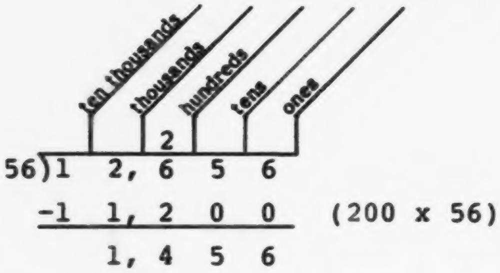
e.g.,

100 x 56 = 5600   5600 < 12656
200 x 56 = 11200   11200 < 12656
300 x 56 = 16800   16800 > 12656

STRATEGY 1: (cont'd)

Say, "What does this information tell us? (Our quotient will be greater than 200 but less than 300.) Ask a student to erase the check mark, write a 2 in the hundreds place, and divide while the other students work in their notebooks.

e.g.,


ten thousands
thousands
hundreds
tens
ones
2
56)1 2, 6 5 6
-1 1, 2 0 0 (200 x 56)
1, 4 5 6

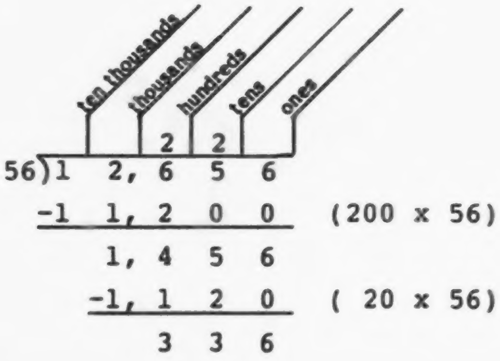
Say, "How can we continue our procedure to find the number that belongs in the tens place?" (We can multiply the divisor by multiples of 10.) Ask a student to record this information on the chalkboard as the other students work in their notebooks.

e.g.,

10 x 56 = 560   560 < 1456
20 x 56 = 1120   1120 < 1456
30 x 56 = 1680   1680 > 1456

Say, "What can we conclude from this information?" (The digit in the tens place will be a 2.) Ask a student to show the computation on the chalkboard as the other students work in their notebooks.

e.g.,


ten thousands
thousands
hundreds
tens
ones
2 2
56)1 2, 6 5 6
-1 1, 2 0 0 (200 x 56)
1, 4 5 6
-1, 1 2 0 (20 x 56)
3 3 6

Follow the same procedure to determine the number in the ones place. Ask a student to complete the work at the chalkboard as the other

6.14.08

STRATEGY 1: (cont'd)

students work in their notebooks.

e.g.,

```
      ten thousands  thousands  hundreds  tens  ones
                        2   2   6
    56)1   2   6   5   6
      -1   1   2   0   0     (200 x 56)
           1   4   5   6
          -1   1   2   0     ( 20 x 56)
               3   3   6
              -3   3   6     (  6 x 56)
                       0
```

Say, "What is the quotient? (226) How can we check our quotient?" (We can multiply the quotient by the divisor. The result should be the dividend.)

e.g.,

```
     check
      226
     x 56
    1 356
  +11 300
   12,656
```

Repeat this procedure to divide other 4- and 5-digit dividends by 2-digit divisors.

STRATEGY 2:

Write the following on the chalkboard: 62)96,543.

Say, "We are going to use the 4-step method of division to solve this example. What is the 4-step method of division? (divide, multiply, subtract, bring down) How can we find the first digit in the quotient?" (We can multiply the divisor by multiples of 1000.) Ask a student to record this information on the chalkboard as the other students write in their notebooks.

STRATEGY 2: (cont'd)

e.g.,

1000 x 62 = 62,000   62,000 < 96,543

2000 x 62 = 124,000   124,000 > 96,543

Say, "What does this information tell us?" (Since 62,000 is less than 96,543 and 124,000 is greater, the quotient will fall between 1000 and 2000. Therefore, its first digit will be a 1 in the thousands column.) Ask a student to show the computation on the chalkboard.

e.g.,

```
     1
62)96543
  -62↓
   345
```

Say, "We can now divide 345 by 62. Since 300 + 60 = 5, let's try multiplying 62 by 5 and by 6." Ask a student to write this information on the chalkboard as the others write it in their notebooks.

e.g.,

5 x 62 = 310   310 < 345

6 x 62 = 372   372 > 345

Say, "Which number belongs in the hundreds place?" (5) Continue the computation at the chalkboard.

e.g.,

```
     15
62)96543
  -62↓|
   345|
  -310↓
    354
```

Say, "Using the same procedure we can place a 5 in the tens column."

e.g.,

```
     155
62)96543
  -62↓||
   345||
  -310↓|
    354|
   -310↓
     443
```

Say, "How can we find the number which belongs in the ones place?" (We can start with 6 because we know 6 x 62 = 372.) Ask a student to record the computation at the chalkboard.

STRATEGY 1: (cont'd)

e.g.,

$6 \times 62 = 372$ $372 < 443$

$7 \times 62 = 434$ $434 < 443$

$8 \times 62 = 496$ $496 > 443$

Say, "Which number belongs in the ones place?" (7) Ask a student to continue the computation at the chalkboard as the other students record it in their notebooks.

e.g.,

$$\begin{array}{rr}
 & 1557 \\
62 & \overline{)96543} \\
 & \underline{-62} \\
 & 345 \\
 & \underline{-310} \\
 & 354 \\
 & \underline{-310} \\
 & 443 \\
 & \underline{-434} \\
 & 9
\end{array}$$

Say, "What is our remainder?" (9) How can we express our remainder as a fraction? $\left(\frac{9}{62}\right)$ What is the quotient for our division example?" $\left(1557\frac{9}{62}\right)$ Ask students to complete the computation in their notebooks. Say, "How can we check if our quotient is correct?" (We can use multiplication and addition.)

e.g.,

$$\begin{array}{r}
1\ 557 \\
\underline{\times\ \ 62} \\
3\ 114 \\
\underline{+93\ 420} \\
96\ 534 \\
\underline{+\ \ \ \ 9} \\
96{,}543
\end{array}$$

6.14.08.02 **create and solve word problems related to skills learned in this module**

STRATEGY 1:

Write the following problem on the chalkboard:

STRATEGY 1: (cont'd)

> The school librarian will distribute books to each class in the school in order to create reading centers. If the library has 2793 books to distribute equally among 19 classes in the school, how many books will each class receive?

Have a student read the problem aloud. Discuss with the class the information needed to solve the problem. Say, "What question does the problem ask? (How many books will each class receive?) What necessary information are we given? (the total number of books and the total number of classes) What basic operation should we perform in order to solve the problem? (division) What would you estimate the answer to be?" (between 100 and 200) Ask a student to solve the problem on the chalkboard as the others write it in their notebooks. Say, "Check your answer."

e.g.,

```
                                check
     147
  19)2793                        147
    -19↓|                       x 19
      89|                       1323
     -76↓                      +1470
      133                       2793
     -133
        0
```

Say, "Do we have a remainder? (no) What is the answer to the problem?" (147 books per class)

Repeat this procedure by asking the students to create, solve and check other examples.

P.O. 6.14.08.01

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

1. The quotient of $27\overline{)2322}$ is between __________.

ⓐ 70 and 80 ⓒ 800 and 900

ⓑ 80 and 90 ⓓ 90 and 100

2. $32\overline{)3\ 1\ 0\ 4}$ = ______.

ⓔ 8 ⓖ 9

ⓕ 4 ⓗ NG

3. Solve. Express the remainder as a fraction.

$47\overline{)1\ 3\ 6\ 9}$

check

4. Solve. Express the remainder as a fraction.

$61\overline{)2\ 3\ 1\ 9}$

check

5. Solve using the 4-step method.

$79\overline{)6\ 4,\ 2\ 7\ 6}$

check

6. Solve using the 4-step method.

$37\overline{)7\ 8,\ 2\ 1\ 8}$

check

Do you remember?

Use the common denominator method to determine whether each pair of fractions is equivalent. Shade the correct choice.

1. $\frac{2}{6}, \frac{5}{15}$ $\frac{2}{6}$ ○= ○≠ $\frac{5}{15}$
2. $\frac{4}{5}, \frac{14}{18}$ $\frac{4}{5}$ ○= ○≠ $\frac{14}{18}$
3. $\frac{18}{30}, \frac{3}{5}$ $\frac{18}{30}$ ○= ○≠ $\frac{3}{5}$
4. $\frac{12}{15}, \frac{5}{6}$ $\frac{12}{15}$ ○= ○≠ $\frac{5}{6}$
5. $\frac{2}{6}, \frac{9}{27}$ $\frac{2}{6}$ ○= ○≠ $\frac{9}{27}$
6. $\frac{4}{10}, \frac{7}{15}$ $\frac{4}{10}$ ○= ○≠ $\frac{7}{15}$

P.O. 6.14.08.02

NAME: ______________________ CLASS: _______ DATE: ____________

Mixed Problem Solving

1. The Intrepid is a floating museum that is docked along the Hudson River. Sailors aboard the Intrepid teach visitors about World War II. Last year 37,428 people visited the ship. Find the average number of visitors each month. solve check

____

2. The Intrepid depends on volunteers to help carry out its many programs. Over a five year period, 85 volunteers worked a total of 78,260 hours, each working the same number of hours. How many hours did each volunteer work? solve check

____

3. There are 37 separate exhibits aboard the Intrepid. If 65 people visit each exhibit every hour, how many visitors are watching the exhibits each hour? solve check

____

4. Last year each visitor to the Intrepid spent about $10.00 on souvenirs. About how much money was spent? solve check

____

6.05.01 DIVISION BY MULTIPLES OF 100

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

*6.05.01.01A divide a multiple of 1000, 10,000, or 100,000 by a multiple of 100 and check;

e.g.,

$$\begin{array}{r} 40 \\ 200\overline{)8000} \end{array} \qquad \begin{array}{r} \underline{\text{check}} \\ 200 \\ \underline{\times\ 40} \\ 8000 \end{array}$$

6.05.01.02A divide a 4-, 5-, or 6-digit dividend by a multiple of 100 and check;

e.g.,

$$\begin{array}{r} 15 \\ 500\overline{)7551} \\ \underline{-500}\downarrow \\ 2551 \\ \underline{-2500} \\ 51 \end{array} \quad \text{or} \quad \begin{array}{rl} 15 & \\ 500\overline{)7551} & \\ \underline{-5000} & \underline{(10 \times 500)} \\ 2551 & \\ \underline{-2500} & \underline{(5 \times 500)} \\ 51 & \end{array} \qquad \begin{array}{r} \underline{\text{check}} \\ 500 \\ \underline{\times\ 15} \\ 2500 \\ \underline{5000} \\ 7500 \\ \underline{+\ 51} \\ 7551 \end{array}$$

The answer is $15\frac{51}{500}$.

6.05.01.03A create and solve word problems related to skills learned in this module.

e.g.,

The Acme Printing Company printed 2460 copies of a world atlas. They shipped these books in cartons of 200 books each. How many full cartons were shipped?

PROBLEM EXPLORATION SKILLS:

- to consider a related but simpler problem
- to state the goals of a problem clearly
- to find approximate solutions
- to use estimation to check the reasonableness of an answer

VOCABULARY

dividend	quotient
divisor	partial quotient

PREVIOUS UNDERSTANDINGS:

- division of 4- and 5-digit dividends by 2-digit divisors

GETTING STARTED:

Display the following:

> Mr. Millstone baked 600 cookies and wanted to separate them into packages of 10. How many packages can he make?

Say, "What operation should we use to find the answer?" (Since Mr. Millstone is dividing a batch of 600 cookies evenly by ten, we will use division.) Display the problem and ask the students to solve and check their answer.

e.g.,

$600 \div 10 = 60$

check

$60 \times 10 = 600$

Say, "Suppose Mr. Millstone decided that each package should contain 20 cookies or 30 cookies. How many packages of cookies would there now be in each case?" Ask the students to find and check the answers at their desks while one or two students work at the chalkboard.

e.g.,

$600 \div 10 = 60$

$600 \div 20 = 30$

$600 \div 30 = 20$

check

$$\begin{array}{rrr} 60 & 30 & 20 \\ \underline{\times 10} & \underline{\times 20} & \underline{\times 30} \\ 600 & 600 & 600 \end{array}$$

Say, "What pattern can you discover when dividing or multiplying multiples of tens? (Answers may vary. Elicit that, in division, the divisor and quotient together have the same number of zeros as the dividend. In multiplication, the factors have the same numbers of zeros together as the product.) Is there another way we can arrive at the same answers? Explain." (Yes. We can eliminate the same number of zeros from the divisor and dividend and then divide.) Ask a student to demonstrate this information on the chalkboard as the other students work in their notebooks.

e.g.,

$60\cancel{0} \div 1\cancel{0} = 60 \div 1 = 60$ 60 tens ÷ 1 ten = 60 ÷ 1 = 60

$60\cancel{0} \div 2\cancel{0} = 60 \div 2 = 30$ 60 tens ÷ 2 tens = 60 ÷ 2 = 30

$60\cancel{0} \div 3\cancel{0} = 60 \div 3 = 20$ 60 tens ÷ 3 tens = 60 ÷ 3 = 20

6.05.01

6.05.01.01A divide a multiple of 1000, 10,000, or 100,000 by a multiple of 100 and check

STRATEGY 1:

Write the following example on the chalkboard:

$$500\overline{)3,000}$$

Say, "What do we know about the numbers in this example? (They are both multiples of 100.) How can we simplify this problem? (We can remove two zeros from the right of both the divisor and the dividend.) What is the meaning of what we have done?" (We have divided both the divisor and the dividend by 100.) Ask a student to do the work at the chalkboard while the other students work in their notebooks.

e.g., $500\overline{)3000} = 5\cancel{0}\cancel{0}\overline{)30\cancel{0}\cancel{0}} = 5\overline{)30}$

Say, "Now we can solve the simpler problem, 30 ÷ 5 = ___. What is its quotient? (6) Therefore, 3000 ÷ 500 is also equal to 6. How can we show our quotient is correct?" (We can check by multiplication. 500 x 6 = 3000)

Say, "When we divide large multiples that end in zeros, we can use this method of crossing out zeros, dividing, and then checking to make sure our quotients are correct." Write the following on the chalkboard:

(a) $60,000 \div 100 = 60,0\cancel{0}\cancel{0} \div 1\cancel{0}\cancel{0} = 600 \div 1 = 600$
(b) $60,000 \div 200 = 60,0\cancel{0}\cancel{0} \div 2\cancel{0}\cancel{0} = 600 \div 2 = 300$
(c) $60,000 \div 300 = 60,0\cancel{0}\cancel{0} \div 3\cancel{0}\cancel{0} = 600 \div 3 = 200$
(d) $60,000 \div 400 = 60,0\cancel{0}\cancel{0} \div 4\cancel{0}\cancel{0} = 600 \div 4 = 150$
(e) $60,000 \div 500 = 60,0\cancel{0}\cancel{0} \div 5\cancel{0}\cancel{0} = 600 \div 5 = 120$
(f) $60,000 \div 600 = 60,0\cancel{0}\cancel{0} \div 6\cancel{0}\cancel{0} = 600 \div 6 = 100$
(g) $60,000 \div 700 = 60,0\cancel{0}\cancel{0} \div 7\cancel{0}\cancel{0} = 600 \div 7 = 85\frac{5}{7}$

Say, "Notice that item (g) has a remainder. What patterns do you notice? (As the dividend remains the same and the divisor increases, the quotient decreases.) Study the patterns in the following sets of problems."

e.g.,

(A)	(B)
$8000\cancel{0}\cancel{0} \div 2\cancel{0}\cancel{0} = 4000$	$800,0\cancel{0}\cancel{0} \div 8\cancel{0}\cancel{0} = 1000$
$800\cancel{0}\cancel{0} \div 2\cancel{0}\cancel{0} = 400$	$800,0\cancel{0}\cancel{0} \div 4\cancel{0}\cancel{0} = 2000$
$80\cancel{0}\cancel{0} \div 2\cancel{0}\cancel{0} = 40$	$800,0\cancel{0}\cancel{0} \div 2\cancel{0}\cancel{0} = 4000$
	$800,0\cancel{0}\cancel{0} \div 1\cancel{0}\cancel{0} = 8000$

STRATEGY 1: (cont'd)

Say, "What patterns do you notice in problem set A? (As the dividend decreases and the divisor remains the same, the quotient decreases. What occurs in problem set B? (As the dividend remains the same, and the divisor decreases, the quotient increases.) Can you discover a pattern about the zeros in each problem?" (Elicit that in set A and set B, the dividend always has the same number of zeros as both the divisor and quotient combined.)

Repeat this procedure to divide and check other multiples of 1000, 10,000, and 100,000 by a multiple of 100 in both horizontal and vertical formats.

6.05.01.02A **divide a 4-, 5-, or 6-digit dividend by a multiple of 100 and check**

STRATEGY 1:

Display the following problem on the chalkboard:

> Two hundred sixth grade students from P.S. 444 raised $4600 selling raffles at their annual school fundraiser. How much money did each student raise if they each raised equal amounts?

Say, "What operation is needed to solve this problem?" (division) Ask a student to write the algorithm on the chalkboard as the other students write in their notebooks.

e.g.,

$$200\overline{)4600}$$

Say, "When we begin to divide by a 3-digit divisor such as 200, we must determine whether the first 3 digits in the dividend of 4600 can be divided by 200. Since these first 3 digits are 4-6-0, we can divide it by 200. How many multiples of 200 are there in 460? (2) The digit 2 is our first partial quotient and is written above the last digit in 460 into which we divided." Demonstrate.

e.g.,

$$\begin{array}{r} 2 \\ 200\overline{)4600} \end{array}$$

STRATEGY 1: (cont'd)

Say, "This division step is the first of four steps necessary in dividing any number successfully. What are the other three steps?" (After dividing, we multiply, then subtract, and then bring down the next digit, repeating, in order, the same steps until the full quotient is found.) Ask a student to complete these steps at the chalkboard as the other students work at their desks.

e.g.,

$$\begin{array}{r} 23 \\ 200\overline{)4600} \\ \underline{-400}\downarrow \\ 600 \\ \underline{-600} \\ 0 \end{array} \qquad\qquad \begin{array}{r} \underline{\text{check}} \\ 200 \\ \underline{\times\ \ 23} \\ 600 \\ \underline{4000} \\ 4600 \end{array}$$

Say, "What is the quotient and what does it represent? (The quotient of 23 represents $23, the amount of money each student raised.) If you recall the patterns which exist in division, predict what the quotient would be if the dividend were 46,000 instead of 4600." (The quotient would be 230 instead of 23.)

Repeat this question for a dividend of 460,000 and record these patterns on the chalkboard.

e.g.,

$$\begin{array}{r} 23 \\ 200\overline{)4600} \end{array} \qquad \begin{array}{r} 230 \\ 200\overline{)46{,}000} \end{array} \qquad \begin{array}{r} 2\ 300 \\ 200\overline{)460{,}000} \end{array}$$

Say, "Let's look at another problem." Write the following problem on the chalkboard and ask the students to record it into their notebooks.

$$300\overline{)24{,}005}$$

Say, "Our divisor is now 300 and our dividend is a 5-digit number. How is the first division step for this problem different from the first step in the other problem?" (The first three digits in this dividend are not large enough for the divisor, 300, to divide into. The first four digits in the dividend must be used to divide by 300.) Demonstrate.

e.g.,

$$\underline{300}\overline{)\underline{2400}5}$$

STRATEGY 1: (cont'd)

Say, "How many multiples of 300 are there in 2400? (8) Where will we write the partial quotient? (Above the last digit in 2400.) What would we do next? (Complete the next 3 steps in the division process and repeat if necessary until a full quotient is obtained.) Ask a student to complete the steps in this example as the other students work in their notebooks to find the quotient.

e.g.,

$$\begin{array}{r} 80\frac{5}{300} \\ 300\overline{)24005} \\ -\underline{2400}\downarrow \\ 5 \end{array}$$

Say, "What is the quotient? ($80\frac{5}{300}$) Can we express the remainder another way?" (Yes. We can reduce $\frac{5}{300}$ to $\frac{1}{60}$.) Ask a student to demonstrate this information on the chalkboard and check the problem.

e.g.,

$$\begin{array}{r} 80\frac{5}{300} = 80\frac{1}{60} \\ 300\overline{)24005} \\ -\underline{2400}\downarrow \\ 5 \end{array}$$

check

$$\begin{array}{r} 300 \\ \underline{\times\ 80} \\ 000 \\ \underline{2400} \\ 24\ 000 \\ \underline{+\qquad 5} \\ 24,005 \end{array}$$

Repeat this procedure using other 4-,5-, and 6-digit dividends divided by multiples of 100.

6.05.01.03A create and solve word problems related to skills learned in this module

STRATEGY 1:

Display the following word problem:

> The Nutty Peanut Butter Company produced 240,000 jars of peanut butter and has 400 trucks to help deliver it to the grocery stores. If each truck delivers the same number of jars, how many jars will each truck hold?

Ask a student to read the problem aloud. Say, "What were we asked to

STRATEGY 1: (cont'd)

find? (We were asked to find how many jars of Nutty peanut butter each truck can carry.) What operation should we use to solve this problem? (division) What is the dividend? (240,000) What do we call the 400?" (the divisor) Write the problem on the chalkboard.

e.g., $400\overline{)240,000}$

Say, "Can we make this into a simpler problem? Explain." (Yes, we can eliminate 2 zeros from both the 400 and the 240,000 and then divide and check. Demonstrate.

e.g.,

$$4\cancel{0}\cancel{0}\overline{)2400\cancel{0}\cancel{0}} \quad = \quad \begin{array}{r} 600 \\ 4\overline{)2400} \\ \underline{-2400} \\ 0 \end{array} \qquad \begin{array}{r} \underline{\text{check}} \\ 600 \\ \underline{\times \quad 4} \\ 2400 \end{array}$$

Say, "How many jars did each truck carry? (600) When we changed the original problem into a simpler one, did this change the quotient?" (No, 240,000 + 400 also equals 600.)

Expand the original problem by writing the following on the chalkboard:

> If the number of jars of peanut butter produced in the month of April was 85,225 and 300 jars were delivered to each store, how many stores received a shipment?

Say, "In this word problem, will our quotient represent the number of jars of peanut butter delivered? Explain." (No, the quotient will represent the number of stores receiving the jars.) Ask a student to solve the problem at the chalkboard as the others solve it in their notebooks.

e.g.,

$$\begin{array}{r} 284\frac{25}{300} \\ 300\overline{)85,225} \\ \underline{-60\ 0}\downarrow\ \ \\ 25\ 22\ \ \\ \underline{-24\ 00}\downarrow \\ 1\ 225 \\ \underline{-1\ 200} \\ 25 \end{array} \quad \text{or} \quad 284\frac{1}{12}$$

Say, "How is this quotient different from the quotient in the prewious

STRATEGY 1: (cont'd)

problem? (It has a remainder.) What does the quotient tell us? (284 stores each received a shipment of 300 jars of peanut butter) What does the 25 remaining jars signify?" (There are 25 jars that were not part of the shipment to each store. There are 25 jars left over.)

Repeat this procedure to solve other word problems, some of which are created by the students, to divide other 4-, 5-, or 6 digit dividends by multiples of 100.

P.O. 6.05.01.01-.02

NAME: ______________________ CLASS: _______ DATE: ______________

I. Draw a line from the division example in Column A to the division example in Column B that will have the same quotient.

A	B
300)180,000	4)280
400)28,000	1800 + 3
7200 + 9	7)560
700)1400	4900 + 7
5600 + 70	90)72,000
70)49,000	7)14

II. Solve and check each problem.

1. 700)56,000 check

2. 600)666,010 check

III. For each division problem, find the quotient. Shade the circle for the correct answer.

1. 400)28,000

ⓐ 7000 ⓒ 70
ⓑ 700 ⓓ NG

---

2. 300)900,000

ⓔ 300 ⓖ 3000
ⓕ 3 ⓗ NG

---

3. 200)4862

ⓐ 24 ⓒ 62 R24
ⓑ 24 R62 ⓓ NG

---

4. 500)93,500

ⓔ 186 ⓖ 187
ⓕ 187 R400 ⓗ NG

---

5. 620,080 + 100 = ☐

ⓐ 620 R80 ⓒ 6200 R8
ⓑ 62 R80 ⓓ NG

CHALLENGE!

Hard Times

1. How many centuries are there in 3744 months? ______________
2. How many centuries are there in 48,000 decades? ______________

P.O. 6.05.01.03

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

Use the chart to answer each question.

CIRCUMFERENCES OF PLANETS IN OUR SOLAR SYSTEM

PLANET	CIRCUMFERENCE	PLANET	CIRCUMFERENCE	PLANET	CIRCUMFERENCE
Mercury	15,000 km	Mars	21,000 km	Uranus	159,500 km
Venus	38,000 km	Jupiter	450,000 km	Neptune	152,600 km
Earth	40,000 km	Saturn	379,000 km	Pluto	94,000 km

Note: Different planets have different atmospheres so a space ship may use varying amounts of fuel to travel the same distance around the various planets.

1. A spaceship traveling around the circumference of Mars can go 700 km on each tank of fuel. How many tanks are needed to go completely around Mars? ____________
2. A spaceship can go about 300 km on a tank of fuel. How many tanks of fuel are needed to go around Neptune? ______________________
3. Saturn's atmosphere is very difficult to travel through. If a spaceship traveling around Saturn uses a tankful of fuel to travel 100 km, how many tanks are necessary to travel it's circumference?

   ____
4. a) Which planet has the greatest circumference? ______________
   b) Which planet has the smallest circumference? ______________
   c) If a spaceship can travel 500 km per tank of fuel, how many tanks of fuel will it need to circle around the planet with the greatest circumference? ______________
   d) How many tanks of fuel will it use to circle around the smallest planet? ______________
5. If a spaceship circling Pluto uses 47 tanks of fuel, how many kilometers can it travel on one tank of fuel? ____________________
   a) How many tanks of fuel must it use to travel around the circumference three times? ____________

6.05.02 DIVISION BY 3-DIGIT DIVISORS

(3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

6.05.02.01A estimate the quotient;

e.g.,

$$317\overline{)9210} \qquad 9210 \div 317 = \square$$

$$\square \times 317 = 9210$$

$$\begin{array}{ll} 10 \times 317 = 3170 & 3170 < 9210 \\ 20 \times 317 = 6340 & 6340 < 9210 \\ 30 \times 317 = 9510 & 9510 > 9210 \end{array}$$

Therefore, $20 < \square < 30$.

6.05.02.02A divide a 4-, 5-, or 6-digit number by a 3-digit divisor and check;

e.g.,

$$\begin{array}{r} 29 \\ 317\overline{)9210} \\ \underline{-634}\downarrow \\ 2870 \\ \underline{-2853} \\ 17 \end{array} \quad \text{or} \quad \begin{array}{rl} 29 & \\ 317\overline{)9210} & \\ \underline{-6340} & \underline{(20 \times 317)} \\ 2870 & \\ \underline{-2853} & \underline{(9 \times 317)} \\ 17 & \end{array} \qquad \begin{array}{r} \underline{\text{check}} \\ 317 \\ \underline{\times\ 29} \\ 2853 \\ \underline{6340} \\ 9193 \\ \underline{+\ 17} \\ 9210 \end{array}$$

The answer is $29\frac{17}{317}$.

6.05.02.03A create and solve word problems related to skills learned in this module.

e.g.,

The 6th grade students of the Northside School collected $1575 for UNICEF. If there are 225 students in the 6th grade, find the average amount collected by each student.

PROBLEM SOLVING SKILLS:

- to generalize procedures
- to use estimation to check the reasonableness of an answer

VOCABULARY

divisor
dividend
quotient

PREVIOUS UNDERSTANDINGS:

- round 4-, 5-, and 6-digit numbers to the left-most place
- division of 4-, 5-, and 6-digit dividends by two place divisors
- division by multiples of 100

GETTING STARTED:

Write the following information on the chalkboard and ask the students to write it in their notebooks:

$$64\overline{)2952}$$

$$\begin{array}{rcr} 10 \times 64 & = & 640 \\ 20 \times 64 & = & 1280 \\ 30 \times 64 & = & 1920 \\ 40 \times 64 & = & 2560 \\ 50 \times 64 & = & 3200 \end{array}$$

Say, "What does the information on the chalkboard tell us? (Our quotient will be greater than 40 but less than 50 since 2952 is between 2560 and 3200.) How does this estimate help us?" (We know that our quotient will have 2 digits and we will know whether or not our quotient is reasonable. Ask a student to solve the problem at the chalkboard as the other students work in their notebooks.

e.g.,

$$\begin{array}{r} 46\frac{8}{64} = 46\frac{1}{8} \\ 64\overline{)2952} \phantom{= 46\frac{1}{8}} \\ -\underline{256\downarrow} \phantom{= 46\frac{1}{8}} \\ 392 \phantom{= 46\frac{1}{8}} \\ -\underline{384} \phantom{= 46\frac{1}{8}} \\ 8 \phantom{= 46\frac{1}{8}} \end{array}$$

6.05.02

6.05.02.01A estimate the quotient

STRATEGY 1:

Write a division example on the chalkboard, e.g., $706\overline{)89042}$. Say, "Before we begin to solve this problem, what can we do to make sure that our answer will be reasonable? (We can estimate the quotient.) How can we begin to estimate the quotient? Explain. (We can multiply the divisor, 706, by multiples of 10 and/or 100 until we discover an estimate that is reasonably close to our dividend.) First we should determine whether the quotient is between 10 and 100 or greater than 100." Write the following on the chalkboard as it is elicited from the students:

10 x 706 = 7060 7060 < 89,042
100 x 706 = 70,600 70,600 < 89,042

Say, "What does the information on the chalkboard tell us? (Our quotient will be greater than 100.) Could it be greater than 1000?" (No, 706 x 1000 = 706,000, which is greater than the dividend) Continue multiplying by multiples of 100.
e.g.,

10 x 706 = 7060 7060 < 89,042
100 x 706 = 70,600 70,600 < 89,042
200 x 706 = 141,200 141,200 > 89,042

Say, "What can we now conclude from the information on the chalkboard? Explain. (100 x 706 is less than the dividend and 200 x 706 is greater than the dividend. Therefore, the quotient will be greater than 100 but less than 200.) Let's further refine our estimate. Will the quotient be closer to 100 or 200? Explain." (The quotient will be closer to 100 because 70,600 is closer to the dividend than 141,200.) Write the following on the chalkboard:

Estimated Quotient: between and 100 and 200, closer to 100

Repeat this procedure to estimate the quotients for other 4-, 5-, or 6-digit dividends with 3-digit divisors. Calculators can later be used to verify these estimates.

6.05.02.02A divide a 4-, 5-, or 6-digit number by a 3-digit divisor and check

STRATEGY 1:

Adapt Strategy 1 or 2, P.O. 6.14.08.01, Unit 22.

6.05.02.03A create and solve word problems related to skills learned in this module

STRATEGY 1:

Write the following word problem on the chalkboard:

> The N.Y.C. Subway system can accommodate 547 riders on each subway train and has to transport 308,880 passengers between Brooklyn and Manhattan each morning during rush hour. How many subway trains will be needed to transport these passengers?

Ask a student to read the problem aloud. Say, "What information does the problem give us? (the number of passengers that can be accommodated on one train, and the total number of passengers that have to be transported) What are we being asked to find? (the number of trains needed) What operation should we use?" (division) Ask a student to write the division problem on the chalkboard as the other students write it in their notebooks.

e.g.,

$$547\overline{)308{,}880}$$

Say, "What can we do to make sure our answer will be reasonable? Explain." (We can estimate the quotient by multiplying the dividend by multiples of 100.) Write the following on the chalkboard as it is elicited from the students:

$100 \times 547 = 54{,}700$ $\quad 54{,}700 < 308{,}880$
$200 \times 547 = 109{,}400$ $\quad 109{,}400 < 308{,}880$
$300 \times 547 = 164{,}100$ $\quad 164{,}100 < 308{,}880$
$400 \times 547 = 218{,}800$ $\quad 218{,}800 < 308{,}880$

$500 \times 547 = 273{,}500 \qquad 273{,}500 < 308{,}880$

$600 \times 547 = 328{,}200 \qquad 328{,}200 > 308{,}880$

Say, "What does this information tell us?" (The quotient will be greater than 500 but less than 600.) Ask a student to solve and check the problem at the chalkboard as the other students solve and check it in their notebooks.

e.g.,

```
           564 372/547
547)308,880
   -273 5↓ |
      35 38|
     -32 82↓
       2 560
      -2 188
         372
```

check

```
    564
   x547
  3 948
 22 560
282 000
308 508
+   372
308,880
```

Say, "What was the answer to the division problem? (564 with a remainder of 372) How many trains are needed? Explain." (565 trains are needed. An additional train is needed for the overflow.)

Repeat this procedure to solve other word problems involving division of 4-, 5-, and 6-digit dividends by 3-digit divisors.

P.O. 6.05.02.01&.03

NAME: ______________________ **CLASS:** ________ **DATE:** ____________

I. Estimate the quotient for problems 1-4.

1) $675\overline{)29,025}$

The quotient is greater than ______ but less than ______.

2) $302\overline{)9786}$

The quotient is greater than ______ but less than ______.

---

3) $219\overline{)43,720}$

The quotient is greater than ______ but less than ______.

4) $520\overline{)498,622}$

The quotient is greater than ______ but less than ______.

II. Solve each of the following.

1) Mt. McKinley in the state of Alaska has an elevation of 20,320 feet. If a mountain climber climbs the mountain at the rate of 475 feet per hour, approximately how long will it take her to reach the top?

Approximately: ______________________________

2) One Friday evening a movie theater grossed $4667.25. If each person paid $5.25, approximately how many people attended the theater that evening?

Approximately: ______________________________

Do you remember?

Compare the following decimals using the symbols >, <, and =.

a. 0.45 ○ 0.7 c. 0.568 ○ 0.80 e. 0.85 ○ 0.850

b. 1.343 ○ 0.69 d. 0.25 ○ 0.155 f. 0.609 ○ 0.690

P.O. 6.05.02.01-.03

NAME: ______________________ **CLASS:** ________ **DATE:** ____________

I. **For examples 1-6, estimate the quotient, then solve and check the problem.**

1. $209\overline{)13298}$ estimate: __________ check

2. $420\overline{)376012}$ estimate: __________ check

3. $118\overline{)9479}$ estimate: __________ check

4. $365\overline{)20592}$ estimate: __________ check

5. $922\overline{)278645}$ estimate: __________ check

6. $815\overline{)39490}$ estimate: __________ check

II. **Solve the following problem.**

The Littletown Daily News sold 212,795 newspapers in 1987. What was the average number of newspapers sold daily? ________________

III. Create and solve your own problem.

____

____

____

SHARPENING YOUR SKILLS

Units 22, 23, 24

Name: ____________ **Class:** ________ **Date:** ______

Directions: For items 1-8, mark your answers on the answer grid below.

1. The quotient of $38\overline{)3542}$ is between ____________.

ⓐ 70 and 80 ⓒ 90 and 100
ⓑ 80 and 90 ⓓ 900 and 1000

2. Solve. Express the remainder as a fraction.

$$48\overline{)63{,}159}$$

ⓔ $1315\frac{39}{48}$ ⓖ $1315\frac{39}{63159}$
ⓕ $1315\frac{48}{39}$ ⓗ Not Here

3. Solve. Express the remainder as a fraction.

$$124{,}563 \div 63 = \square$$

ⓐ $10{,}997\frac{12}{63}$ ⓒ $1977\frac{12}{63}$
ⓑ $19{,}770\frac{12}{63}$ ⓓ Not Here

4. Find the missing digit.

$$\begin{array}{r} 5\square 76 \\ 83\overline{)421{,}308} \end{array}$$

ⓔ 0 ⓖ 5
ⓕ 3 ⓗ Not Here

5. $300\overline{)7248}$

ⓐ 204 R48 ⓒ 240 R4
ⓑ 24 R48 ⓓ Not Here

6. $600\overline{)42{,}308}$

ⓔ 700 R38 ⓖ 70 R308
ⓕ 70 R38 ⓗ Not Here

For items 7 and 8, estimate the quotient.

7. $156\overline{)284{,}129}$

The quotient is between _____ and _____.

ⓐ 1700, 1800 ⓒ 1900, 2000
ⓑ 1800, 1988 ⓓ Not Here

8. $243\overline{)563{,}148}$

The quotient is between _____ and _____.

ⓔ 2300, 2400 ⓖ 2100, 2200
ⓕ 2200, 2300 ⓗ Not Here

Answer Grid

1. ⓐ ⓑ ⓒ ⓓ
2. ⓔ ⓕ ⓖ ⓗ
3. ⓐ ⓑ ⓒ ⓓ
4. ⓔ ⓕ ⓖ ⓗ
5. ⓐ ⓑ ⓒ ⓓ
6. ⓔ ⓕ ⓖ ⓗ
7. ⓐ ⓑ ⓒ ⓓ
8. ⓔ ⓕ ⓖ ⓗ

6.12.01 STATISTICS - MODE AND MEDIAN

(1 lesson)

PERFORMANCE OBJECTIVES

The student will be able to...

*6.12.01.01A find the mode of a given set of data;
e.g.,

The mode is the number which appears most often in a set of data.

3, 9, 4, 2, 9, 8, 9

The mode in this set of data is 9.

6.12.01.02A find the median of a given set of numerical data.
e.g.,

For an odd number of items, the median is the middle number when the numbers are arranged in order from least to greatest.

2, 4, 5, 6, 7
↓
median

For an even number of items, we will define the median as the mean of the two middle terms when the terms are in numerical order.

7, 8, 12, 14, 17, 19

(12 + 14) + 2 = 13

The median is 13.

PROBLEM EXPLORATION SKILLS:

- to adopt a different point of view
- to compare and contrast
- to state the goal of a problem clearly
- to select a procedure appropriate to the goal of the problem

VOCABULARY

statistics
mode
median
range

PREVIOUS UNDERSTANDINGS:

- arranging numbers in increasing order
- finding the mean (average) of a series of numbers
- finding the range of a series of numbers

GETTING STARTED:

Materials:

- chart shown below

Display the following chart:

Date	Low Temperature	High Temperature
Nov. 10	56°	67°
Nov. 11	52°	62°
Nov. 12	60°	65°
Nov. 13	60°	63°
Nov. 14	57°	64°
Nov. 15	53°	59°
Nov. 16	54°	68°

Ask the students to examine the information presented on the chart. Say, "What data does this chart show?" (It shows the high temperature and the low temperature for each of seven days in November.)

How would we find the mean (average) high temperature for the week?" (We would find the total of the high temperatures for the week and divide that number by 7.) Ask the students to find the mean high temperature for the week shown.

Repeat this procedure to find the mean low temperature for the week shown.

GETTING STARTED: (cont'd)

Say, "What was the mean high temperature? (64°) ...mean low temperature for the week? (56°) What was the range in temperature for the week? (16°) How did you find the range? (by finding the difference between the highest temperature, 68°, and the lowest temperature, 52°, for the week) What other questions could we ask based on the data shown on this chart?" (Accept all reasonable answers.) You may wish to allow a student who asks a question based on the chart to call upon another student to answer that question, or the students may prefer to collect all such questions on a worksheet to be assigned as classwork for groups, homework for individuals, or a "bonus" practice sheet.

6.12.01.01A find the mode of a given set of data

STRATEGY 1:

Materials:
- chart used in Getting Started

Display the temperature chart.
e.g.,

Date	Low Temperature	High Temperature
Nov. 10	56°	67°
Nov. 11	52°	62°
Nov. 12	60°	65°
Nov. 13	60°	63°
Nov. 14	57°	64°
Nov. 15	53°	59°
Nov. 16	54°	68°

Say, "We have talked about the data on this chart and found the mean high temperature, the mean low temperature, and the range in temperature for the week. Look at the column showing the low temperature for each day of the week. In your notebook, list these temperatures in order from lowest to highest." Ask one student to write this on the chalkboard.
e.g.,

52° 53° 54° 56° 57° 60° 60°

Say, "When we write the numbers in increasing order, it is easy to see that one number, or temperature, occurs more often than the others. What number is that? (60°) Yes, 60° occurred twice during the week and each of the other temperatures occurred only once. In examining a series of numbers, whether they relate to temperatures or other numerical data, the number that occurs most often is called the mode. In this case, 60° is the mode." Below the numbers written on the

STRATEGY 1: (cont'd)

chalkboard write: mode = 60°. Ask the students to write it in their notebooks along with the definition of mode.

Say, "I will call the names of ten students. As I call your name, say your age aloud. I will write the ages on the chalkboard. Then we will find the mode of this set of data." Call the students names and record the data.

e.g., 11 11 10 11 12 10 11 11 12 12

Say, "What can we do to make it easier to find the mode?" (We should arrange the numerical data in increasing order.) Ask a student to rewrite the numbers in increasing order on the chalkboard as the other students write them in their notebooks.

e.g., 10 10 11 11 11 11 11 12 12 12

Say, "What numbers appear in this list? (10, 11, and 12) Which number is the mode? (11) Why?" (The mode is the number that appears most often in a set of data.)

Say, "What problems do you think you might encounter in trying to find the mode? (There might be a tie for which number appears most often.) Can you provide an example of a set of numbers where this happens? (Accept every reasonable answer.) If, in a group of numbers such as 8, 9, 9, 10, 10, 11, 12, no single number occurs most frequently, we say that the set has more than one mode. Here, 9 and 10 are both modes. We say that this set is bimodal because it has two modes."

Say, "Look at the temperature chart again. Let's see if we can find the mode of the high temperatures for the week." Ask a student to list the high temperatures in increasing order on the chalkboard as the other students do the same in their notebooks.

e.g., 59° 62° 63° 64° 65° 67° 68°

Say, "What do you notice about this set of numerical data? (There is no number that appears more than once.) Since no number appears more often than any other, we say that there is no mode for this set of numbers."

Repeat this procedure to find the modes of other sets of numerical data.

STRATEGY 1: (cont'd)

Say, "Why is knowing the mode of a set of data useful? (It tells us the data that occurs most often. This helps us see trends or patterns in data.) We can see this easily if we imagine that we are the managers of a shoe store. It is more important for us to know the mode of sizes sold rather than the average size sold. To reorder the sizes that were sold most frequently, we need to know the mode. If everyone in the neighborhood were size 8 or size 10, the mean would be some number between 8 and 10 (such as 8½, 9, 9½); if we ordered shoes of one of these intermediate sizes no one would buy them! Can you describe other situations in which knowing the mode would be important?" (Accept all reasonable responses.)

6.12.01.02A find the median of a given set of numerical data

STRATEGY 1:

Materials:
- chart used in Getting Started

Display the temperature chart. Say, "What types of information have we gotten from this chart? (We have found the average high temperature, the average low temperature, the range in temperature, and the mode of the low temperatures.) Why didn't we find the mode of the high temperatures? (No number in this set occurred more than the others, so there was no mode.) Look at the low temperatures for the week. Let's examine this data once again. This time we will be looking for the median." On the chalkboard write: For an odd number of items, the median is the middle number when the numbers are arranged in order from least to greatest. Ask the students to write this definition in their notebooks after one student reads it aloud.

Ask a student to rewrite the low temperatures for the week in increasing order on the chalkboard as the other students write them in their notebooks.

STRATEGY 1: (cont'd)

e.g.,

52° 53° 54° 56° 57° 60° 60°

Say, "What is the middle number in this set of data?" (56°) Ask a student to circle 56° on the chalkboard. Say, "56° is the median for this set of data." On the chalkboard write: The median is 56°. Ask the students to write this in their notebooks.

Say, "Lets find the median high temperature for November 10 through November 13. What is the first thing we should do?" (List the numbers in increasing order.) Ask a student to list the data on the chalkboard as the other students write it in their notebooks.

e.g.,

62° 63° 65° 67°

Say, "Can we find the median by just examining the data? Explain. (No, this is not an odd number of items, so there is no middle number.) When this occurs, we define the median to be the average, or mean, of the two middle terms." Write the following on the chalkboard and ask the students to write it in their notebooks: For an even number of items, the median is the mean, or average, of the two terms that are in the middle when the terms are listed in numerical order.

Ask the students to find the median for the set of four temperatures listed on the chalkboard. Ask one student to work at the chalkboard.

e.g.,

$$\begin{array}{r} 63 \\ +65 \\ \hline 128 \end{array} \qquad \begin{array}{r} 64 \\ 2\overline{)128} \end{array}$$

The median is 64°.

Say, "What is the median of 62°, 63°, 65°, and 67°?" (The median is 64°, which is the mean of 63° and 65°.)

Repeat this procedure to find the median of other sets of numerical data.

Say, "What conclusions would you draw if you knew the median of a set of numbers?" (Half the numbers in the group are greater than this value and half are less than the median.)

STRATEGY 1: (cont'd)

Say, "Suppose you were going on vacation for a month in a city where the following conditions held: median temperature 75°, temperature range 40°, mode temperature 90°. What weather would you expect? How would you pack? List a set of temperature values that might be recorded during your stay." (Accept all reasonable answers.)

P.O. 6.12.01.01-.02

NAME: ______________________ CLASS: _______ DATE: ____________

A. For each set of numbers below, find the mode and the median.

	Mode	Median
① 2, 3, 8, 4, 6, 62, 1, 2, 2		
② 14, 21, 60, 35, 11, 21, 14, 21		
③ 75, 80, 90, 85, 80		
④ 100, 112, 98, 210, 98, 114, 98, 104, 112		
⑤ 66, 72, 48, 39, 60, 66, 73, 45		

B. Use the chart to answer each question.

High-Jumpers Basketball Team	
Player	Height
Sal	70 in.
Marty	75 in.
Bob	69 in.
Andy	70 in.
Lou	68 in.

1. What is the median height for the High-Jumpers basketball team? _______________

2. How much taller than the median height is Marty? _______________

3. Whose height is one inch less than the mode for this set of data? _______________

Do you remember?

Find the equivalent for each of the given units of measure.

1) 52 g = ______kg

2) 280 mm = ______dm

3) 200cm = ______m

4) 50 mg = ______g

5) 2.4 L = ______mL

6) 4.25 km = ______m

6.14.09 REVIEW: GRAPHS

(1 lesson/ongoing)

PERFORMANCE OBJECTIVES

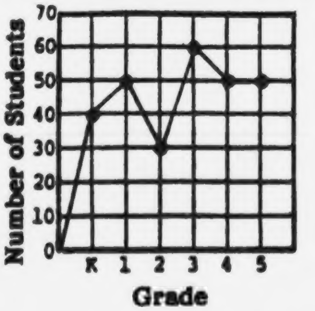
The student will be able to...

6.14.09.01 read and interpret information from a line graph;
e.g.,

How many students are in the 1st grade?

Which grade has the most students?

NUMBER OF STUDENTS IN EACH GRADE

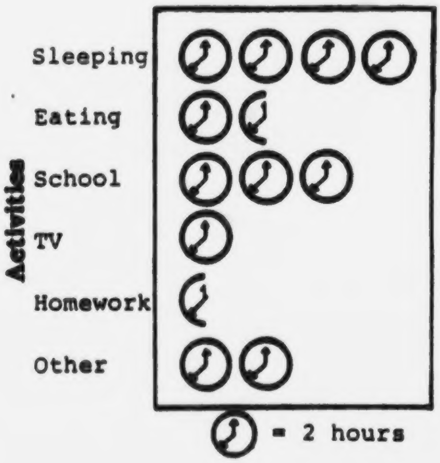


6.14.09.02 read and interpret information from a picture graph.
e.g.,

HOW WE SPENT OUR TIME

How many more hours were spent sleeping than eating?

How many minutes were spent in school?



PROBLEM EXPLORATION SKILLS:

- to use analogy
- to compare and contrast

VOCABULARY

line graph
circle graph
picture (picto)graph
double bar graph
vertical axis

SYMBOLS

horizontal axis
point of origin
data

PREVIOUS UNDERSTANDINGS:

- read and interpret information from line, circle, picture, bar, and double bar graphs
- construct graphs from given data

GETTING STARTED:

Materials:
- grid paper
- rulers

Say the following to the students:

> The sixth grade at P.S. 777 just completed its annual candy sale. Class 6-240 sold 19 boxes of chocolates, 15 boxes of caramels, and 11 boxes of cherries. Class 6-330 sold 18 boxes of chocolates, 10 boxes of caramels, and 17 boxes of cherries.

Say, "Can you remember all the information mentioned?" Elicit that it is helpful to record factual information if we wish to analyze it.

Say, "What are some ways we can record a great deal of related information, or data? (We can make a table, a graph, or a chart.) What is the importance of organizing data?" (Organized data is easier to read and to understand. Required information can be located more efficiently.) Display the following chart:

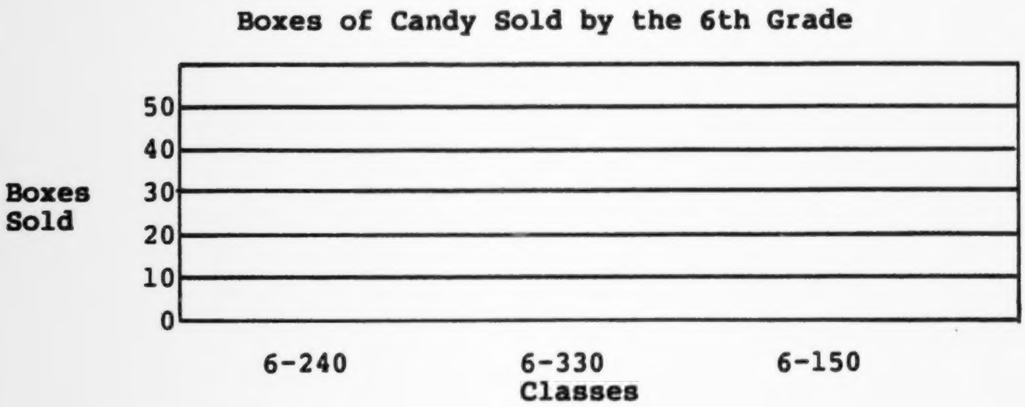
Boxes of Candy Sold by the 6th Grade

Class	Chocolates	Caramels	Cherries	Total
6-240	19	15	11	45
6-330	18	10	12	40
6-150	17	16	17	50

GETTING STARTED: (cont'd)

Say, "How can we provide a pictorial representation of the information contained on the chart? (We can construct bar, double bar, circle, or picture graphs.) Why wouldn't a line graph be appropriate? (A line graph is usually used to show changes in data with respect to a changing factor, such as a period of time. Since there is no natural order or quantified relationship between the classes or candy types, a line graph would not be appropriate.) Let's construct a bar graph showing the total number of boxes sold by each class using the above data. What are the components necessary to construct a bar graph?" (a title for the graph, subtitles for each axis, and the statistical data)

Display the following:


Boxes of Candy Sold by the 6th Grade

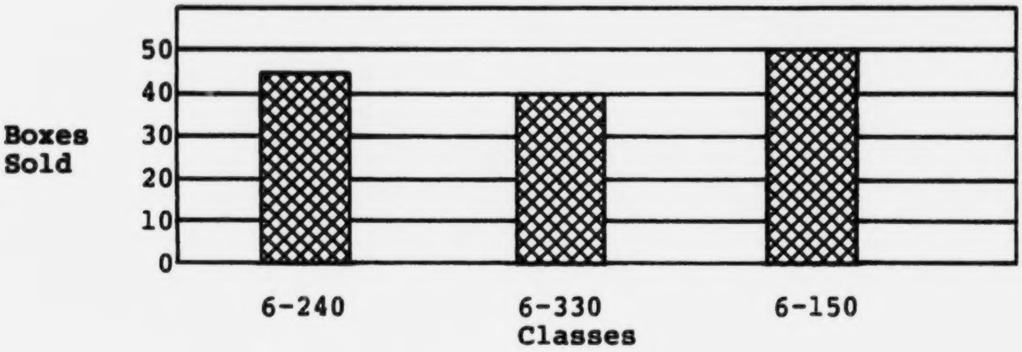
Say, "We must decide how many boxes of candy each interval on the numerical scale should represent. The lowest line will represent zero boxes. How many boxes shall we choose to be the value of each line up from this base line?" (Accept any suggestion reasonable for the area where the graph is to be drawn. As illustrated, 10 boxes per interval would be appropriate.) Mark the scale on the vertical axis.

Ask several students to complete the bar graph at the chalkboard as the others work at their desks, using grid paper, or lined paper.

GETTING STARTED: (cont'd)

e.g.,

BOXES OF CANDY SOLD BY THE 6TH GRADE CLASSES



Ask the students to create and solve problems using the data presented in this bar graph. You may wish to have the students make separate bar graphs for each type of candy. After Unit 27 is taught, you may ask them to make a multi-bar graph showing all three types.

6.14.09

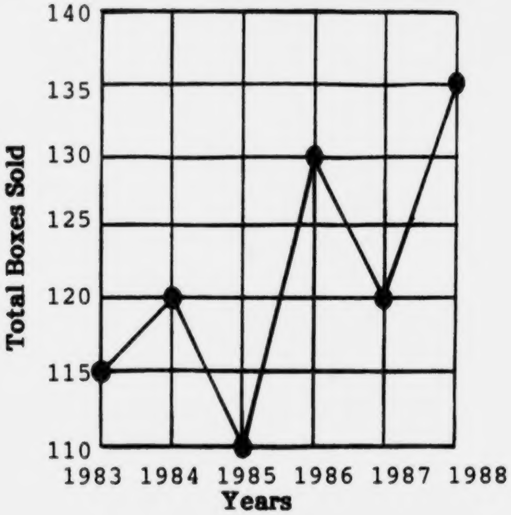
6.14.09.01 **read and interpret information from a line graph**

STRATEGY 1:

Materials:
- line graph shown below

Display the following line graph:

P.S. 777
GRADE 6 YEARLY CANDY SALE RESULTS



Say, "What kind of graph is shown here? (a line graph) What does it tell us? (It shows the total number of boxes of candy the 6th grade of P.S. 777 sold each year from 1983 to 1988.) What is the scale on the vertical axis? (Each interval represents 5 boxes.) Why doesn't this scale start at zero? (Because the numbers are so large that if the scale started at zero, it would be very hard to see the changes. They would be all flattened out.) How do the lines which connect the points help us? (They help us see the changes in the number of boxes sold from year to year.) How many boxes were sold in 1984? (120 boxes) ...1986? (130) In which year were 115 boxes sold? (1983) How many more boxes of candy were sold in 1988 than the year before? (15 more) In which year was the least number of boxes of candy sold? (1985) ...the most? (1988) From 1984 to 1985 did sales increase or decrease? (decrease) Between which two years was the greatest increase in sales made? (between 1985 and 1986) What other questions could you ask about this data? (Accept all responses and write them on the board.) Which of these questions can be answered without additional information?"

Repeat this procedure to provide students with other opportunities to read and interpret information from line graphs.

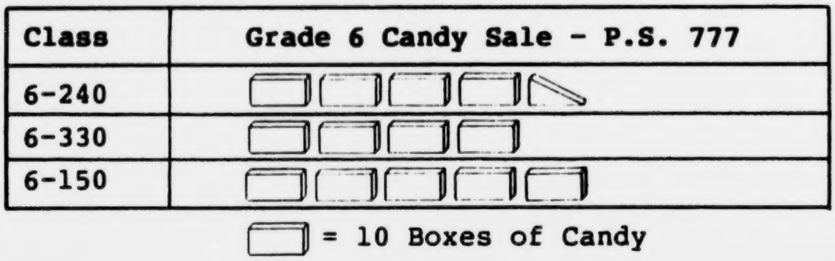
Ask students to bring in graphs found in newspapers and magazines and have them write questions a) which can be answered from the data presented in the graphs and b) which are raised by the graphs but would require additional data to answer.

6.14.09.02 read and interpret information from a picture graph

STRATEGY 1:

Materials:
- picture graph using data from Getting Started

Display the following picture graph:



Say, "What kind of graph is this? (a picture graph) What does the graph show us? (the number of boxes of candy sold by three 6th grade classes in P.S. 777) How is this graph different from the bar graph we made to show the same information? (It uses repeated symbols, or pictures, instead of the lengths of bars to represent the amounts of candy sold.) What do we call the information written directly below the graph? (a key) What is its function?" (The key shows us how to interpret each symbol. In this case, each box on the graph represents 10 boxes of candy.)

Say, "How many boxes of candy did class 6-330 sell? Explain. (40 boxes, since 10 x 4 is 40) How many more boxes were sold in class 6-150 than in class 6-330? (10) How many more boxes were sold by class 6-240 than 6-330? Explain. (Five boxes more were sold by Class 6-240, because the last symbol for 10 boxes in the top row is only half a symbol and half of ten is five.) How many boxes were sold by these three sixth grade classes?" (135 boxes)

This lesson can be enhanced and extended with additional data to stimulate the students to explore the existing data with a fresh point of view, e.g., "If each box of candy cost $3.00, how much money was earned by class 6-330? ...by these three sixth grade classes?"

In a similar fashion, provide students with other opportunities to read and interpret picture graphs.

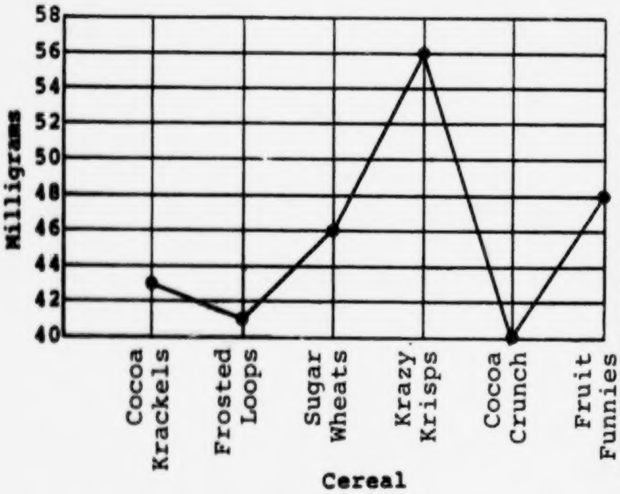
P.O. 6.14.09.01-.02

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

HOW SALTY IS IT?

Below are a line graph and a picture graph. Both of these graphs show the same data but in different ways.

MILLIGRAMS OF SODIUM IN CEREALS



MILLIGRAMS OF SODIUM IN CEREALS

Cocoa Krackels	○○○○○○○○○○○○○○ ○○○○○○○◓
Frosted Loops	○○○○○○○○○○○○○○ ○○○○○○◓
Sugar Wheats	○○○○○○○○○○○○○○ ○○○○○○○○○
Krazy Krisps	○○○○○○○○○○○○○○ ○○○○○○○○○○○○○○
Cocoa Crunch	○○○○○○○○○○○○○○ ○○○○○○
Fruit Funnies	○○○○○○○○○○○○○○ ○○○○○○○○○○

Each ○ = 2 milligrams

Use either graph to answer each question, and check your answer using the other graph.

1. Which cereal has the most sodium? ____________
2. Which cereal has the least sodium? ______________
3. How many milligrams of sodium are in Cocoa Krackels? _________ Is this more or less than the amount of sodium in Sugar Wheats? ______
4. Which cereal has 10 fewer milligrams of sodium than Krazy Krisps? ______________
5. Which two cereals have nearly the same amount of sodium? ___________ and _____________ Which graph, in your opinion, makes it easier to find this information? ________________
6. How many more milligrams of sodium does Fruit Funnies have than Sugar Wheats? ________
7. What is the average amount of sodium in the six cereals? __________ What is the range in the amount of sodium found in the six cereals? ____________

6.12.02 DOUBLE BAR GRAPHS

(1-2 lessons)

PERFORMANCE OBJECTIVES

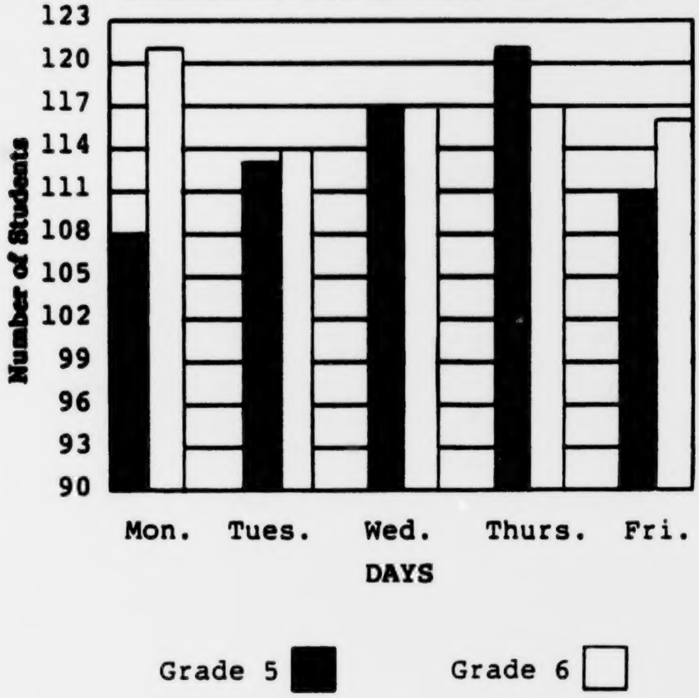
The student will be able to...

*6.12.02.01A construct a double bar graph from a set of data; e.g.,

ATTENDANCE FOR WEEK OF APRIL 16

	Grade 5	Grade 6
Mon.	108	121
Tues.	113	114
Wed.	117	117
Thurs.	121	117
Fri.	111	116

ATTENDANCE FOR WEEK OF APRIL 6



6.12.02.02A analyze data displayed in double bar graph format;

e.g.,

What was the mean (average) 6th grade daily attendance for the week?

What was the range in attendance for the 5th grade?

Which grade had better attendance for the week?

Note: Additional information is needed to answer the third question: total registration for each grade, definition of better.

6.12.02.03A create and solve word problems based on data presented in double bar graph format.

e.g.,

How many more 6th graders were present on Monday than 5th graders?

Which grade had the higher average attendance?

PROBLEM EXPLORATION SKILLS:

- to draw conclusions
- to compare and contrast
- to make a diagram

VOCABULARY

vertical axis
horizontal axis
range

PREVIOUS UNDERSTANDINGS:

- reading and interpreting bar graphs
- collecting data
- constructing bar graphs
- reading and interpreting double bar graphs
- finding the mean (average) and range of a series of numbers

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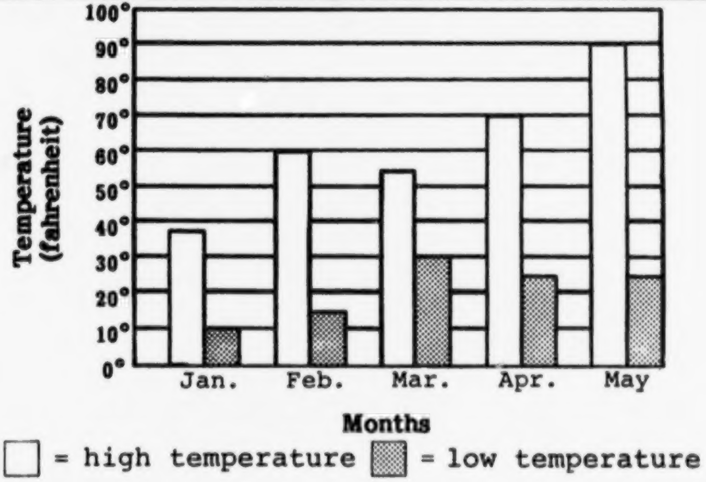
GETTING STARTED:

Display the following double bar graph:

HIGH AND LOW TEMPERATURES IN STUDYVILLE, JAN.-MAY



Say, "Look at the bar graph. What does it show? (It shows high and low temperatures in Studyville for January through May.) What is listed on the horizontal axis? (the months) ...the vertical axis? (temperatures from 0° to 100° Fahrenheit) How does the graph differentiate between high and low temperatures? (The low temperatures are represented by shaded bars, the highs by unshaded bars.) What is the lowest temperature recorded on the graph? (10°F) What is the highest temperature recorded by the graph? (90°F) Is January the coldest month of the year in Studyville? (We don't know because all the months are not shown.) Is it the coldest month for the period shown? (Probably, since both the maximum and minimum temperatures have their lowest values in January.) What is the range

GETTING STARTED: (cont'd)
of temperatures for this 5 month period? (80°) How did you determine this?" (The range is the difference between the highest recorded temperature, 90°F, and the lowest recorded temperature, 10°F.) Repeat this procedure by asking other questions about the graph. Some questions should be created by the students.

6.12.02.01 construct a double bar graph from a set of data

STRATEGY 1:

Materials:

- graph paper

Distribute one or two sheets of graph paper to each student or group of students. Say, "We have learned how to read and interpret bar graphs. Now we are going to make our own. Let's look at mathematics quiz scores earned by two different students over a period of one month." Write fictional scores for two fictional students.
e.g.,

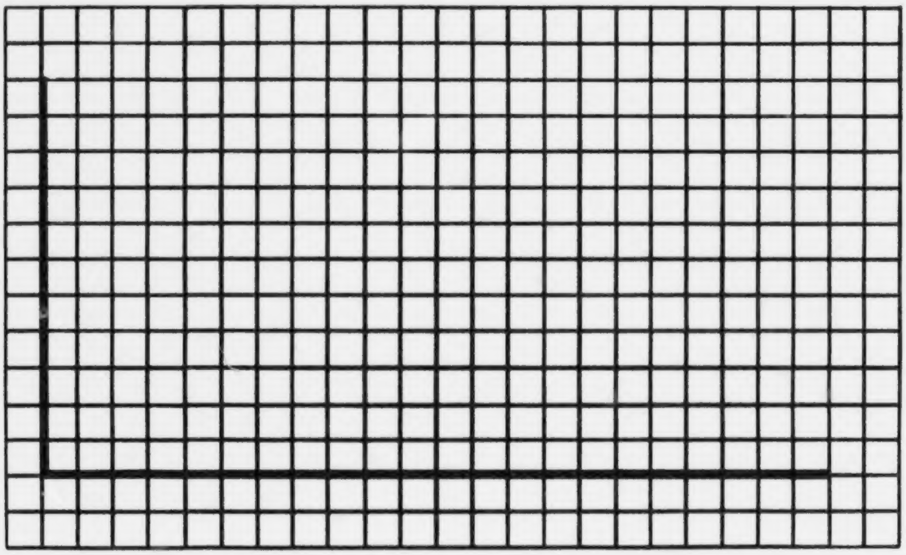
MATHEMATICS QUIZ SCORES		
	Bonnie	Joann
Quiz 1	85	60
Quiz 2	70	90
Quiz 3	75	95
Quiz 4	80	75

Say, "What type of graph could we use to show a comparison of the students quiz scores? (a double bar graph) What scale should we use for the scores? (Number by fives, starting at 0.) Where should we list the scores? (To make a vertical bar graph, we should list the scores on the vertical axis.) What should we show on the horizontal axis? (We should list the four quizzes.) How will we know the difference between Bonnie's scores and Joann's scores? (We can shade the bar for Bonnie's scores and draw stripes in the bars for Joann's scores. We will have to include a key to explain the symbols we select.) How many horizontal boxes are required? (We need two bars for each of four quizzes or eight boxes plus a space between each pair of quizzes, for a total of at least eleven spaces across.) If you prefer to use wider bars, made up of two or more boxes, you may. However, you must be sure to keep all the bars the same width. How many vertical spaces are needed? (If we start at 0 and count by fives

STRATEGY 1: (cont'd)

to 100, we will need 20 intervals along the vertical axis.) On the graph paper, place the axes to allow a rectangle that is at least eleven boxes long and twenty boxes high." Demonstrate this at the chalkboard.

e.g.,



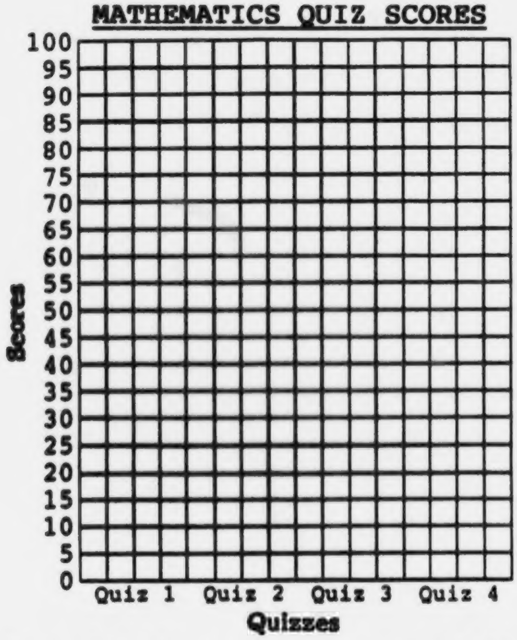
Say, "What should we do next? (We should call the vertical axis "Scores" and number it from 0 to 100 by fives.) What should we label the horizontal axis? (Quizzes) What title should we give the graph?" (We can call it "Mathematics Quiz Scores".)

Write these labels on the chalkboard graph while the students do so at their desks.

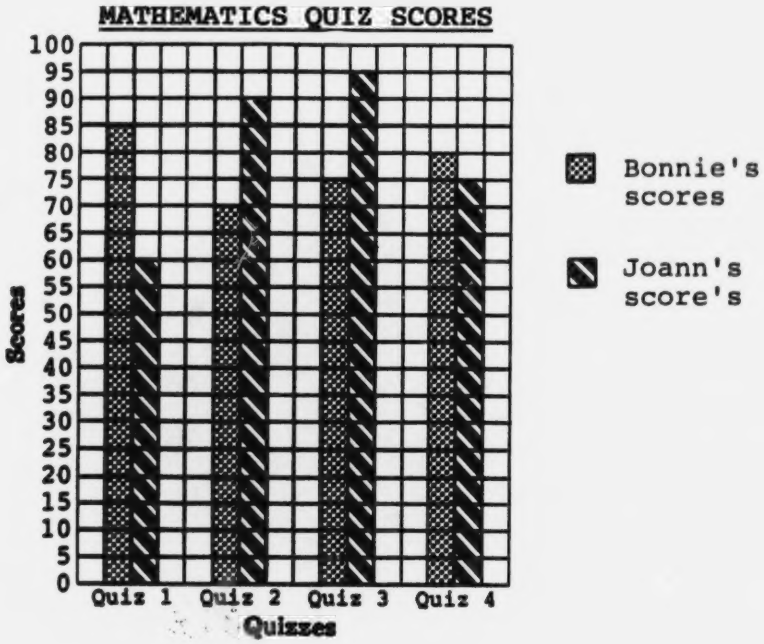
Say, "Let's use the first two columns for quiz 1, then skip a column, use the next two columns for quiz 2, and so on." Demonstrate on the chalkboard.

STRATEGY 1: (cont'd)

e.g.,



Say, "Now let's follow the chart of scores and fill in our graph. We should have a key on the side to show whose scores they are. Remember, Bonnie's are shaded, Joann's are striped." Ask one student to fill in the chalkboard graph while the students work at their desks.



6.12.02

6.12.02.02A analyze data displayed in double bar graph format

STRATEGY 1:

Display a copy of the graph the students made in Strategy 1, P.O. 6.12.02.01.

Say, "Look at your bar graph. What does it show? (It shows four quiz scores for Bonnie and Joann.) How can we tell the difference between the girls' grades? (The key tells us that Bonnie's scores are shaded and Joann's scores are striped.) Which of the girls scored higher on Quiz 1? (Bonnie received the higher score.) How much higher did she score? (Since Bonnie scored an 85 and Joann scored a 60, Bonnie scored 25 points higher than Joann.) What is Joann's highest score? (95.) What is her lowest score? (60) What is the range for Joann's grades? (The range for Joann's scores is 35.) Which girl had a greater range of scores? (Joann) What was Bonnie's best score? (85, on quiz 1) Which girl had the higher mark most often? (Neither; each had the higher mark twice.) How can we find the mean of Bonnie's scores?" (We can find the total of all her scores and then divide by the number of scores, in this case, 4.) Ask two students to find the mean scores for the two girls at the chalkboard while the other students work at their desks.

e.g.,

Bonnie

$$\begin{array}{r} 85 \\ 70 \\ 75 \\ +\ 80 \\ \hline 310 \end{array}$$

$$\begin{array}{r} 77\frac{2}{4} \\ 4\overline{)310} \\ -28\downarrow \\ \hline 30 \\ 28 \\ \hline 2 \end{array} \quad \text{or} \quad 77\frac{1}{2}$$

Joann

$$\begin{array}{r} 60 \\ 90 \\ 95 \\ +\ 75 \\ \hline 320 \end{array}$$

$$\begin{array}{r} 80 \\ 4\overline{)320} \\ -32\downarrow \\ \hline 0 \\ -0 \\ \hline \end{array}$$

Say, "Who had the higher average for the 4 quizzes? (Joann's average was higher.) What was the class average on these quizzes?" (We do not have enough information to find the class average. We only know the two girls' scores.)

Continue this procedure by considering other questions about the graph, some of which have been created by the students.

6.12.02

6.12.02.03A create and solve problems based on data presented in double bar graph format

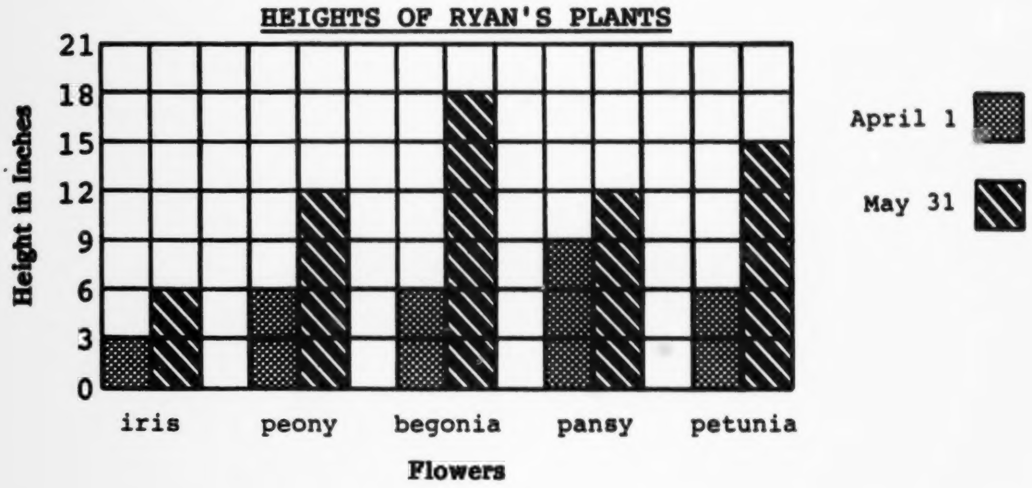
STRATEGY 1:

Materials:
- graph shown below

Prepare a graph based on the following story:

> On April 1, Ryan planted an iris, a peony, a begonia, a pansy, and a petunia in his yard. He measured them as soon as they were planted and again at the end of May. He compared the results and drew a graph to show his class.

Display the graph.
e.g.,



Say, "How many inches did the peony grow from April 1st to the end of May? (The peony grew 6 inches.) What question might be asked in reference to the different growth rates of the plants? (We might ask to find whether the plants that were the same height when planted showed the same growth rate?) How does the graph help us to solve that problem? (The graph shows us that the peony, the petunia, and the begonia were the same height when planted but their growth rates were different.) Which plant grew the tallest? (The begonia grew the

STRATEGY 1: (cont'd)

tallest.) Would you have expected the begonia to be the tallest? (No, the pansy was tallest when planted. We would have expected the pansy to be the tallest.) Why do you think this happened?" (Accept any reasonable responses, e.g., the pansy may not be inclined to grow as quickly.)

Repeat this procedure to allow students to create and solve other problems based on the graph.

P.O. 6.12.02.01

NAME: ______________________________ **CLASS:** ________ **DATE:** ______________

Use graph paper to construct double bar graphs based on the data below.

1.

Number of Students in Class 6-306 and 6-301
Whose Height Is Between 5'0" and 5'3"

6-306	6-301
5'0" - 6	5'0" - 5
5'1" - 4	5'1" - 5
5'2" - 2	5'2" - 1
5'3" - 2	5'3" - 3

Remember: Title, Axis labels, Key

- -

2.

Computers Based in P.S. 391 and P.S. 615

	P.S. 391	P.S. 615
Atari	4	0
Commodore	9	12
I.B.M.	7	15

Remember: Title, Axis labels, Key

CHALLENGE!

Use the information from the chart in number 1 to answer the following question:

What is the average height of all the students in the given set of data?

____

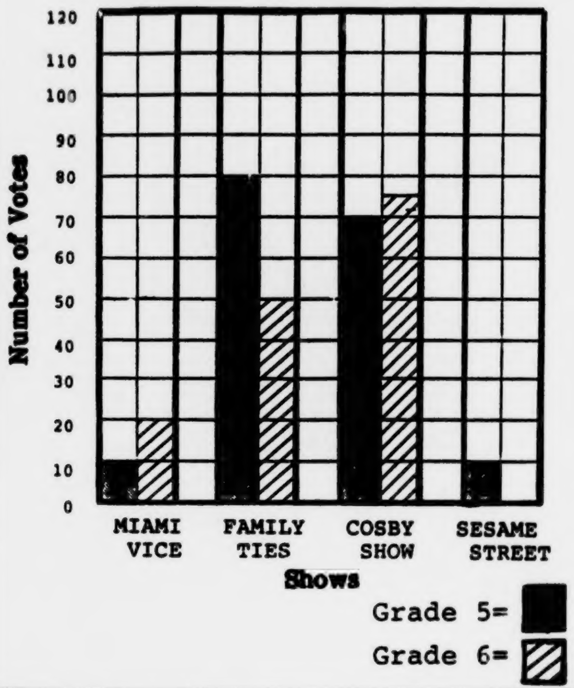
P.O. 6.12.02.02-.03

NAME: ______________________ **CLASS:** _______ **DATE:** ______________

Use the graph to answer the following questions:

Favorite Television Shows

Grades 5 and 6



1. Which show was most popular with grade 5? __________

2. What was the range in grade 6? __________

3. What was the mean in grade 5? __________

4. What shows were below the mean in grade 5? __________________

5. How many students in grade 6 voted? __________________

Create and answer 3 questions based on the graph above.

I. Question: ______________________ Answer: ______________

II. Question: ______________________ Answer: ______________

III. Question: ______________________ Answer: ______________

SHARPENING YOUR SKILLS

Units 25, 26, 27

Name: ____________________ **Class:** ____________ **Date:** ________

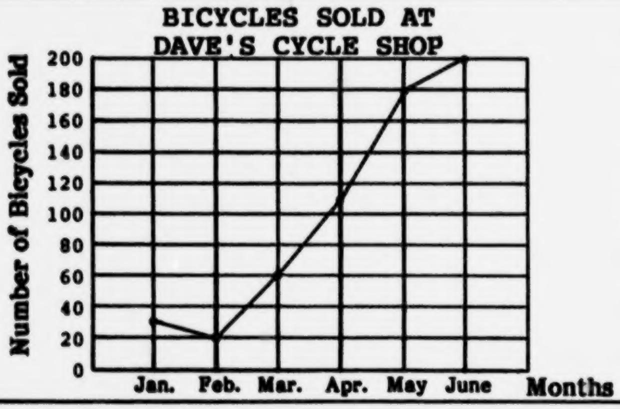
1. Find the mode and the median for the following data:

HIGH TEMPERATURES FOR WEEK OF JANUARY 10	
Sun.	29°
Mon.	24°
Tues.	30°
Wed.	26°
Thurs.	27°
Fri.	30°
Sat.	23°

mode = _____ median = _____

2. The median of 126, 132, 118, and 104 is _______.

Use the line graph below to answer questions 3 - 6.


BICYCLES SOLD AT DAVE'S CYCLE SHOP

3. How many more bicycles were sold in May than in April?

ⓐ 180 ⓒ 120

ⓑ 60 ⓓ Not Here

4. Which month had the least increase from the previous month?

ⓔ January ⓖ June

ⓕ March ⓗ Not Here

5. How many bicycles were sold during this 6-month period?

ⓐ 610 ⓒ 510

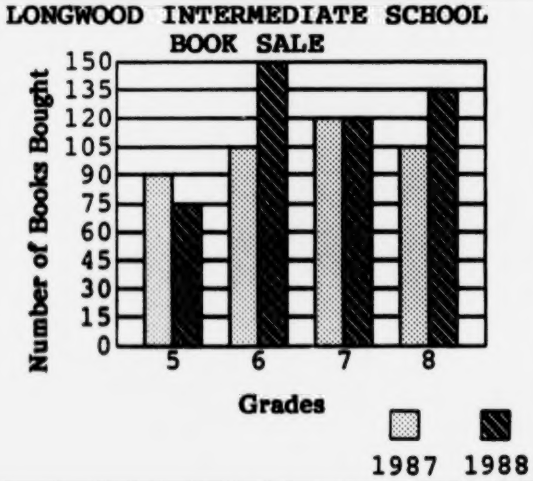
ⓑ 200 ⓓ Not Here

6. What was the average number of bikes sold each month?

ⓔ 85 ⓖ 100

ⓕ 110 ⓗ Not Here

Use the double bar graph below to answer questions 7 - 10.


LONGWOOD INTERMEDIATE SCHOOL BOOK SALE

7. How many more books were sold in 1988 than in 1987? ______

8. Which grade bought the most books during the two years?

____

9. In 1987, how many more books were bought by the fifth grade than by the seventh grade?

____

10. What was the average number of books bought by each grade in 1987? ________

Unit 28

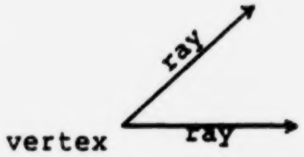
6.09.01 ANGLES AND ANGLE MEASUREMENT

(3 lessons)

PERFORMANCE OBJECTIVES

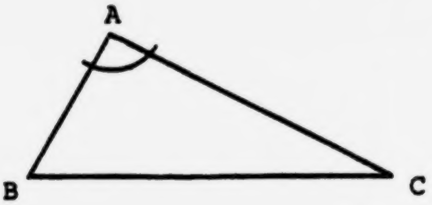
The student will be able to...

*6.09.01.01A identify the parts of an angle; e.g.,



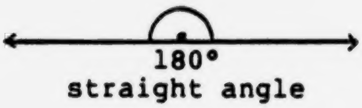
Two rays with a common endpoint form an angle.

*6.09.01.02A name angles using the angle symbol; e.g.,



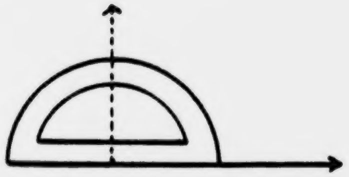
∡A	
∡BAC	These are different ways
∡CAB	to name the same angle.

6.09.01.03A identify and measure straight angles; e.g.,



*6.09.01.04A measure and construct right angles using a protractor;

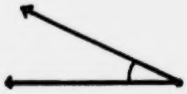
e.g.,



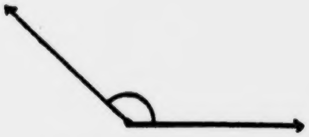
A right angle measures exactly 90°.

6.09.01.05A identify acute and obtuse angles;

e.g.,



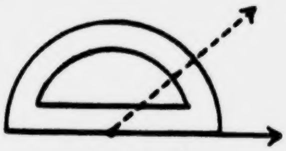
The measure of an acute angle is less than 90°.



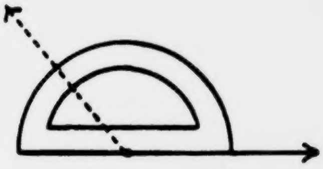
The measure of an obtuse angle is greater than 90°, but less than 180°.

*6.09.01.06A measure and construct acute and obtuse angles using a protractor;

e.g.,



acute angle

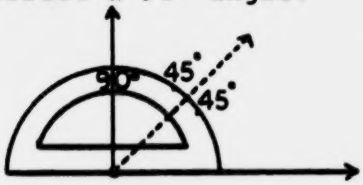


obtuse angle

*6.09.01.07A bisect an angle using a protractor.

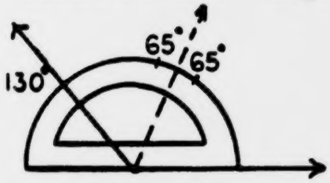
e.g.,

Bisect a 90° angle.



each angle measures 45°

Bisect a 130° angle.



each angle measures 65°

PROBLEM EXPLORATION SKILLS:

- to estimate (angle measurement)
- to systematically account for all possibilities

VOCABULARY

vertex
rays
endpoint
protractor
acute
right
obtuse
straight
bisect

SYMBOLS

° (degrees)

PREVIOUS UNDERSTANDINGS:

- recognize and identify points, lines, line segments, and rays
- identify the parts of an angle
- name angles using the angle symbol
- identify right angles

GETTING STARTED:

Say, "We are going to study a branch of mathematics called geometry. What skills have we learned within the topic of geometry?" (We have learned to identify and discuss points, lines, line segments, rays, planes, geometric figures, and angles. We have learned to measure lengths of line segments and areas of some regions.)

On the chalkboard, write:

line segment ray

Say, "How are a line segment and a ray similar? (Each is part of a line.) How are they different?" (A line segment has two endpoints while a ray has only one endpoint. We can measure the length of a line segment but not that of a ray.)

On the chalkboard, write:

square triangle

Say, "How are a square and a triangle similar? (Both are polygons.) How are they different?" (Accept all reasonable responses, e.g., a square has four sides of equal length; a triangle has three sides which may or may not be equal in length.)

Repeat this procedure to compare and contrast the following:

- a rectangle and a square
- a circle and a square
- parallel lines and perpendicular lines
- a point and a line

6.09.01

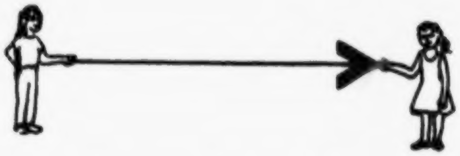
6.09.01.01A identify the parts of an angle

STRATEGY 1:

Materials:
- two pieces of string (each at least 6 feet long)
- cardboard arrows

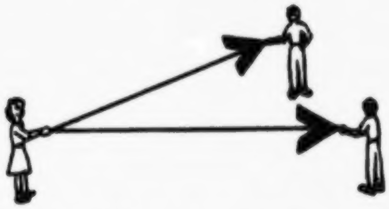
Display the two pieces of string. Staple a cardboard arrow to one end of each string and ask two students to hold one string stretched out from end to end.

e.g.,



Say, "What geometric figure does this string represent? Explain." (It represents a ray because it has one endpoint and it goes on and on in one direction.) Repeat this using the second string. Ask another student to tie the two strings together at the ends without the arrows and to hold the knot while two other students hold the arrow ends of the strings.

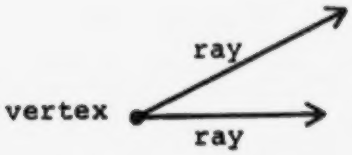
e.g.,

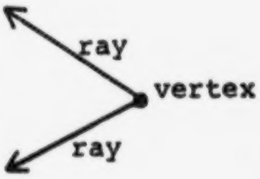


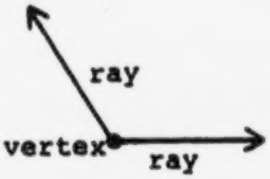
Say, "What does the knot represent? (The knot represents an endpoint shared by the two rays.) What is represented by the strings? (The strings represent rays.) A pair of rays with a common endpoint form an angle." Write this definition on the chalkboard and ask the students to write it in their notebooks. e.g., Two rays with a common endpoint form an angle. Say, "The common endpoint is called the vertex of the angle. How can we use the strings to form larger angles? (We can ask the two students holding the arrow ends of the strings to walk farther away from each other.) How can we use the strings to form smaller angles?" (The two students holding the arrow ends would move closer to each other.) Ask the students holding the

STRATEGY 1: (cont'd)

arrow ends of the strings to demonstrate by walking closer together to form smaller angles and farther apart to form larger angles. Draw representations of these angles on the chalkboard and ask the students to draw them in their notebook. Label the parts of each angle.
e.g.,







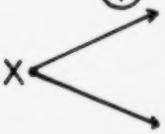
Repeat this procedure using the models created by each group.

6.09.01.02A name angles using the angle symbol

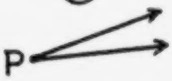
STRATEGY 1:

Draw several angles on the chalkboard.
e.g.,

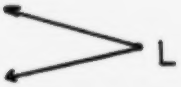
①



②

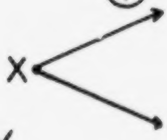


③



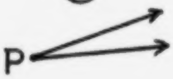
Say, "One way to name an angle is to use the letter at the vertex. What are the names for angles 1, 2, and 3?" (Angle 1 is labeled X, angle 2 is P, and angle 3 is L.) Write the name for each angle on the chalkboard using the angle symbol.
e.g.,

①



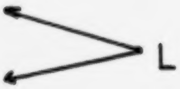
∡ X

②



∡ P

③

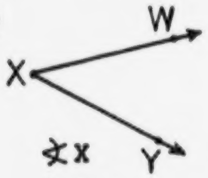


∡ L

Say, "There is also another way to name an angle. We can label one point on each of the rays and use the three letters to name the angle." Demonstrate.

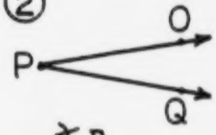
STRATEGY 1: (cont'd)

e.g., ①



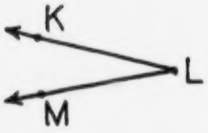
∡X
∡WXY
∡YXW

②



∡P
∡OPQ
∡QPO

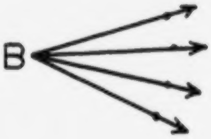
③



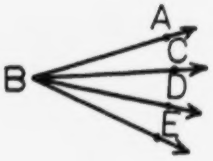
∡L
∡KLM
∡MLK

Say, "What do you notice regarding the placement of the letters? (The vertex is the middle letter when naming an angle.) Why is it important to learn the three-letter procedure for naming angles?" (It is necessary for clarity of communication when several angles have the same point as vertex.)

Draw the following on the chalkboard:



Say, "What would be the difficulty encountered if you were asked to trace angle B? (Since there are several rays eminating from the vertex, point B, it would be difficult to find the appropriate angle to be traced.) How can we avoid the confusion?" (We can label a point on each ray to clearly identify each angle.) Demonstrate. e.g.,



Ask several students to name different angles while a student volunteer records the information on the chalkboard and another student traces each angle named at the chalkboard. Ask the students to list the names of all distinct angles that appear in this figure. Say, "How many angles did you name with the vertex at point B?" (6) Ask a student to list them on the chalkboard.

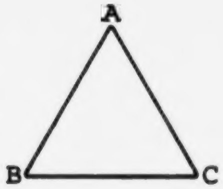
STRATEGY 1: (cont'd)

e.g.,

∢ABC
∢ABD
∢ABE
∢CBD
∢CBE
∢DBE

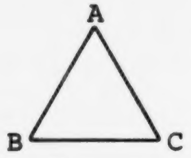
Using a ruler, construct and label a triangle at the chalkboard. Label its vertices A, B, and C

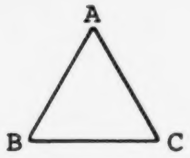
e.g.,

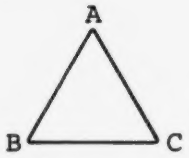


Say, "How can we name angle B three different ways?" (Angle B can be identified as angle B, angle ABC, and angle CBA.) Ask a student to name each angle similarly and develop a list at the chalkboard while the others write the angle names in their notebooks.

e.g.,







∢B	∢A	∢C
∢ABC	∢BAC	∢BCA
∢CBA	∢CAB	∢ACB

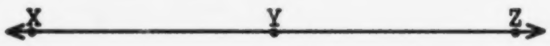
Repeat this procedure naming the angles of other geometric shapes.

6.09.01.03A identify and measure straight angles

STRATEGY 1:

Materials:
- chalkboard protractor

Draw the following on the chalkboard:

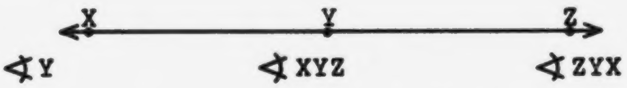


Say, "The geometric figure drawn on the chalkboard is a type of angle.

STRATEGY 1: (cont'd)

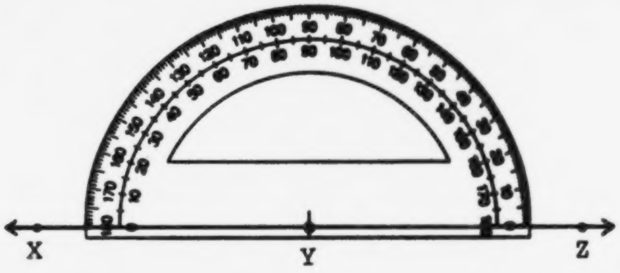
We call this a straight angle. Name the rays that make up this angle. (ray YX and ray YZ) What does point Y represent on the straight angle? (the vertex of the angle) How can we identify this straight angle?" (It can be identified as angle Y, angle XYZ or angle ZYX.) Ask a student to write this information underneath the angle on the chalkboard.

e.g.,



Say, "A straight angle always measures 180°. How can we verify the measurement of angle XYZ?" (We can use a protractor.) Display the chalkboard protractor. Ask a student to come to the chalkboard and use the protractor to measure angle XYZ. Say, "Place the protractor so that the centermark is on point Y and ray YZ is on the 0° mark."

e.g.,



Say, "Through which numbers does ray YX pass? (0° and 180°) Which is the correct measure?" (180°).

Repeat this procedure by asking students to identify and measure other straight angles.

6.09.01.04A measure and construct right angles using a protractor

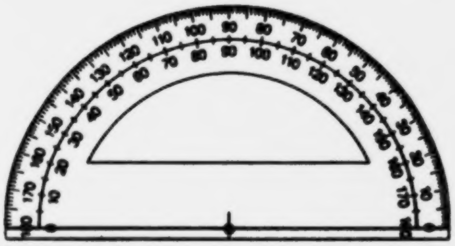
STRATEGY 1:

Materials:
- plastic protractor
- protractor for each student
- chalkboard protractor
- overhead projector

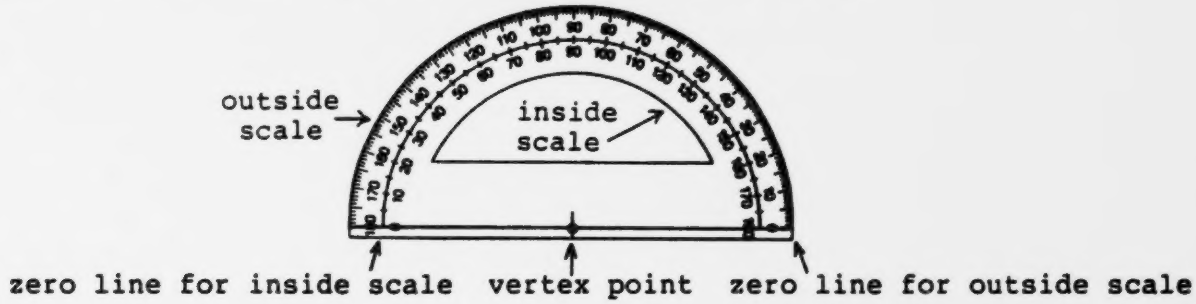
Display the plastic protractor on the overhead projector.

STRATEGY 1: (cont'd)

e.g.,

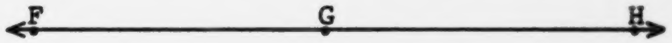


Say, "What is this instrument called and how is it used?" (It is a protractor and its purpose is to measure and/or construct angles.) Name and label the parts of the protractor as shown:


outside scale
inside scale
zero line for inside scale
vertex point
zero line for outside scale

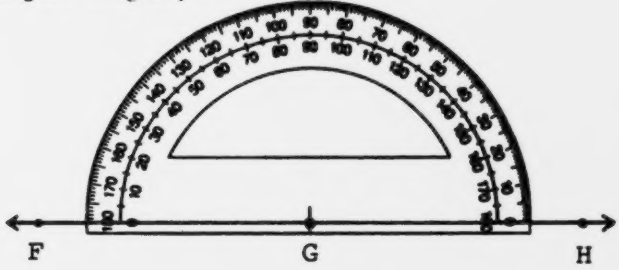
Say, "How is each scale numbered? (Each scale is numbered by tens from 0° through 180°.) What do we call an angle that measures exactly 180°? (a straight angle) What does it look like?" (It looks like a straight line but it has one point on it which is identified as the vertex).

Draw the following on the chalkboard:



Say, "How can we verify the measurement of angle FGH?" (Place the protractor so that the center mark is on G, which is the vertex of the straight angle, and ray GH is on the 0° mark. This ray will be one side of the straight angle).

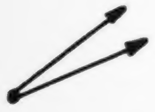
e.g.,



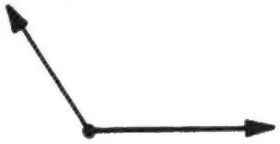
STRATEGY 1: (cont'd)

Say, "Through which numbers does ray GF pass? (0° and 180°) Which is the measure of the angle? (180°) What is a right angle?" (an angle which measures exactly 90°, which is exactly half that of a straight angle) Ask the students to identify examples of straight angles and right angles around the classroom. Draw several angles on the chalkboard, including one right angle.

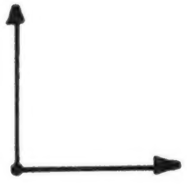
e.g.,



A



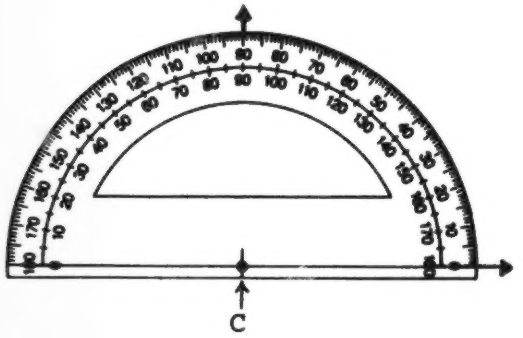
B



C

Say, "Which drawing represents a right angle? Explain. (Angle C appears to be a right angle because it looks like half of a straight angle so it probably measures 90°.) How can we verify whether or not it is a right angle? (We can measure angle C using a protractor.) To use the protractor correctly, place the vertex point of the protractor on the vertex of angle C and the 0° mark along one ray of the angle." Ask a student to place the chalkboard protractor at the vertex of angle C.

e.g.,

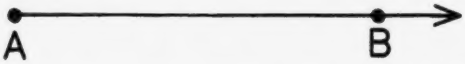


Say, "What measure does the other ray indicate on the protractor scale? (It crosses at the point marked 90°.) In what units do we measure angles? (degrees) How is the measure of angle C expressed? (Angle C measures 90° degrees.) How can angle C be described in another way?" (Angle C is a right angle.) Let's try measuring angle C another way. Again, place the vertex point at point C. This time, place the other 0° mark along the other ray of the angle. Read the protractor. What is the measure of the angle?" (Again, the protractor shows the measure to be 90°.)

STRATEGY 1: (cont'd)

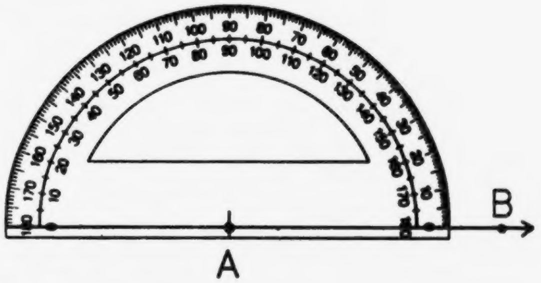
Say, "We can also use a protractor to construct a right angle." Using the straight edge of the chalkboard protractor, draw ray AB.

e.g.,



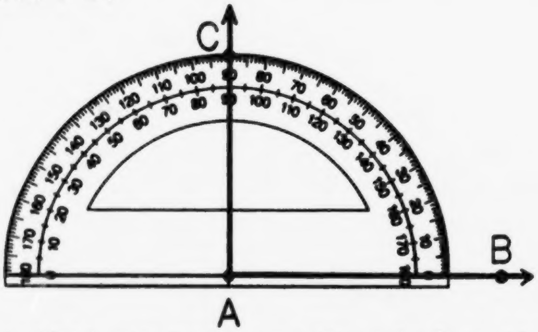
Say, "We are going to construct a right angle with its vertex at point A. How shall we place the center point of the protractor?" (Place the center point of the protractor on point A and the 0° line of the protractor on ray AB.) Ask a student to demonstrate this at the chalkboard with the large protractor.

e.g.,



Say, "Through which marking on the protractor scale should the other side of the right angle pass?" (90°) Ask a student to place a point on the chalkboard at the 90° mark, label it point C, and construct a ray from point A through point C.

e.g.,



Say, "How can we be certain that we have constructed a right angle? (It measures 90°. We can match it against a known right angle, such as the corner of a page in a book) What are the correct names for this angle?" (We can call it angle A, angle BAC, or angle CAB.) Ask a student to write the angle names on the chalkboard as the others write them in their notebooks.

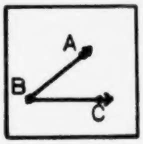
Repeat the procedure by constructing additional right angles.

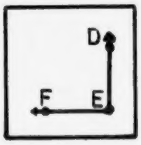
6.09.01.04A identify acute and obtuse angles

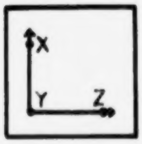
STRATEGY 1:

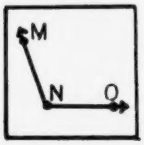
Materials:
- cards which show acute, right, and obtuse angles
- protractors
- chalkboard protractor
- narrow oaktag strips
- paper fasteners

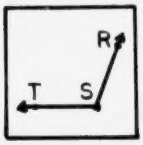
Display the following angle cards:

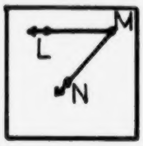






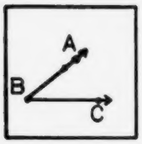
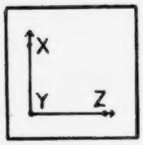
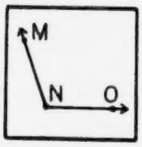
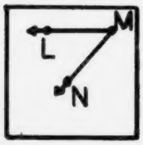
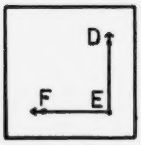
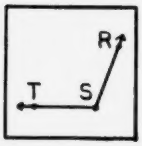






Say, "What can we say about the sizes of these right angles?" (Angle XYZ and angle DEF appear to be right angles. Angle ABC and angle LMN are each smaller than a right angle. Angle MNO and angle RST are each larger than a a right angle.) Ask a student to sort the cards into these three different groups.

e.g.,

Angles Smaller than Right Angles	Right Angles	Angles Larger than Right Angles
Group 1	Group 2	Group 3
		
		

STRATEGY 1: (cont'd)

Say, "What can we say about the angles in Group 2? (All the cards showing right angles are in Group 2.) How can we be certain that each angle in Group 2 represents a 90° angle?" (We can use a protractor to measure each angle.) Ask a student to measure the angles in Group 2 to verify that they are right angles. Say, "Which cards are in Group 1? (Those cards showing angles which appear to be less than 90 degrees are placed in Group 1.) We call these angles acute angles." Above Group 1 write: Acute Angles. Say, "Which cards are placed in Group 3? (Angles which appear to be greater than 90° are placed in Group 3.) We call these angles obtuse angles." Above Group 3 write: Obtuse Angles. Write the following on the chalkboard:

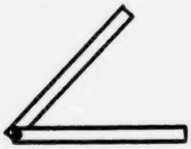
> Acute angles are smaller than right angles and measure less than 90°.
>
> Obtuse angles are larger than right angles and measure more than 90°, but less than 180°.

Ask the students to write these definitions in their notebooks. Ask several students to draw acute angles on the chalkboard. Ask several other students to draw obtuse angles on the chalkboard.

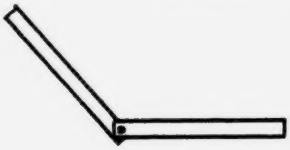
Say, "Which angles have we identified and measured? (We have identified and measured acute angles, obtuse angles, and right angles.) Distribute two narrow oaktag strips joined at one end by a paper fastener. Say, "Construct a representation of each angle described." Ask students to display their model once it has been constructed.

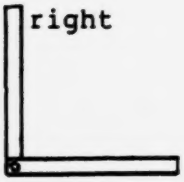
e.g.,

acute



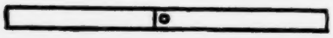
obtuse





Ask a student to open the oaktag strips until they are in a straight line.

e.g.,



Say, "What kind of angle have you modeled?" (a straight angle)

6.09.01.05A **measure and construct acute and obtuse angles using a protractor**

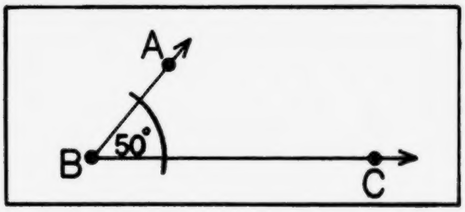
STRATEGY 1:

Materials:
- cards which show acute, obtuse, and right angles
- protractors for each student or group of students
- chalkboard protractor

Refer to the sorted sets of cards used in P.O. 6.09.01.04A, Strategy 1. Distribute the cards to small groups of students. Say, "How can we check the correctness of the classification groups? (We can use a protractor to measure each angle or we can compare them to a known right angle.) How can we then classify the various angles? (We can classify them as acute, obtuse, or right angles.) An obtuse angle is larger than a right angle and measures more than 90°. How did we use a protractor to measure right angles? (We placed the zero edge of the protractor along one ray of the angle, making sure that the center of the zero edge was on the vertex of the angle. Then we looked at the degree mark through which the other ray passed, that is, 90°.) We will use this same procedure to measure acute angles and obtuse angles, but now we will have to be concerned with the inside scale and the outside scale." Ask the group with angle ABC to display this angle on the chalkboard ledge. Say, "Look at angle ABC. Will it measure more than 90° or less than 90°?" (less than 90°) On the chalkboard, write: ∢ ABC measures less than 90°. Ask a student to place the zero edge of the protractor on one ray of angle ABC. Say, "It does not matter which ray we use first. Through which markings on the protractor does the other ray pass? (50° and 130°) To determine whether to read the measure as 50°, the outside scale, or 130°, the inside scale, we must recall what kind of angle ABC is. Since it is an acute angle and must measure less than 90°, which measure would be accurate, 50° or 130°?" (50°) Ask a student to label the angle measurement inside the angle.

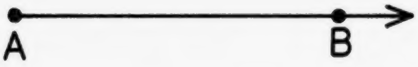
STRATEGY 1: (cont'd)

e.g.,



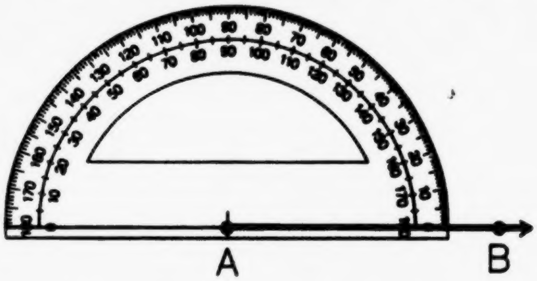
Say, "How can we construct an acute angle of a specific size?" (We can use a protractor to construct an angle which measures less than 90°.) Ask a student to come to the chalkboard. Say, "Draw ray AB."

e.g.,



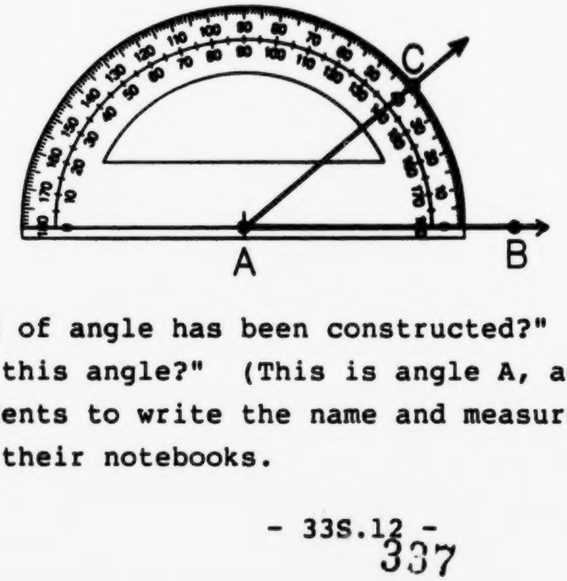
Say, "Place the center mark of the protractor on point A. Where will ray AB lie on the protractor?" (Ray AB will be placed on the 0° line.)

e.g.,



Say, "Which scale will we use to measure a 40° angle? (We will read the numbers on the outside scale, because they are numbers less than 90°, and we will place point C at the 40° mark.) Use the straight edge of the protractor to connect points A and C." Demonstrate.

e.g.,



Say, "What kind of angle has been constructed?" (an acute angle) How would you name this angle?" (This is angle A, angle CAB, or angle BAC.) Ask students to write the name and measurement of the angle constructed in their notebooks.

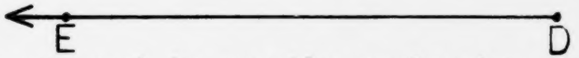
STRATEGY 1: (cont'd)

e.g.,

∢A measures 40° ∢ CAB measures 40° ∢BAC measures 40°

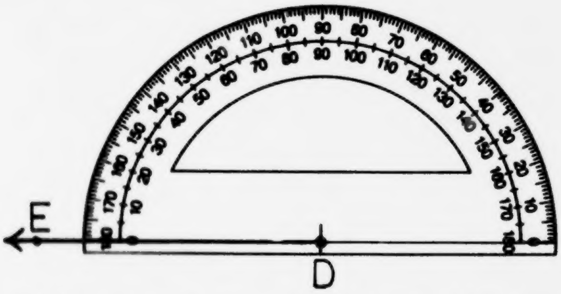
Say, "Let's see how this would work if we wanted the angle to face the other way." Draw a ray pointing in the other direction.

e.g.,



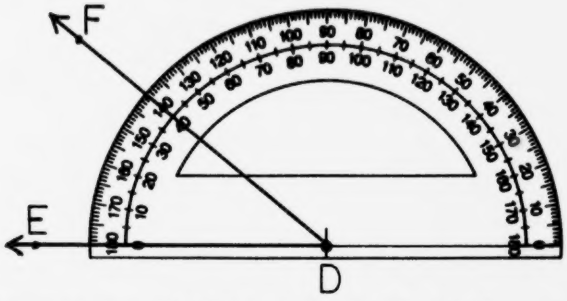
Say, "Again, we want to use the protractor to construct a 40° angle. This time, we want to have the vertex at point D." Place the protractor appropriately.

e.g.,



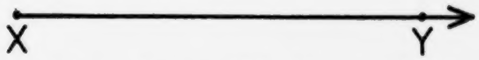
Say, "Which scale will we use to measure a 40° angle? (This time, we need to use the numbers on the inside scale, because those are the numbers less than 90 and we know we want an acute angle.) Place point F at the 40° mark and connect points D and F.

e.g.,



Say, "Let's construct an obtuse angle of 120°. How should we start?" (Draw a ray.)

e.g.,

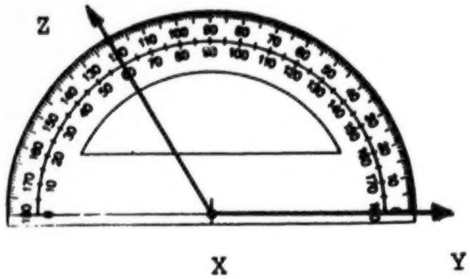


Say, "Which scale of the protractor will be used to construct an obtuse angle? (Since we are constructing an angle greater than 90°, but less than 180°, we read the scale where the 120° mark will give an opening greater than 90°. In this case, that is the outside scale.)

STRATEGY 1: (cont'd)

Place a point at the 120° mark, label it point Z. Draw a straight line which connects point Z to point X."

e.g.,



Say, "How can we name the 120° angle?" (angle X, angle YXZ, or angle ZXY) Ask students to write the three angle names and indicate the correct measurement.

e.g.,

∢X measures 120°
∢XYZ measures 120°
∢ZXY measures 120°

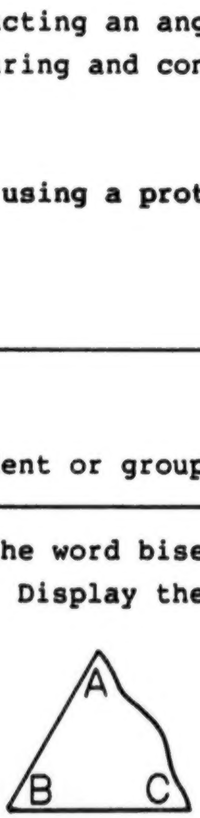
As before, demonstrate constructing an angle opening the other way. Repeat this procedure by measuring and constructing other obtuse and acute angles.

6.09.01.07A bisect an angle using a protractor

STRATEGY 1:

Materials:
- chalkboard protractor
- protractors for each student or group of students

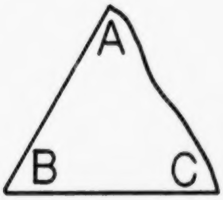
Say, "What is the meaning of the word bisect?" (Bisect means to divide into two equal parts.) Display the following on a large sheet of paper:

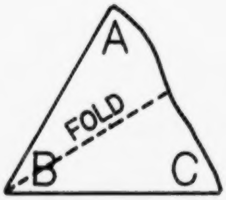


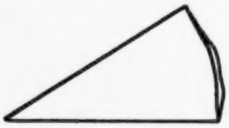
STRATEGY 1: (cont'd)

Say, "How can we bisect angle B?" (We can bisect angle B by folding the sheet of paper in half through angle B.) Ask a student to demonstrate.

e.g.,

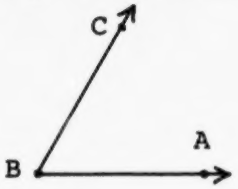






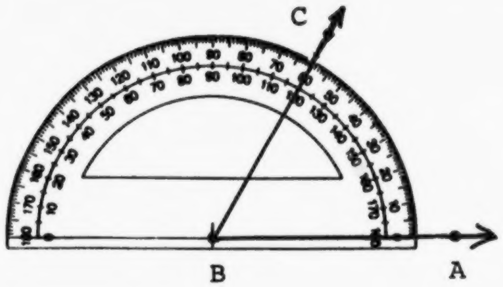
Say, "How does each part of the fold compare? (Angle B has been divided into two equal parts.) We have bisected angle B. How can we bisect an angle using a protractor?" (We can measure an angle with a protractor and then divide its measure by 2. Then we can use the protractor again to locate the ray that exactly bisects it.) Ask a student to draw acute angle ABC on the chalkboard.

e.g.,



Say, "How can we measure angle ABC?" (Place the vertex point of the protractor at the vertex and place ray BA on the 0° mark on the protractor. Then read the measure on the appropriate scale.) Demonstrate.

e.g.,

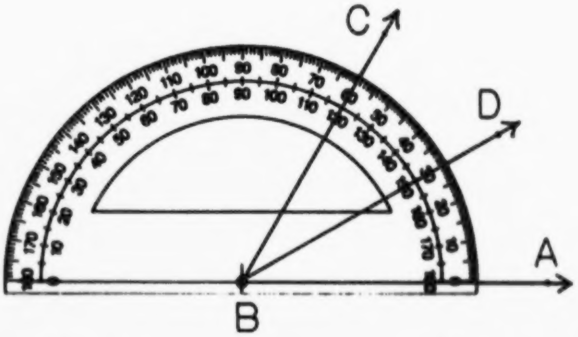


Say, "What is the measure of angle ABC? (Angle ABC measures 60 degrees.) In order to bisect an angle of 60° degrees, we must first find half of 60°. What is half of 60°? (30°) Without moving the protractor, we must construct a 30° angle inside ∢ABC having one of its sides along ray AB. How can we do that? (We can place a point at

STRATEGY 1: (cont'd)

the 30° mark, since that is half of 60° degrees. We will then draw a straight line from point B through the new point D and form ray BD.) Demonstrate.

e.g.,



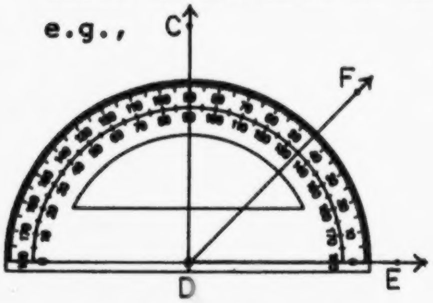
Say, "How does angle ABD compare to angle ABC? (Its measure is half the measure of angle ABC.) How does the measure of angle CBD compare to that of angle DBA?" (They are equal. Each measures 30°.) Ask a student to write these relationships on the chalkboard as the others write them in their notebooks.

e.g.,

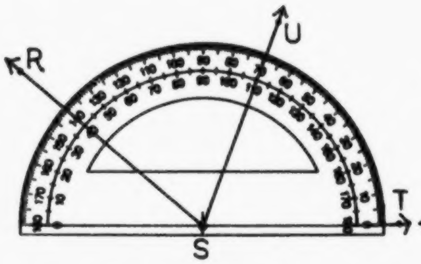
The measure of ∢ ABC = 60°
The measure of ∢ ABD = 30°
The measure of ∢ DBC = 30°

Say, "How can we bisect a right angle, an obtuse angle, or a straight angle?" (We measure the original angle, take one-half of this quantity, and draw a new ray which forms two equal angles, each of which is one-half of the original angle.) Demonstrate.

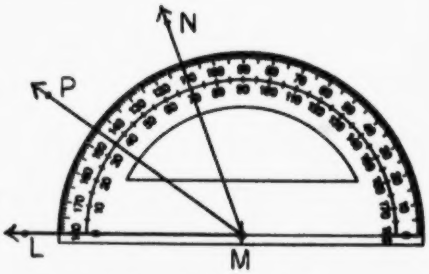
e.g.,



right angle CDE
measures 90°
∢FDE measures 45°
∢CDF measures 45°



obtuse angle RST
measures 140°
∢RSU measures 70°
∢UST measures 70°



acute angle LMN
measures 70°
∢LMP measures 35°
∢PMN measures 35°

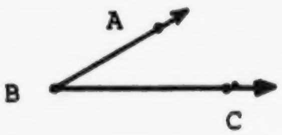
Repeat this procedure by bisecting other obtuse, acute, and right angles.

P.O. 6.09.01.01-.02

NAME: ______________________________ **CLASS:** _______ **DATE:** ____________________

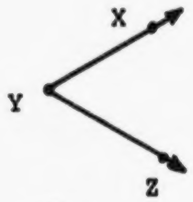
I. Complete each sentence.

Ⓐ



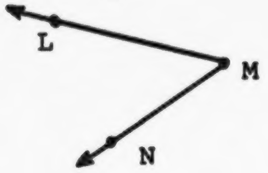
The two _________ that meet at B, together form a(an) _________. ____ is the vertex of the angle.

Ⓑ



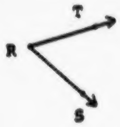
A(n) _______ is a figure formed by _____ different rays which have the same endpoint. ____ and ____ are both rays of the angle formed at Y.

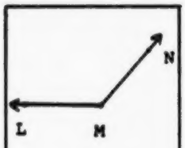
Ⓒ

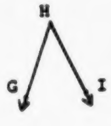


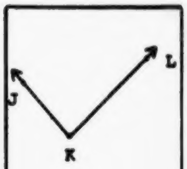
The endpoint for rays ML and MN is called the ________. An angle may be named by its _______. We may also name an angle by three points with the vertex in the _______. The model to the left may be named as ∡M, ∡LMN or ________.

II. Draw a line through the angle name(s) which do not belong.





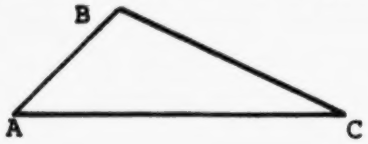




angle R	angle M	angle H	angle K
∡R	∡LMN	∡GHI	∡K
∡TRS	∡M	∡I	∡JLK
∡STR	∡NLM	∡H	∡JKL
∡SRT	∡LNM	∡IGH	∡LKJ
∡TSR	∡NML	∡IHG	∡LJK

III. Name each of the angles in the triangles three different ways.

Ⓐ

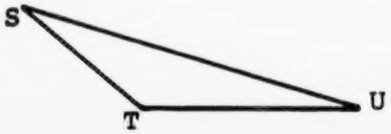


a) angle A _____ _____ _____

b) angle B _____ _____ _____

c) angle C _____ _____ _____

Ⓑ



a) angle S _____ _____ _____

b) angle T _____ _____ _____

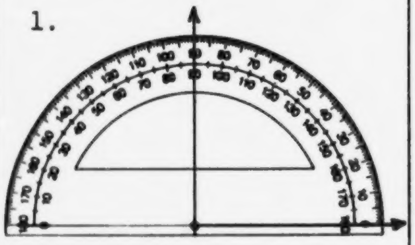
c) angle U _____ _____ _____

P.O. 6.09.01.03 & .05

NAME: ______________________ **CLASS:** _______ **DATE:** ____________

I. Find the measure of each angle and identify the type of angle formed.

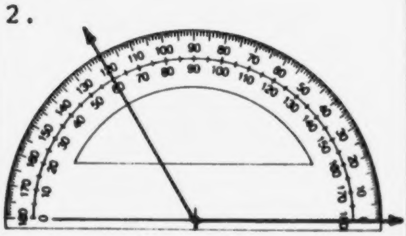
1.



angle measurement: 90°

angle type: right

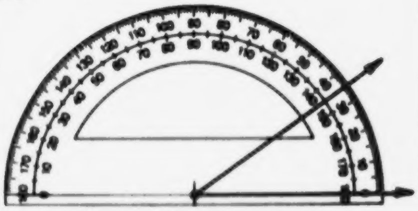
2.



angle measurement: ___

angle type: _______

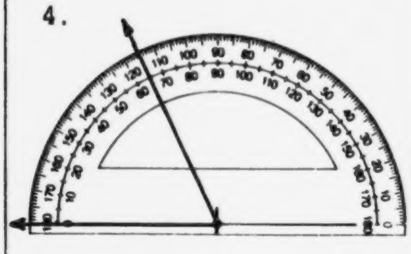
3.



angle measurement: ____

angle type: _______

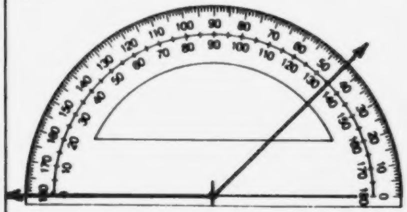
4.



angle measurement: ___

angle type: _______

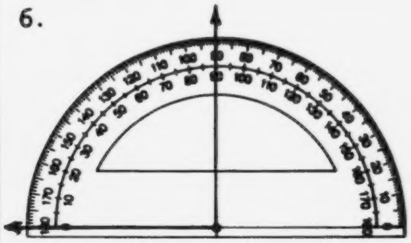
5.



angle measurement: ___

angle type: _______

6.

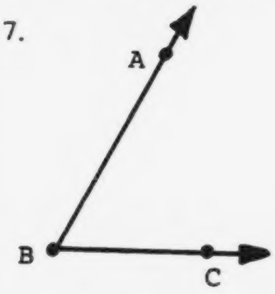


angle measurement: ____

angle type: _______

II. Estimate and measure each angle. Check which type of angle is formed in each case.

7.



___ degrees (est.)

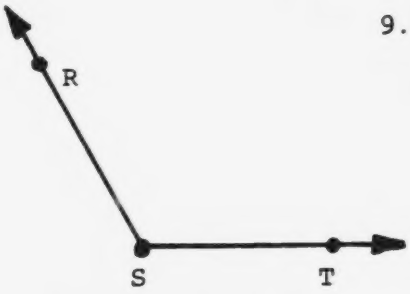
___ degrees measured

___ right angle

___ obtuse angle

___ acute angle

8.



___ degrees (est.)

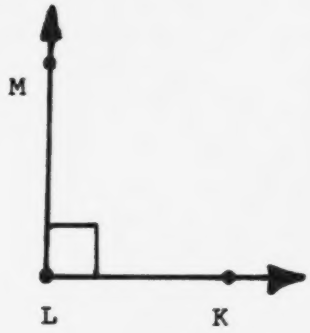
___ degrees measured

___ right angle

___ obtuse angle

___ acute angle

9.



___ degrees (est.)

___ degrees measured

___ right angle

___ obtuse angle

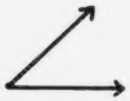
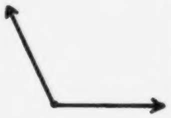
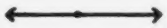
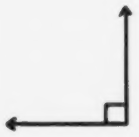
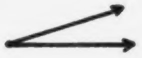
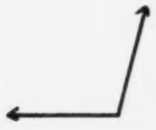
___ acute angle

P.O. 6.09.01.04&.06-.07

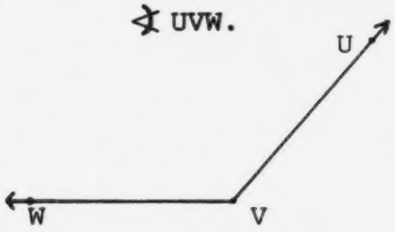
NAME: ______________________________ **CLASS:** _______ **DATE:** ______________

Follow the directions below.

I. Estimate whether the following angles are right, acute, obtuse, or straight angles.

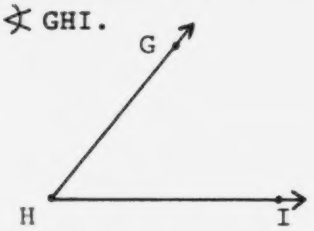
					
a.	b.	c.	d.	e.	f.

II. 1. Construct ray VS to bisect ∡UVW.



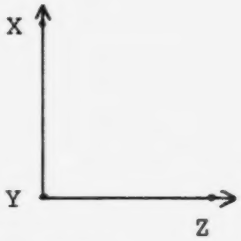
measure of ∡UVW = ______
measure of ∡UVS = ______

2. Construct ray HJ to bisect ∡GHI.



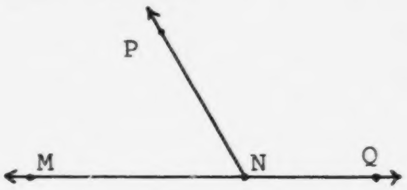
measure of ∡GHI = ______
measure of ∡JHI = ______

3. Bisect ∡Y.



New angle name: __________
New angle measurement: __________

4. Construct ray NR to bisect ∡PNQ.



measure of ∡PNM = ______
measure of ∡PNQ = ______

CHALLENGE!

Draw a triangle, then use your protractor to bisect each of its angles.

6.09.02 TRIANGLES AND ANGLES

(2-3 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

6.09.02.01A classify triangles according to the length of their sides;

e.g.,

- Equilateral means that all sides are equal in length.
- Isosceles means that two sides are equal in length.
- Scalene means that no sides are equal in length.

6.09.02.02A classify triangles according to their angles;

e.g.,

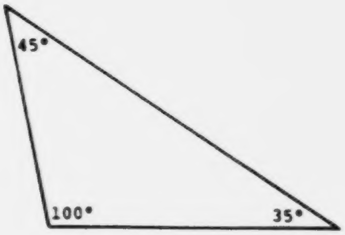
A right triangle contains one right (90°) angle.

An obtuse triangle contains one obtuse (more than 90° but less than 180°) angle.

An acute triangle contains three acute (less than 90°) angles.

*6.09.02.03A state that the sum of the measures of the angles of any triangle is 180°;

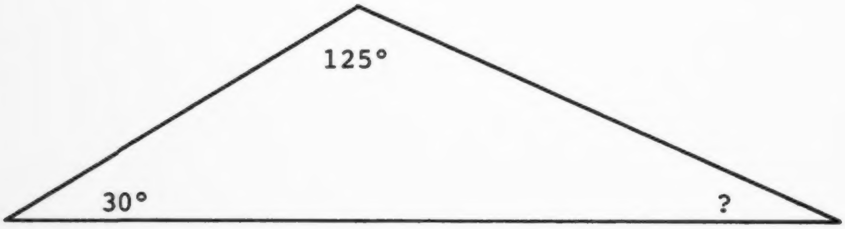
e.g.,



$100° + 45° + 35° = 180°$

6.09.02.04A find the measure of the third angle of a triangle when given the measures of the other two angles.

e.g.,



$$\begin{aligned} 125^\circ + 30^\circ + \square &= 180^\circ \\ 155^\circ + \square &= 180^\circ \\ \square &= 180^\circ - 155^\circ \\ \square &= 25^\circ \end{aligned}$$

PROBLEM EXPLORATION SKILLS:

- to compare and contrast
- to find an alternate solution

VOCABULARY

equilateral
isosceles
scalene
right triangle
obtuse triangle
acute triangle

SYMBOLS

° (degree)

PREVIOUS UNDERSTANDINGS:

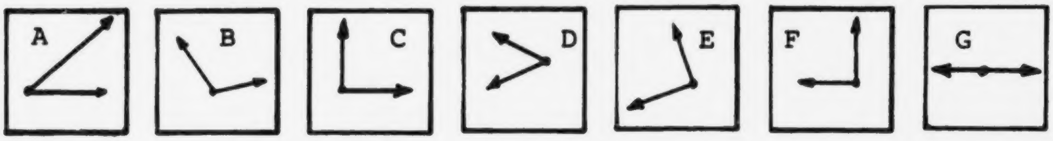
- defining and naming rays, lines, line segments, points
- recognizing and naming angles
- identifying parts of an angle

GETTING STARTED:

Materials:

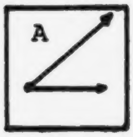
- a set of cards which show pictorial representations of various angles

Display the following cards on the chalkboard ledge:



Select one of the cards from the groups.

e.g.,



Say, "What type of angle is shown on card A? (an acute angle) How many degrees do you estimate it contains? (Answers will vary.) How could we determine whose estimate was closest?" (We could use a protractor to measure the angle.)

Repeat this procedure for cards B through G.

6.09.02

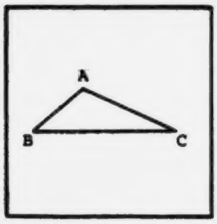
6.09.02.01A classify triangles according to the length of their sides

STRATEGY 1:

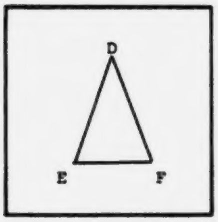
Materials:
- 12-inch rulers
- drawings of a scalene, an isosceles, and an equilateral triangle as shown below
- felt-tipped marker

Display the following charts on the chalkboard:

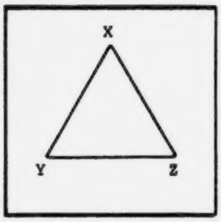
1.



2.



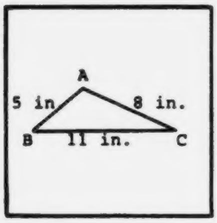
3.



Ask a student to use a ruler to measure each side of triangle ABC. Ask another student to record the lengths next to each side using the felt-tipped marker.

e.g.,

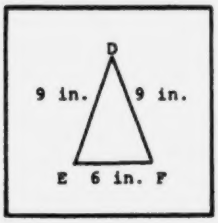
1.



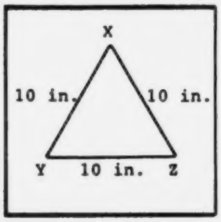
Repeat this procedure for triangle DEF and triangle XYZ.

e.g.,

2.



3.



Say, "Look at the lengths of the sides that have been recorded for each triangle. How would you describe the sides of triangle ABC. (The three sides have three different lengths.) ...triangle DEF? (Two sides have the same measure, one side is different.) ...triangle XYZ? (All three sides have the same length.) Mathematicians use special words to describe each type of triangle shown here. Any triangle

STRATEGY 1: (cont'd)

whose sides have three different lengths is called a **scalene** triangle. Which of the three triangles would be classified as a scalene triangle? (triangle ABC) On the chalkboard write: A scalene triangle has sides each of which measures a different length. Say, "Any triangle that has two sides with the same measure is called an **isosceles** triangle. Which triangle would be classified as an isosceles triangle? (triangle DEF) On the chalkboard write: an isosceles triangle has two sides equal in length. Say, "Any triangle whose three sides have the same measure is called an **equilateral** triangle. Which triangle would be classified as an equilateral triangle?" (triangle XYZ) On the chalkboard write: An equilateral triangle has three sides which are equal in length. Ask the students to write the three definitions in their notebooks. Say, "Is it possible for an isosceles triangle to contain a right angle?" (yes) Is it possible for an isosceles triangle to contain an obtuse angle? (yes) These labels give you information about the lengths of the sides of a triangle, not the measures of its angles."

Ask a student to use the felt-tipped marker to write the classification of triangle ABC on the chart on which that triangle is drawn. Repeat this with the two other triangles. These may now be displayed permanently in the classroom.

Ask the students to draw, measure, and classify other triangles according to the lengths of their sides.

6.09.02.02A classify triangles according to their angles

STRATEGY 1:

Materials:
- drawings of triangles on cards which show acute, right, and obtuse triangles

Place the cards in a "triangle" box and draw three wide columns on the chalkboard.

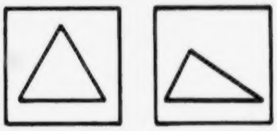
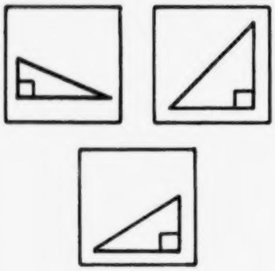
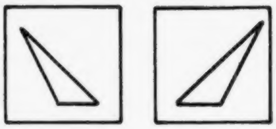
STRATEGY 1: (cont'd)

e.g.,

Column I	Column II	Column III

Say, "Watch me carefully." Draw a card from the box and tape it in one of the columns. Repeat this procedure until each column has two or three triangles.

e.g.,

Column I	Column II	Column III
		

Say, "Can anyone explain what logic I am using to determine in which column to place each card? What did the triangles in each column have in common? (Answers will vary.) Students should be encouraged to respond to each others conjectures. If necessary, add more examples to each column to help the students discover a pattern. Say, "What pattern do you see in column two? (Each of the triangles in Column II contains a 90 degree angle.) How does Column II compare to the other two groups? (None of the other groups has right angles in their triangles.) What properties do each of the triangles in these groups contain? Explain." (The triangles in Column I all have angles which are acute, that is, each of the angles in the triangles is less than 90 degrees. The triangles in Column III each has one obtuse angle, that is an angle greater than 90 degrees.) Ask a student to come to the chalkboard to label each column.

STRATEGY 1: (cont'd)

Column I	Column II	Column III
triangles which contain three acute angles	triangles which contain one right angle	triangles which contain one obtuse angle

Say, "Yesterday, we classified triangles according to the lengths of their sides. We can also classify triangles according to their angles. A right triangle contains one right angle. An obtuse triangle contains one obtuse angle. If all three angles of the triangle are acute, we say the triangle is acute." Write these definitions on the chalkboard and ask the students to write them in their notebooks.

Place all the cards back in the box. Repeat this procedure by asking students to come to the chalkboard to draw a card from the box, announce its classification according to the angles it has, and then to place it in its proper column.

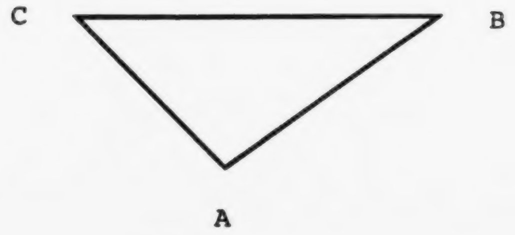
6.09.02.03A state that the sum of the measures of the angles of any triangle is 180°

STRATEGY 1:

Materials:
- protractor
- model of triangles

Display a model of a triangle.

e.g.,



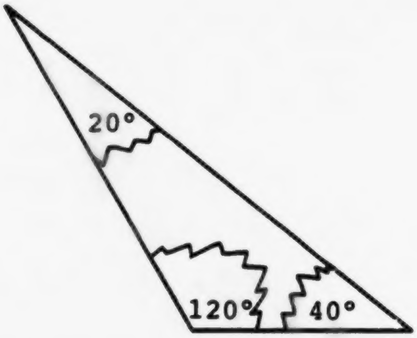
STRATEGY 1: (cont'd)

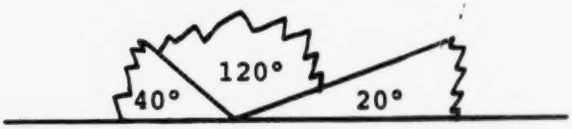
Say, "How can we find the sum of the measures of the three angles?" (We could use a protractor to measure each of the three angles and then add them together.) Ask a student to come to the chalkboard to measure and record the measure of the three angles of the triangle. e.g.,

angle	measure
A	100°
B	35°
C	45°

Say, "What are the measures of the three angles?" (The measures of the angles are 100°, 35°, and 45°.) Ask other students to come to the chalkboard to repeat this procedure measuring other models of triangles.

After the students have completed their measurements, say, "What have you discovered about the measures of the angles of your triangles? (The sum of the measures of the three angles in each of the triangles was 180 degrees.) How can we use an alternate way to show that the sum of the measures of the angles of any triangle is 180°?" (Since a straight angle always measures 180° you could cut off the three angles in the model and join them together. They should form a straight angle.) Demonstrate.
e.g.,





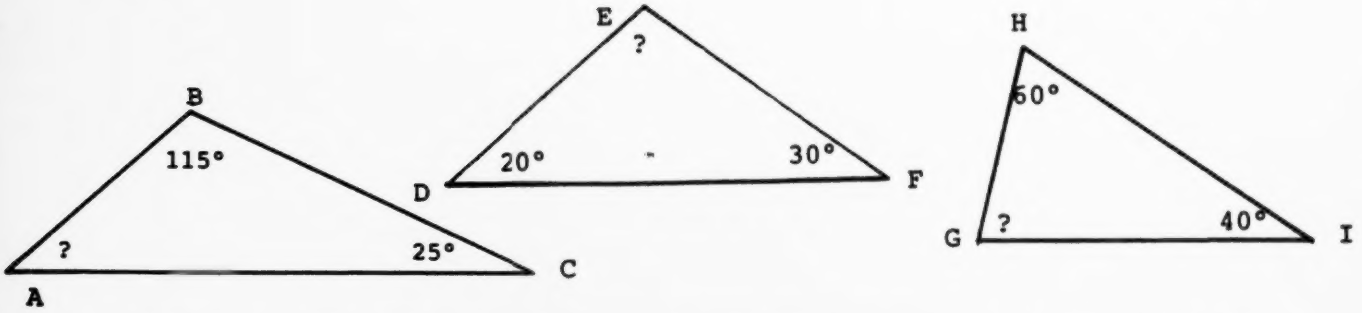
Say, "This fact is true for every triangle." Write the following on the chalkboard and ask the students to copy it in their notebooks:

FACT: The sum of the measures of the three angles in any triangle is 180°.

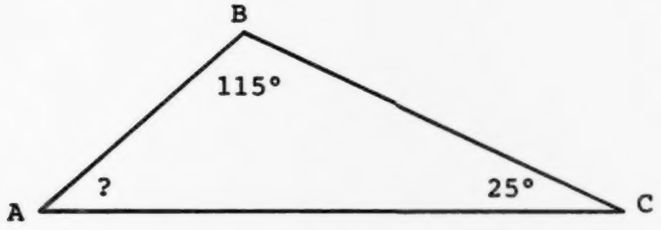
6.09.02.04A find the measure of the third angle of a triangle when given the measures of the other two angles

STRATEGY 1:

Draw models of triangles which display the measure of only two angles.
e.g.,



Say, "How can we find the measure of the unknown angle in each of the triangles? (We could use our protractor to measure the unknown third angle.) How else can we find the measure of each unknown angle?" (We know that the sum of the measures of the three angles of a triangle is 180 degrees. In each case, we can find the sum of the two known angles and then subtract this sum from 180°.) Demonstrate.
e.g.,

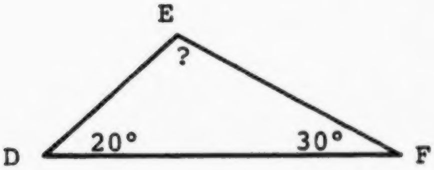


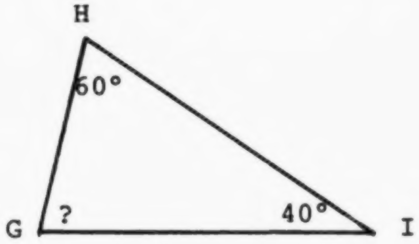
$\measuredangle A + \measuredangle B + \measuredangle C = 180°$
$\measuredangle A + 115° + 25° = 180°$
$\measuredangle A + 140° = 180°$
$\measuredangle A = 180° - 140°$
$\measuredangle A = 40°$

Say, "What is the measure of angle A?" (Angle A must measure 40°.) Repeat this procedure for triangle DEF and for triangle GHI as the students do the same in their notebooks.

STRATEGY 1: (cont'd)

e.g.,





$\measuredangle E + 20^\circ + 30^\circ = 180^\circ$
$\measuredangle E + 50^\circ = 180^\circ$
$\measuredangle E = 180^\circ - 50^\circ$
$\measuredangle E = 130^\circ$

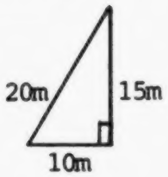
$\measuredangle G + 60^\circ + 40^\circ = 180^\circ$
$\measuredangle G + 100^\circ = 180^\circ$
$\measuredangle G = 180^\circ - 100^\circ$
$\measuredangle G = 80^\circ$

Repeat this procedure to find the measure of the third angle of other triangles when given the measures of two of their angles.

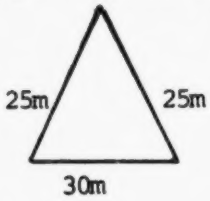
P.O. **6.09.02.01-.02**

NAME: ______________________ **CLASS:** ________ **DATE:** ____________

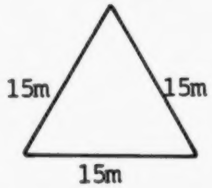
I. Classify each triangle according to the lengths of its sides.



1. ____________



2. ____________



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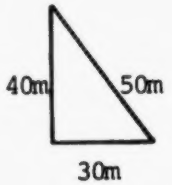

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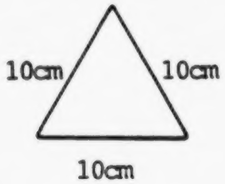
10cm
10cm
10cm

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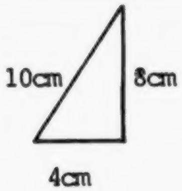
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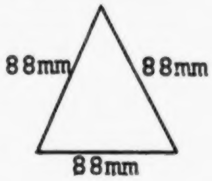
6. ____________


88mm
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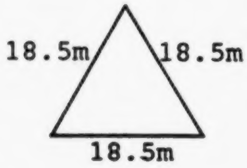
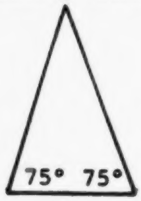
7. ____________

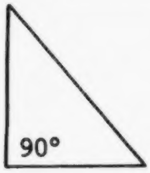

18.5m
18.5m
18.5m

8. ____________

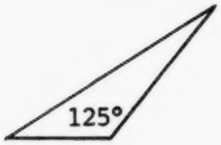
II. Classify each triangle according to the measures of its angles.



1. ____________



2. ____________



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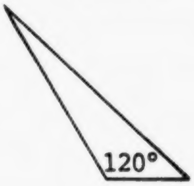

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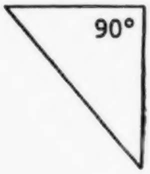

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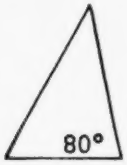
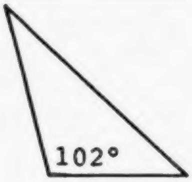
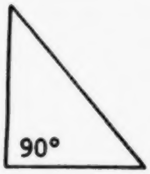


6. ____________



7. ____________



8. ____________

III. Write the missing word.

a) A triangle which has sides that are all equal in length is classified as a(n) ____________________ triangle.

b) A(n) ______________ triangle has no two sides equal in length.

c) A triangle which has two sides equal is a(n) __________ triangle.

d) An __________ triangle has an angle greater than 90°.

P.O. **6.09.02.03-.04**

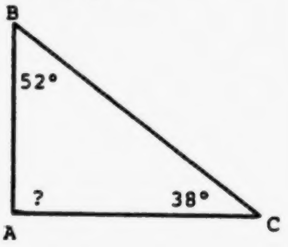
NAME: ______________________ **CLASS:** ________ **DATE:** ____________

I. What is true about the measures of the three angles of any triangle? ________________________________

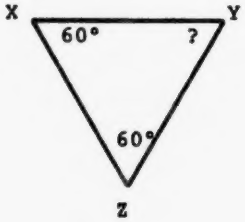
____

____

II. Find the measure of the missing angle. Two angles are completed for you.



1. __________



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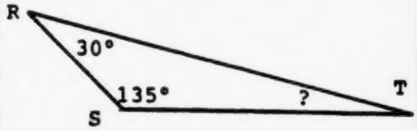

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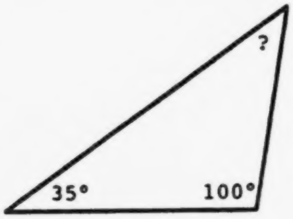

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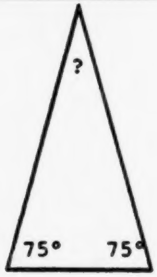
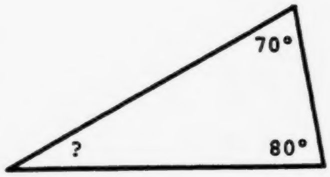
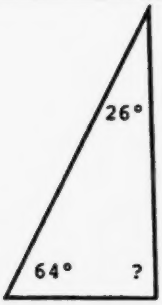


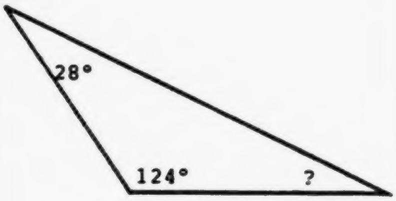
5. __________



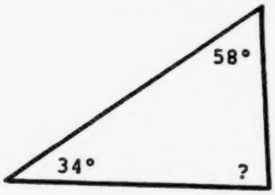
6. __________



7. __________



8. __________



9. __________

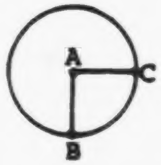
6.14.10 REVIEW: CIRCLE CONCEPTS

(1 lesson/ongoing)

PERFORMANCE OBJECTIVES

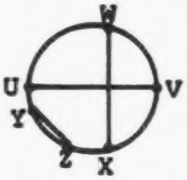
The student will be able to...

6.14.10.01 identify a radius of a circle;
e.g.,



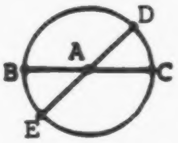
$\overline{AB}$ and $\overline{AC}$ are radii of the circle. A radius is a line segment from the center to the outer edge of the circle.

6.14.10.02 identify a chord of a circle;
e.g.,



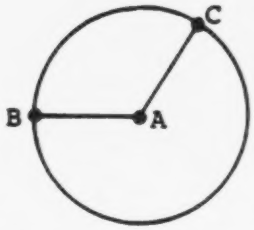
$\overline{UV}$, $\overline{WX}$, and $\overline{YZ}$ are chords. Chords are line segments whose endpoints are on the outer edge of the circle.

6.14.10.03 identify a diameter of a circle;
e.g.,



$\overline{BC}$ and $\overline{ED}$ are special chords called diameters of the circle. A diameter is a chord which passes through the center of the circle. The diameters are the longest chords in a circle. The length of each is twice the length of a radius.

6.14.10.04 identify a central angle of a circle as an angle with its vertex at the center of the circle; e.g.,



∡BAC is a central angle of circle A

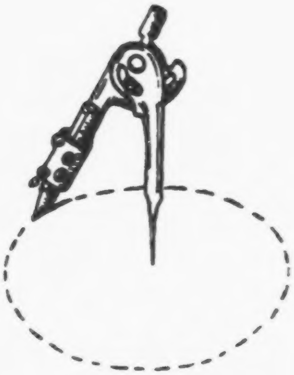
6.14.10.05 define circumference and explain the relationship between diameter and circumference and between radius and circumferemce; e.g.,

The circumference is the distance around a circle.

The circumference is approximately three times the diameter

The circumference is approximately six times the radius.

6.14.10.06 construct a circle using a compass. e.g.,



PROBLEM EXPLORATION SKILLS:

- to make a diagram
- to find a real or abstract model

VOCABULARY

radius
radii
chord
diameter
arc
compass
central angle

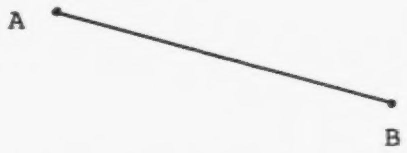
PREVIOUS UNDERSTANDINGS:

- identification of points, line segments, and angles

GETTING STARTED:

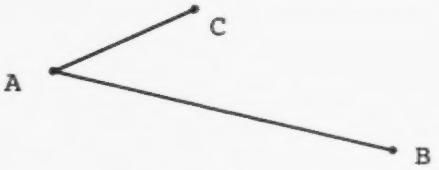
Draw and label point A on the chalkboard. Say, "How would you identify this geometric figure?" (It is called point A.) Ask another student to draw and label point B on the chalkboard and to connect the two points.

e.g.,



Say, "How would you identify this geometric figure?" (It is called line segment AB.) Ask a student to draw line segment AC.

e.g.,



Say, "How would you identify this geometric figure? (It is called angle CAB or angle BAC.) What do we call the point at which the two line segments meet to form the angle? (We call that point the **vertex** of the angle.) If we connect point B and point C, what geometric figure will be formed? (a triangle) How do we know it will be a triangle?" (It will be a closed figure made up of three line segments.) Ask a student to draw segment BC so that triangle ABC is formed.

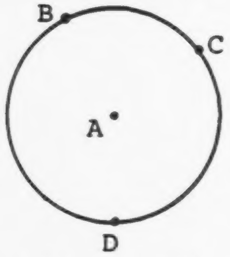
Say, "Imagine a closed figure with four sides. What figure are you imagining?" (Accept all reasonable responses, e.g., a square, a rectangle, a parallelogram, a quadrilateral, ...) As each new response is given, ask a student to differentiate it from the other types of four-sided figures which have already been named. Say, "What closed figure are we familiar with that has no sides?" (a circle)

6.14.10.01 identify a radius of a circle

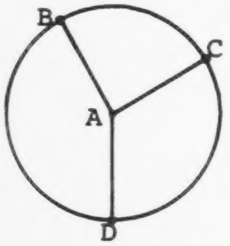
STRATEGY 1:

Materials:
- chalkboard compass
- ruler

Draw a large circle on the chalkboard. Say, "All of these points are the circle." Mark the center point of the circle and label it A. Say, "Point A is the center of this circle. It is not a point of the circle itself." Designate points B, C, and D on the edge of the circle.
e.g.,



Ask a student to come to the chalkboard and connect points A and B, points A and C, and points A and D.
e.g.,



Ask three students to measure the three line segments and record each measure near the line segment. Say, "What is true about the lengths of the line segments we drew? (They are all exactly the same length.) Each of these line segments is a radius of the circle because a radius is a line segment joining the center to some point of the circle." Record this definition on the chalkboard and ask the students to write it in their notebooks.

Ask a student to select any point E of the circle and draw line segment AE. Say, "What can you say about line segment AE? (It is a radius of the circle.) Why is segment AE a radius of the circle? (It is also a line segment connecting the center of the circle with the outer edge of the circle.) What do you know about the length

STRATEGY 1: (cont'd)

of segment AE? (It will have the same measure as segments AB, AC, and AD.) How many radii have we drawn in this circle? (4) How many radii do you think there can be in any circle? Explain." (There can be infinitely many radii in any circle because the circle is made up of infinitely many points to which a radius can be drawn.)

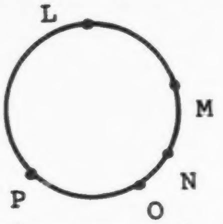
6.14.10.02 identify a chord of a circle

STRATEGY 1:

Materials:
- chalkboard compass

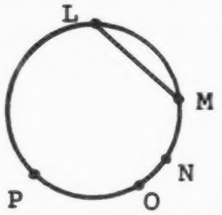
Using the chalkboard compass, draw a large circle on the chalkboard. Designate and label several points of the circle.

e.g.,



Ask a student to draw a line segment to connect two of the points of the circle, for example, point L and point M.

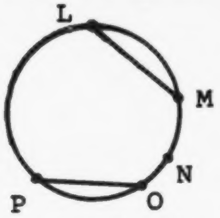
e.g.,



Say, "Is line segment LM a radius of the circle? Explain. (No, segment LM is not a radius because it is not drawn from the center of the circle to a point on the circle.) We call segment LM a chord of the circle. A chord is a line segment whose endpoints are both on the circle." (Remember, the center is not a point of the circle.) Write this definition on the chalkboard and ask the students to write it in their notebooks.

STRATEGY 1: (cont'd)

Ask another student to come to the chalkboard and draw another chord of the circle using two of the designated points.
e.g.,



Say, "Why is segment PN a chord of this circle?" (Its endpoints are on the circle.)

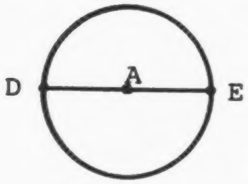
Repeat this procedure, asking the students to draw and identify other chords of the circle.

6.14.10.03 identify a diameter of a circle

STRATEGY 1:

Materials:
- chalkboard compass
- ruler

Using the chalkboard compass, draw a large circle on the chalkboard and label the center point A. Ask a student to use the ruler to draw a chord that passes through point A and to label the endpoints.
e.g.,

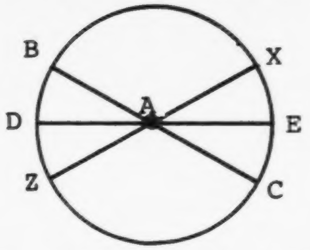


Say, "Why is segment DE a chord? (Its endpoints are on the circle.) How is this chord different from those we have already seen? (It passes through the center of the circle.) A chord that passes through the center of a circle is given a special name. It is called a **diameter of the circle**." Write this definition on the chalkboard and ask the students to write it in their notebooks.

STRATEGY 1: (cont'd)

Say, "What prediction can you make about the length of the chord that is a diameter of the circle as compared to other chords of the circle? (The diameter will be the longest chord of the circle because it passes through the widest part of the circle.)

Ask several other students to draw other diameters of the circle.
e.g.,



Say, "How do you think the diameter compares to the radius in length? Explain. (The diameter will be twice as long as the radius because we can think of it as two radii end-to-end.) Can we explain this relationship another way?" (Yes, a radius is one-half the diameter.)

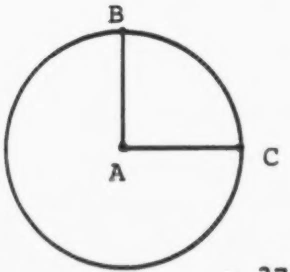
Say, "How many diameters do you think there can be in a circle? Explain." (There can be infinitely many diameters in a circle because there are infinitely many points on the outer edge.)

6.14.10.04 identify a central angle of a circle as an angle with its vertex at the center of the circle

STRATEGY 1:

Materials:
- chalkboard compass

Draw a large circle on the chalkboard with two radii, $\overline{AB}$ and $\overline{AC}$.
e.g.,



STRATEGY 1: (cont'd)

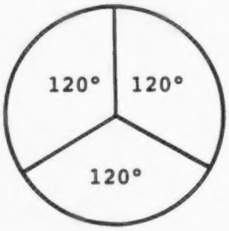
Say, "How would you describe the illustration shown on the chalkboard? (It is a circle with radii AB and AC.) Can you describe the geometric figure formed by the two radii? (Yes, it is an angle.) What is the special name of point A with respect to the angle? (Point A is the vertex of the angle.) What is the special name of point A with respect to the circle? (Point A is the center of the circle.) How would you name this angle? (angle BAC or angle CAB) This type of angle is called a **central angle of the circle.** When an angle is formed with its vertex at the center of a circle, we call it a central angle of the circle." Record this information on the chalkboard and ask the students to write it in their notebooks.

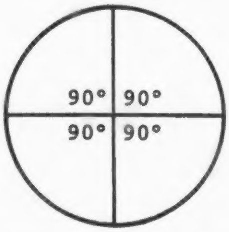
e.g., **central angle of a circle** - an angle with its vertex at the center of the circle.

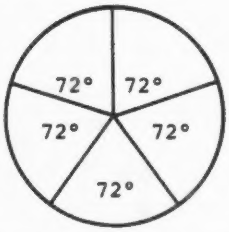
Say, "How many central angles does a circle have? Explain. (A circle has infinitely many central angles because any two radii may be selected as sides for angles.) How could I draw an angle with a specified number of degrees?" (I would use a protractor and a compass.)

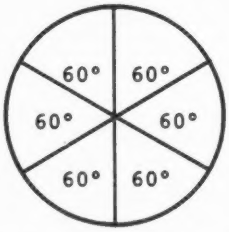
To provide students with the opportunity to practice using these concepts and using the protractor and compass, the teacher might at this point ask students to draw several equal circles and then, using the protractor to create appropriate central angles, draw radii in each so that one circular region is subdivided into thirds, another fourths, another fifths, and another sixths.

e.g.,









Students might enjoy working on this activity in groups. The circles might be colored and used to decorate the walls, or, if drawn or mounted on cardboard, cut out and used as fractional parts when exploring ideas about fractions.

Be sure that students select a radius large enough to permit themselves to use the protractor comfortably.

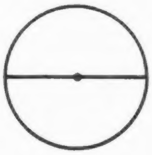
6.14.10.05 **define circumference and explain the relationship between diameter and circumference and between radius and circumference**

STRATEGY 1:

Materials:
- large cardboard circles of various sizes each showing one diameter
- several lengths of string
- yardstick

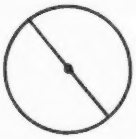
Tape the cardboard circles to the chalkboard. Ask a student to select a piece of string, measure the distance around circle X, and then use the yardstick to measure the amount of string used to the nearest half-inch. Record this amount under circle X. Then ask the student to measure the diameter of this circle and to record this data. Repeat this procedure using other students to measure circles Y and Z. e.g.,

circle X



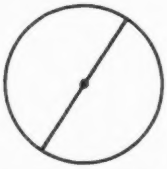
approximate distance around: $15\frac{1}{2}$ in.
length of diameter: 5 in.

circle Y



approximate distance around: $12\frac{1}{2}$ in.
length of diameter: 4 in.

circle Z



approximate distance around: 18 in.
length of diameter: 6 in.

Say, "Look at the information shown on the chalkboard. How does the diameter for each circle compare with the distance around the circle? (Elicit that in each case the distance around the circle is approximately three times as large as the diameter.) The distance around a circle is called its circumference. As we have noted the circumference of a circle is approximately three times as large as a diameter of that same circle. How would a radius of a circle compare with its circumference? Explain." (Since a diameter of a circle is twice as large as a radius, the circumference of the circle will be

STRATEGY 1: (cont'd)

approximately six times as large as a radius of a circle.) Record this information on the chalkboard and ask the students to write it in their notebooks.

e.g.,

- The circumference is the distance around a circle.
- The circumference of a circle is approximately three times the length of a diameter.
- The circumference of a circle is approximately six times the length of a radius.

Say, "If circle X on the chalkboard has a diameter of 5 inches, what would be the length of the radius? (about half the length of the diameter or about $2\frac{1}{2}$ inches) Is the circumference of circle X six times longer than a radius? Explain." (Yes, because 6 times $2\frac{1}{2}$ inches is 15 inches, which is the approximate circumference of circle X.)

Repeat this procedure and record this data for circles Y and Z.

6.14.10.06 construct a circle using a compass

STRATEGY 1:

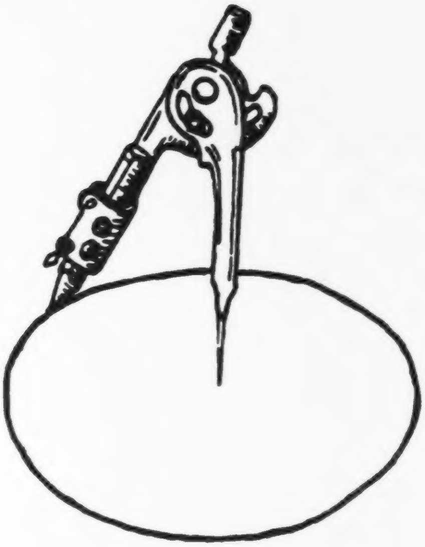
Materials:
- chalkboard compass
- compass for each student
- unlined sheets of paper

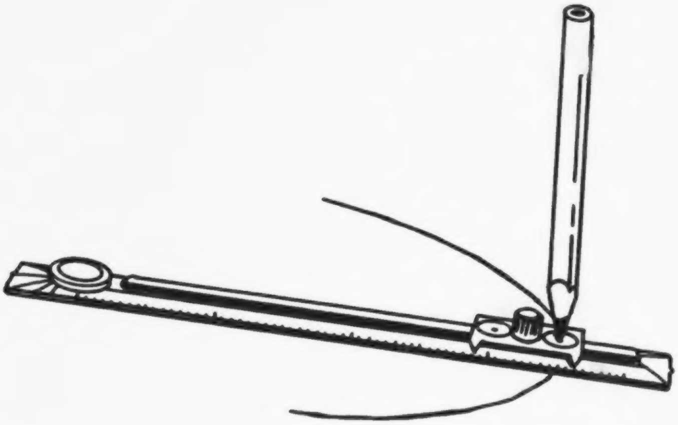
Distribute a compass and several unlined sheets of paper to each student. Say, "What instrument do we usually use to construct circles of different sizes?" (a compass) Use the chalkboard compass to demonstrate the procedure for drawing a circle. Say, "What does the point of the compass do? (It keeps still at the center of the circle.) How can I draw circles of different sizes using the compass?" (Circles of different sizes can be drawn by adjusting the opening of

STRATEGY 1: (cont'd)

the compass, the distance between the center point and the pencil point.) Ask the students to construct circles of different sizes on the unlined paper by adjusting the opening of the compass. Remind students that once they have the desired opening, they should lock the compass, place the point on the paper without moving it, and then carefully swing the pencil around to form the circle.

e.g.,



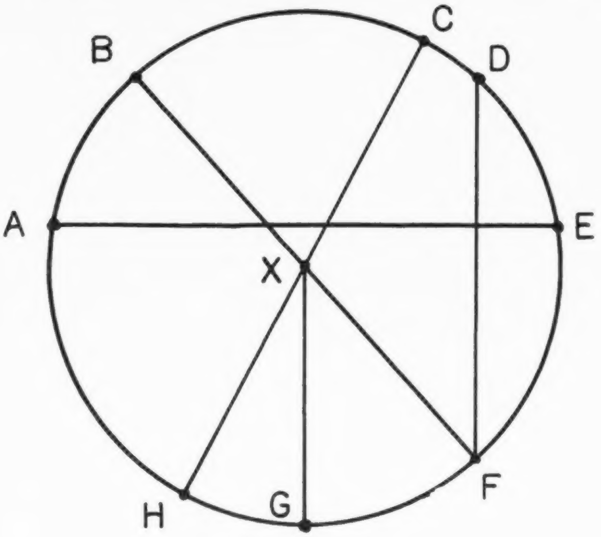


Say. "I want to draw a circle with a $1\frac{1}{2}$ inch radius. How can I accomplish this without using a ruler. Explain." (The compass has a scale with markings on it that indicate the length of the radius. The compass must be adjusted to show $1\frac{1}{2}$ inches on this scale in order to draw a circle with a $1\frac{1}{2}$ inch radius.) Students should check this by setting the compass using the markings on its scale, and then measuring the opening with a ruler. Emphasize that accuracy rather than speed is the goal. Ask students to continue practicing constructing circles by adjusting the compass each time for a specific radius and then drawing the circle.

P.O. **6.14.10.01-.03**

NAME: ______________________ **CLASS:** ________ **DATE:** ____________

Using the diagram, identify each line segment as a radius, chord or diameter of the circle and then fill in the missing term in each sentence.



Point X is the center of the circle.

1. $\overline{AE}$ is a ______________ of the circle
2. $\overline{BX}$ is a ______________ of the circle
3. $\overline{CH}$ is a ______________ of the circle
4. $\overline{CX}$ is a ______________ of the circle
5. $\overline{DF}$ is a ______________ of the circle
6. $\overline{FB}$ is a ______________ of the circle
7. $\overline{GX}$ is a ______________ of the circle
8. $\overline{HX}$ is a ______________ of the circle

Do you remember?

Find the mode and median for the following set of data.

Bowling Scores of 6-204's Bowling Team:
90,89,90,89,86,92,90,92,88

Mode: __________ Median: __________

P.O. 6.14.10.04-.06

NAME: ______________________ **CLASS:** ______ **DATE:** ______________

Match each diameter for a circle with the approximate circumference for the circle.

length of diameter	approximate circumference
a. 12 in.	25.1 in.
b. 5 in.	78.5 in.
c. 8 in.	37.1 in.
d. 25 in.	62.8 in.
e. 15 in.	15.7 in.
f. 20 in.	47.1 in.

Complete each sentence.

1. The circumference is the ____________ around a circle.
2. The circumference is approximately three times the length of the ____________.
3. The circumference is approximately six times the length of the ____________.

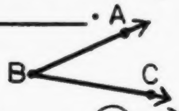
Use a compass to construct a circle with a 2-inch radius. Label the center point A. Then use your protractor to construct central angle MAC having a measure of 155°.

SHARPENING YOUR SKILLS

Units 28, 29, 30

Name: ____________________ **Class:** ____________ **Date:** ________

1. The angle below may be named ______________.

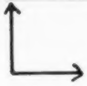


ⓐ ∡BAC ⓒ ∡A
ⓑ ∡ABC ⓓ Not Here

2. A straight angle measures _______ degrees.

ⓐ 90° ⓒ 180°
ⓑ 360° ⓓ 100°

3. On the line below each angle, write acute, obtuse, right, or straight to describe the angle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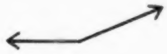
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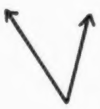
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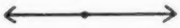
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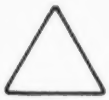
4. An obtuse angle measures more than ____ degrees but less than 180°.

5. On the line below each triangle, write equilateral, isosceles, or scalene to describe the triangle.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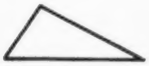
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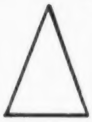
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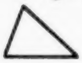
6. Classify each triangle below as right, acute, or obtuse.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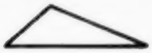
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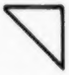
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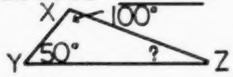
7. The sum of the measures of the angles of any triangle is _____.

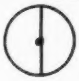
ⓐ 90° ⓒ 360°
ⓑ 100° ⓓ 180°

8. In the triangle below, ∡Z measures _____.



ⓔ 30° ⓖ 180°
ⓕ 50° ⓗ Not Here

9. In which circle below has a radius been drawn?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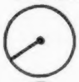
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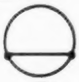
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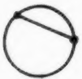
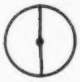

ⓓ

10. In which circle below has a diameter been drawn?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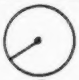
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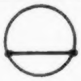
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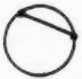
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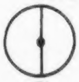
11. In which circle below has a chord been drawn?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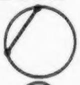
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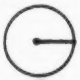
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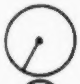

ⓓ

12. The distance around a circle is called the ________________.

6.12.03 CIRCLE GRAPHS

(2 lessons)

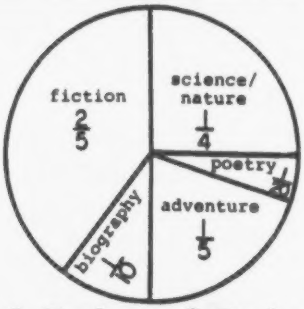
PERFORMANCE OBJECTIVES

The student will be able to...

6.12.03.01A read and interpret circle graphs where the sectors are identified as fractions of the whole;

e.g.,

BOOKS IN THE PARKER SCHOOL LIBRARY



Total Number of Books: 4000

What fractional part of the books in the library are fiction?

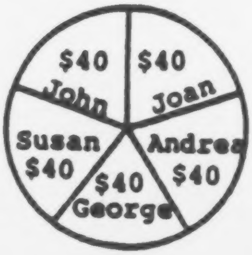
How many adventure books are in the library?

How many more science/nature books are there than biographies?

*6.12.03.02A use a protractor to construct circle graphs where the sectors represent equal unit fractions;

e.g.,

Sharing $200



The whole circle represents $200 which George, Joan, Andrea, Susan, and John earned while working after school. They shared the money equally.

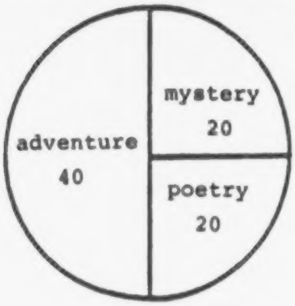
$\frac{1}{5}$ of 360° = 72°

The central angle for each section is 72°.

*6.12.03.03A use a protractor to construct circle graphs where the sectors represent fractions with unlike denominators.

e.g.,

Books on the Shelf



	Part	Fraction	Central Angle
	20 books	$\frac{20}{80} = \frac{1}{4}$	$\frac{1}{4}$ of $360^\circ = 90^\circ$
	20 books	$\frac{20}{80} = \frac{1}{4}$	$\frac{1}{4}$ of $360^\circ = 90^\circ$
	40 books	$\frac{40}{80} = \frac{1}{2}$	$\frac{1}{2}$ of $360^\circ = 180^\circ$
Total	80 books		

PROBLEM EXPLORATION SKILLS:

- to generalize conclusions carefully
- to compare and contrast
- to consider the consequences of an idea
- to make a model

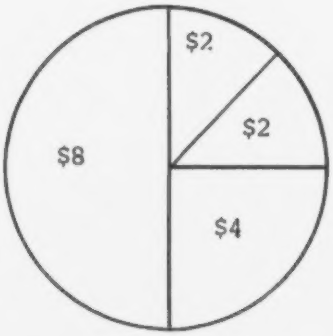
VOCABULARY	**SYMBOL**
circle graph	AB (arc AB)
central angle	
radius	CD (line segment CD)
diameter	
arc	
sector	

PREVIOUS UNDERSTANDINGS:

- constructing a circle graph with a compass
- reading and interpreting graphs
- identifying angles
- finding a fractional part of a whole

GETTING STARTED:

Display the following circle graph:

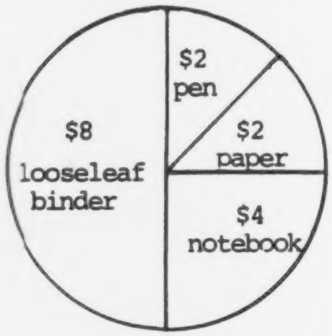


Say, "This is a circle graph which shows how Jessica spent the money her mother gave her to buy school supplies." Label the graph, 'Jessica's School Supplies.' Say, "How much money did Jessica's mother give her? ($16) Explain. (The sum of the money amounts found in the four sections is $16.) Half the money spent was for a looseleaf binder. Where is this information on the graph and how should we label it? Explain. (Half of $16 is $8 which is also half the circle. We can write 'looseleaf binder' in the section for $8.) If half of the remaining money was spent on a notebook, where would we write 'notebook'? (Half of the remaining $8 is $4 so 'notebook' should be written in the $4 section.) What can we say about the sizes of the remaining two sections?" (Since the number amounts are equal, so are the sizes of the sections.) Label these sections 'pen' and 'paper'.

GETTING STARTED: (cont'd)

e.g.,

JESSICA'S SCHOOL SUPPLIES



Say, "What cost $2 more than the pen? (notebook) ...$4 more than the notebook? (looseleaf binder) ...the same as the sum of the amounts of the pen and the paper? (notebook) Which two items together cost $12? (binder and notebook) Which two items cost the same amount?" (pen and paper)

Say, "Why is a circle graph a good way to represent information of this type?" (It gives us a clear, organized representation for the given data so that we can answer different questions more easily. It also emphasizes the relative sizes of the different parts of a whole.)

6.12.03

6.12.03.01A read and interpret circle graphs where the sectors are identified as fractions of the whole

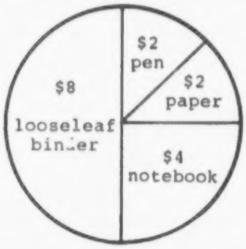
STRATEGY 1:

Materials:
- circle graph from Getting Started
- circle graph showing fractional parts with different denominators

Display the circle graph from the Getting Started activity.

e.g.,

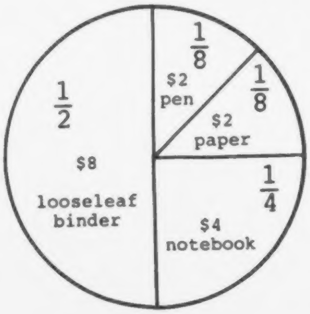
JESSICA'S SCHOOL SUPPLIES



Say, "The sections in this circle graph, as in all circle graphs, have a special name. We call them sectors. They are formed by 2 radii and the included arc. You are familiar with the term 'fractional part of a circle.' A fractional part of a circle is the same as a sector. What fraction should we use for the sector which contains the amount of money spent on the notebook? (Since $4 is $\frac{1}{4}$ of $16, the sector is $\frac{1}{4}$ of the circle.) ... the looseleaf binder? (Since $8 is $\frac{1}{2}$ of $16, the sector is $\frac{1}{2}$ of the circle.) ... the pen and paper?" (Since 2 is $\frac{1}{8}$ of $16, the sector is $\frac{1}{8}$ of the circle.) Ask a student to record the information on the graph.

e.g.,

JESSICA'S SCHOOL SUPPLIES



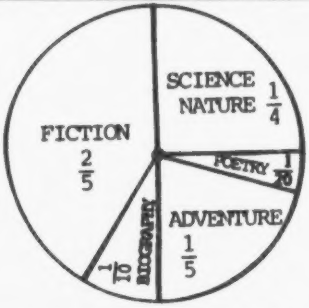
STRATEGY 1: (cont'd)

Say, "If this method of finding a fractional part of a circle is accurate, we need only two sources of information to find the value of any sector of a circle:

1) the amount, or value, of the whole circle
2) the fraction name for each sector

Display the following graph:

BOOKS IN OUR SCHOOL LIBRARY



Total number of books = 2000

Say, "This circle graph represents the 2000 books in our school library, which is divided into five categories. (The sectors represent the five categories.) Let's find out how many 'adventure books' are in our library." Ask a student to write a number sentence that would answer this question while the other students solve it at their seats.

e.g.,

$\frac{1}{5}$ of 2000 books $\qquad \frac{1}{5} \times 2000 = \square$

$$\frac{1}{5} \times \frac{2000}{1} = \frac{2000}{5} = 400$$

Say, "What does this 400 represent?" (There are 400 adventure books in the school library. The sector which is $\frac{1}{5}$ of the circle represents 400 books.)

Continue by asking the students to find how many books there are in each category, using the same procedure.

e.g.,

$\frac{1}{4} \times 2000 = \frac{2000}{4} =$ 500 (Science-Nature)

$\frac{2}{5} \times 2000 = \frac{4000}{5} =$ 800 (Fiction)

STRATEGY 1: (cont'd)

$$\frac{1}{10} \times 2000 = \frac{2000}{10} = 200 \text{ (Biography)}$$

$$\frac{1}{5} \times 2000 = \frac{2000}{5} = 400 \text{ (Adventure)}$$

$$\frac{1}{20} \times 2000 = \frac{2000}{20} = 100 \text{ (Poetry)}$$

2000 (Total)

Say, "How many more science books are there than adventure books? (There are 100 more science books than adventure books.) ...How many biography and poetry books are there in all? (300) If we add the number of books in each category, how many books should there be altogether? (2000) If we add the fractions for each sector, what should they total? Explain." (1 whole. All of the fractions together represent the whole circle.)

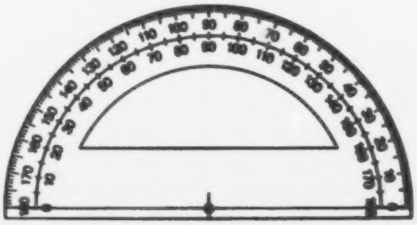
Repeat this procedure to read and interpret other circle graphs where the sectors are represented by fractions.

6.12.03.02A use a protractor to construct circle graphs where the sectors represent equal unit fractions

STRATEGY 1:

Materials:
- protractor and compass for each student or group of students
- large demonstration protractor and compass
- unlined paper

Display the demonstration protractor.



STRATEGY 1: (cont'd)

Distribute the unlined paper, compasses, and protractors to each student or group of students. Say, "How many degrees are there in a protractor? (180°) What geometric shape is a protractor? (a sector) What is the number of degrees in its central angle?" (180°) How many degrees would there be in the central angle for a whole circle (360°) Why is this information important? (If we know a whole circle contains 360°, we could multiply 360° by any fraction of that circle to find the number of degrees contained in the central angles of the sector that represents that fractional part. Then we could measure and construct that sector of the circle.) How could we find the number of degrees in a circle divided into fourths? (Divide 360° by 4.) ...divided into fifths? (Divide 360° by 5.) ...by sixths?" (Divide 360° by 6.) Record the following information on the chalkboard and ask several students to find the number of degrees in the central angle of each sector represented as a fraction.

e.g.,

$$\frac{1}{4} \text{ of } 360° = \frac{1}{4} \times \frac{360}{1} = \frac{360}{4} = 90°$$

$$\frac{1}{5} \text{ of } 360° = \frac{1}{5} \times \frac{360}{1} = \frac{360}{5} = 72°$$

$$\frac{1}{6} \text{ of } 360° = \frac{1}{6} \times \frac{360}{1} = \frac{360}{6} = 60°$$

Say, "With this information we can use a protractor to construct a circle graph in which the number of degrees in each sector is precisely correct for the fractional part of the circle that the sector represents. Let's begin by constructing a circle graph in which all sectors are equal." Present the following problem:

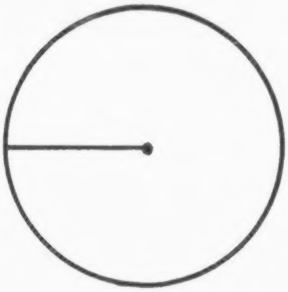
> Maria and seven friends were given $200 prize money for winning first place in a contest. They will share the money equally.

Say, "How many sections, or sectors will the circle graph show? (8) What could we write for the title of the graph?" (Elicit an answer such as "Sharing the Money".) Ask the students to use their compasses to draw a circle while another student uses the demonstration compass to draw it on the chalkboard. Say, "How many sectors will we need? (We will need 8 sectors for Maria and seven friends.) "Use your

STRATEGY 1: (cont'd)

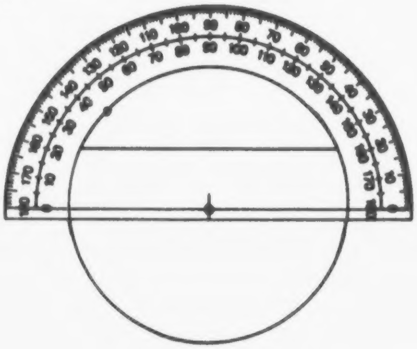
protractor to measure off the central angles of the sectors. How would we determine the number of degrees for each sector?" (Since there are eight sectors, dividing 360 degrees by 8 gives us 45 degrees in each central angle of each sector.) Remind the students that, when constructing sectors, we first locate the center of the circle, then draw a radius at any starting place.

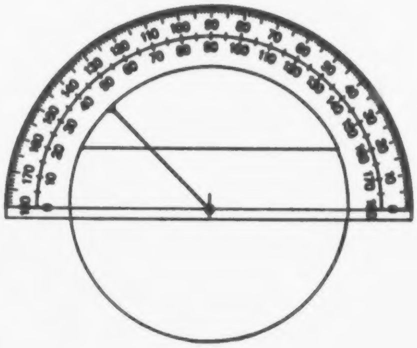
e.g.,



Say, "Begin by drawing the radius that will be the starting place. Then, put the hole, circle, or dot found on the protractor over the center of the circle, lining up the zero line of the protractor with the starting radius at the same time. Now we will construct a 45° angle. Measure off 45 degrees on the protractor scale. At the 45 degree line, draw a small point on the paper. Remove the protractor and draw a radius connecting this point to the center point of the circle."

e.g.,



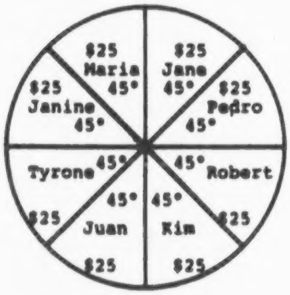


Say, "We have now formed the first 45° sector." Repeat these directions until the circle is divided completely and labeled.

STRATEGY 1: (cont'd)

e.g.,

Sharing The Money



Total = $200

Say, "How much money is represented by the whole circle? ($200) How much prize money did each student receive? ($25) What fraction of the prize did Juan receive? ($\frac{1}{8}$ of $200 or $25.) What fractional part of the circle represents the part of the prize won by the boys? Explain." (There are 3 boys so they received $\frac{3}{8}$ of the prize.)

Repeat this procedure to construct other circle graphs where the sectors represent equal unit fractions.

6.12.03.03A **use a protractor to construct circle graphs where the sectors represent fractions with unlike denominators**

STRATEGY 1:

Materials:
- protractors
- compasses
- unlined paper

Distribute protractors, compasses, and unlined paper to each student or group of students.

STRATEGY 1: (cont'd)

Display the following chart:

FAVORITE PETS OF STUDENTS IN THE LANDERSVILLE ELEMENTARY SCHOOL	
Pet	Number of Students
Dogs	200
Cats	100
Fish	50
Birds	50

Say, "We want to show this information on a circle graph. How many sectors will we need? (4) How will the sectors be different from those on the previous circle graph we created? Explain. (The sectors will be different sizes because the number of students varied for each pet.) In order to determine the size of each sector, we will have to compare the number of students in each sector to the whole. How many students were included in the set of data?" (400) Write the following on the chalkboard:

Pet	Number of Students	Fractional Part of the Whole	Central Angle of the Sector
Dogs	200		
Cats	100		
Fish	50		
Birds	50		

Say, "How can we express, as a fraction of the number of students in this school, the number of students who chose dogs as their favorite pet? Explain." (We know that there were 400 students altogether. 200 of the 400 students liked dogs as their favorite pet. As a fraction we would write this as $\frac{200}{400}$ or $\frac{1}{2}$.) Ask a student to record this information on the chart.

STRATEGY 1: (cont'd)

e.g.,

Pet	Number of Students	Fractional Part of the Whole	Central Angle of the Sector
Dogs	200	$\frac{200}{400} = \frac{1}{2}$	
Cats	100	$\frac{100}{400} = \frac{1}{4}$	
Fish	50	$\frac{50}{400} = \frac{1}{8}$	
Birds	50	$\frac{50}{400} = \frac{1}{8}$	

Say, "How would we express, as a fraction, the number of students who chose cats as their favorite pet? ($\frac{100}{400}$ or $\frac{1}{4}$) ...fish? ($\frac{50}{400}$ or $\frac{1}{8}$) ...birds?" ($\frac{50}{400}$ or $\frac{1}{8}$) Ask a student to record this information on the chart. Say, "We now know the fraction name for each sector. What else do we need to know in order to begin constructing our circle graph? (We need to know the number of degrees for the central angles of each sector.) How can we determine the number of degrees for the central angle of each sector?" (We can multiply the fractional part by 360°.) Ask a student to come to the chalkboard and calculate the number of degrees for each central angle. Ask another student to record this information on the chart.

e.g.,

$\frac{1}{2} \times 360° = 180°$

$\frac{1}{4} \times 360° = 90°$

$\frac{1}{8} \times 360° = 45°$

$\frac{1}{8} \times 360° = 45°$

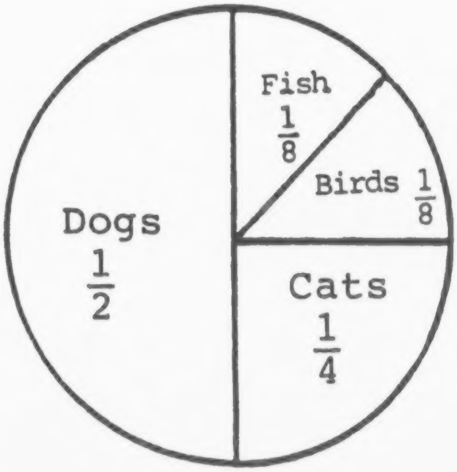
STRATEGY 1: (cont'd)

Pet	Number of Students	Fractional Part of the Whole	Central Angle of the Sector
Dogs	200	$\frac{200}{400} = \frac{1}{2}$	180°
Cats	100	$\frac{100}{400} = \frac{1}{4}$	90°
Fish	50	$\frac{50}{400} = \frac{1}{8}$	45°
Birds	50	$\frac{50}{400} = \frac{1}{8}$	45°

Say, "Do we have all of the information we will need to construct our circle graph?" (yes)

Adapt Strategy 1, P.O. 6.12.03.02A to construct the graph.

e.g.,



P.O. **6.12.03.01-.02**

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

I.A. Find the amount of money in each example.

$\frac{1}{5}$ of \$250 = ____ $\frac{2}{11}$ of \$440 = ____

$\frac{2}{6}$ of \$36 = ____ $\frac{3}{5}$ of \$75 = ____

$\frac{2}{3}$ of \$480 = ____ $\frac{4}{9}$ of \$81 = ____

B. Find the number of degrees in each example.

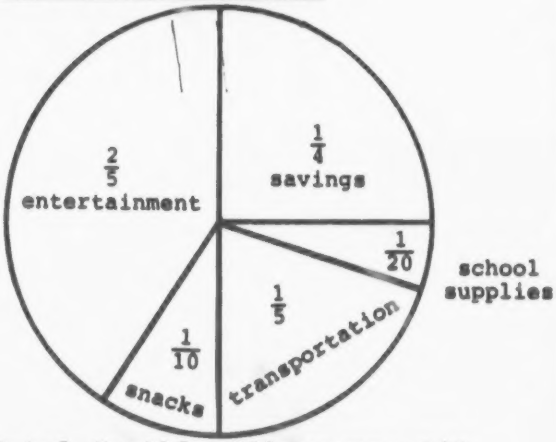
$\frac{1}{5}$ of 360° = ____ $\frac{6}{18}$ of 360° = ____

$\frac{3}{4}$ of 360° = ____ $\frac{5}{9}$ of 360° = ____

$\frac{1}{2}$ of 360° = ____ $\frac{5}{6}$ of 360° = ____

II. Use this circle graph to answer the questions below.

MARY K.'s ALLOWANCE



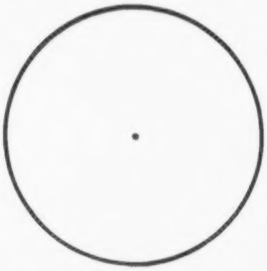
Total Monthly Allowance: \$40

Mary spent the following amount of money in each category:

1. entertainment ________
2. transportation ________
3. school supplies ________
4. snacks ________
5. How much more was spent on entertainment than on snacks and school supplies combined? ______

III. Below are circles A and B. Divide Circle A into 4 equal sectors. Divide Circle B into 6 equal sectors.

Circle A



Circle B

Answer the following questions about the circles above.

1. If circle A represents a total of \$240, how much money does one sector represent? ________
2. If circle B represents a total of \$240, how much money do 5 sectors represent? ________
3. How many degrees are there in the central angle of a sector in circle A? ________
4. How many degrees are in a central angle in a sector in circle B? ________

P.O. 6.12.03.01&.03

NAME: ______________________________ CLASS: _______ DATE: ____________________

The table below shows data about P.S. 76's "After School Program". Complete the data and construct a circle graph below. (Use your protractor!)

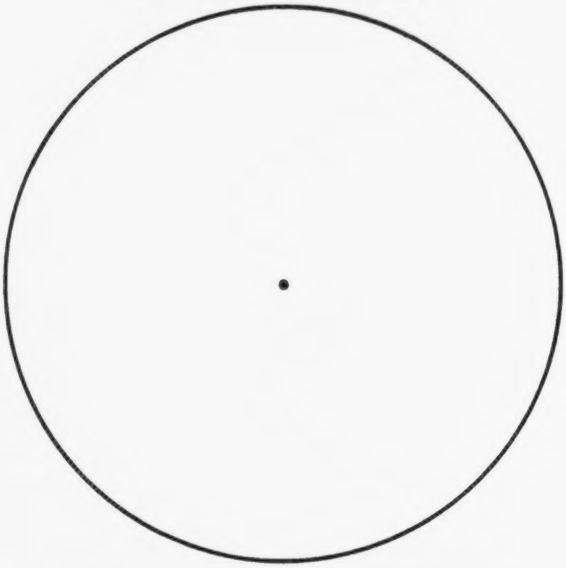
After School Program

P.S. 16

Activity	Number of Students	Fraction (to lowest terms)	Central Angle
Basketball	40	$\frac{40}{150} = \frac{4}{15}$	$\frac{4}{15}$ of 360° = 96°
Ping Pong	20		
Homework	25		
Tutoring	35		
Computers	15		
Games	10		
Arts & Crafts	5		
Total	150		

P.S. 76's AFTER

SCHOOL PROGRAM



TOTAL : 150 STUDENTS

6.14.11 REVIEW: ORDER OF OPERATIONS

(1 lesson/ongoing)

PERFORMANCE OBJECTIVES

The student will be able to...

6.14.11.01 explain and utilize the correct sequence of operations;
e.g.,

$$8^2 + 6 \div 2 - 3 \times 4 = 64 + 3 - 12 = 67 - 12 = 55$$

Note: The indicated operations should be performed in the following order:

1. **All exponentiations first.**
2. **All multiplications and divisions next, in sequence from left to right.**
3. **All additions and subtractions last, in sequence from left to right.**

6.14.11.02 recognize that in the conventional sequence of operations, any calculations indicated within parentheses must be performed first.
e.g.,

$$7^2 + (7 - 4) \times 3 = \square$$

$$49 + 3 \times 3 = \square$$

$$49 + 9 = 58$$

PROBLEM EXPLORATION SKILLS:

- to keep good records
- to maintain an overview
- to recognize a convention

6.14.11

VOCABULARY

exponents
parentheses

PREVIOUS UNDERSTANDINGS:

- exponents

GETTING STARTED:

Display a word problem requiring two operations on the chalkboard.
e.g.,

> Sarah bought 4 souvenirs at the Statue of Liberty and 7 more at the World Trade Center. She gave 9 souvenirs to her friends. How many did she keep for herself?

Ask the student to read the problem aloud. Say, "What do we have to find? (We must find the number of souvenirs Sarah kept for herself.) How can we do this? (We can add 4 plus 7. Then we can subtract 9.) What is the answer?" (She saved 2 souvenirs for herself.) Ask a student to write the solution on the chalkboard.
e.g.,

$$\begin{array}{r} 4 \text{ souvenirs} \\ \underline{+\ 7 \text{ souvenirs}} \\ 11 \text{ souvenirs} \\ \underline{-\ 9 \text{ souvenirs}} \\ 2 \text{ souvenirs} \end{array}$$

Say, "To solve this problem, first we added, then we subtracted. Was the order in which we worked important? (Yes, because if we did not add the 4 and 7 first, we would not have been able to subtract.) Sometimes, the order in which we work to solve a problem is very important. We must do our operations in a certain order to arrive at the correct solution."

Ask a student to suggest or consider other real life situations where this is true.
e.g.,

> Elsie and Eva are earning the same salary. Elsie is given a $10 raise, and then her salary is doubled.
> Eva's salary is doubled, and then she is given a $10 raise.
> Are they still earning the same salary?

6.14.11

6.14.11.01 explain and utilize the correct sequence of operations

STRATEGY 1:

Display a numerical expression requiring several operations.

e.g.,

$$3 + 6^2 \times 2 - 18 \div 3 = \square$$

Ask the students to write the example into their notebooks and to solve it. Say, "What is your answer?" (Students may try to solve the problem by moving through the operations from left to right. In that case, the solution will be 20.) Display this incorrect solution.

e.g.,

$$3 + 6^2 \times 2 - 18 \div 3 = \square$$
$$3 + 36 \times 2 - 18 \div 3 = \square$$
$$39 \times 2 - 18 \div 3 = \square$$
$$78 - 18 \div 3 = \square$$
$$60 \div 3 = 20$$

Say, "This solution is not correct." Write "Incorrect Solution" beside it. Say, "The order in which we do our operations is important. We don't necessarily solve by moving from left to right." Write the rules for the sequence of operations on the chalkboard. Ask the students to copy it into their notebooks.

e.g.,

Correct Sequence of Operations for Evaluating Expressions

1. all exponentiations first
2. all multiplications and divisions, in sequence from left to right
3. all additions and subtractions, in sequence from left to right

Say, "Let's try solving the problem following these rules. What should we do first? (We should solve 6^2 and get 36.) What should we do next? (The next step is to multiply 36 by 2 and get 72.) What is our third step?" (The third step would be to divide 18 by 3 and get

STRATEGY 1: (cont'd)

6.) Write this partial solution on the chalkboard next to the incorrection solution to help students compare and contrast the two. e.g.,

Correct Solution

$3 + 6^2 \times 2 - 18 \div 3 = \square$

$3 + 36 \times 2 - 18 \div 3 = \square$

$3 + 72 - 18 \div 3 = \square$

$3 + 72 - 6 = \square$

$75 - 6 = 69$

Say, "How should we complete the example? (We should first add 3 + 72 and get 75. Then we should subtract 6 and get 69.) What is the final solution to the problem? (The final solution is 69.) Did following the rules for sequence of operations make a difference in the result? (Yes, the first solution we got was 20, but the second solution was 69.) Which is the correct response? (According to the rules, the correct response was 69.) What is the correct sequence of operations?" (First, we do the exponents. Next, we do any multiplication or division, in order from left to right. Last, we do any addition or subtaction, in order from left to right.)

Repeat this procedure with several other examples.

6.14.11.02 **recognize that in the conventional sequence of operations, any calculations indicated within parentheses must be performed first**

STRATEGY 1:

Display the following example on the chalkboard:

$72 \div 6^2 + 18 - 5 \times 4 = \square$

Ask the students to copy the example into their notebooks and solve it, following the rules for the correct sequence of operations.

Say, "What answer did you get?" (My answer was 0.) How did you get 0 as an answer?" (I evaluated 6^2 first to get 36. I divided 72 by 36 and got 2. Then I multiplied 5 by 4 to get 20.) Ask a student to display this partial solution on the chalkboard:

STRATEGY 1: (cont'd)

e.g.,

$$72 + 6^2 + 18 - 5 \times 4 = \square$$
$$72 + 36 + 18 - 5 \times 4 = \square$$
$$2 + 18 - 20 = \square$$

Say, "What did we do next?" (We added 2 and 18 to get 20 and then took 20 away from 20, that's how we got 0.)

Say, "Sometimes situations occur which require that the operations be done in an order that is different from the conventional sequence we have learned. Suppose 4 people each bought 5 red pens and 3 black pens. How would we find the total number of pens bought?" (We would add 5 plus 3 and then multiply by 4.) On the chalkboard, write: 5 + 3 x 4. Say, "If we evaluate this expression according to the order of operations we have learned, what is the answer? (17) Is that the correct number of pens bought? Explain. (No, 4 people bought a total of 8 pens each. The total number of pens bought is 32.) For situations such as these, mathematicians have developed a way to indicate when we should follow an order operations that is different from the one we have learned. By using parentheses, we can show which operation should be done first." On the chalkboard, place parentheses around 5 + 3.

e.g.,

$$(5 + 3) \times 4$$

Say, "The parentheses indicate that, in this case, we should perform the addition before we multiply. Using this order, what will the solution be?" (32, the correct number of pens bought by the 4 people.)

Say, "Let's look back at the expression we evaluated before." Place parentheses around the 18 minus 5.

e.g.,

$$72 + 6^2 + (18 - 5) \times 4 = \square$$

Say, "How does our new rule change things?" (Now we must do the 18 - 5 first.) Ask a student to begin working on the solution on the chalkboard while the others work at their desks. Review the example as the students proceed.

STRATEGY 1: (cont'd)

e.g.,

$$72 \div 6^2 + (18 - 5) \times 4 = \square$$
$$72 \div 36 + 13 \times 4 = \square$$
$$2 + 52 = 54$$

Say, "What steps did we follow to get a solution of 54? (First, we solved the part in the parentheses, 18 - 5, to get 13. Next, we evaluated 6^2 and the result was 36. Then, from left to right, we did the division, 72 ÷ 36 = 2, and the multiplication, 13 x 4 = 52. Last, we did the addition, 52 + 2 = 54. That was our solution.) It is sometimes helpful to have an easy way to remember correct sequence." Write the following sentence on the chalkboard: "Please Excuse My Dear Aunt Sally." Underline the first letter of each word. Say, "Look at the underlined first letters. What do you think they might stand for? (They stand for Parentheses, Exponents, Multiplication, Division, Addition, Subtraction.) Remembering this sentence may help you to remember the correct sequence. Or you may think only of the word PEMDAS to help you remember it better." Caution the students that, if they use this memory aid, they run the risk of being misled to think that multiplications are always done before divisions and additions before subtractions. Review many examples to be sure that the students can explain the correct ideas.

Repeat this procedure with several other examples involving at least three operations in any number sentence. Include examples with and without parentheses, with and without exponents.

P.O. 6.14.11.01-.02

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

Use the correct order of operations to solve each example. Use your answers to solve the riddle.

① $3^2 - 4 \times 2 + 6 \times 5 =$ ☐

Solution = ______ Ⓐ

② $3 \times 8^2 + 2 + 2 =$ ☐

Solution = ______ Ⓢ

③ $(3^2 - 4) \times 2 + 6 \times 5 =$ ☐

Solution = ______ Ⓒ

④ $3 \times (8^2 + 2) + 2 =$ ☐

Solution = ______ Ⓘ

⑤ $6^2 + 7 \times (2 + 8) - 6 =$ ☐

Solution = ______ Ⓝ

⑥ $63 - 17 \times 2 - 7 \times 4 =$ ☐

Solution = ______ Ⓣ

⑦ $5 \times 10 + (7^2 + 1) - 1 =$ ☐

Solution = ______ Ⓐ

⑧ How are people who solve this riddle like people who root for the Yankees and the Mets?

They are

__ __ __ - __ __ __ __ __ __

0 31 100 1 31 193 1 99 40

6.11.01 OPEN SENTENCES

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

*6.11.01.01A solve one-step open sentences with one variable by using the inverse operation;

e.g.,

$n - 20 = 10$ $a + 2 = 20$

$n = 30$ $a = 40$

6.11.01.02A write the open sentence that can be used to solve a given word problem.

e.g.,

Jerry had $315.00 in his savings account at the end of last month. He added more money to his account this month and his savings now total $408.00. How much money did he put into his savings account this month?

$N + 315 = 408$

$N = 93$

PROBLEM EXPLORATION SKILLS:

- to make a diagram
- to adopt a different point of view
- to look for a pattern

VOCABULARY

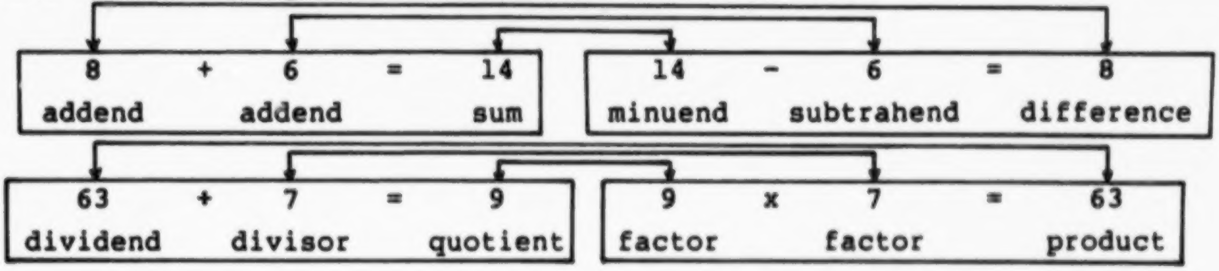
algebra
related facts
inverse operations
open sentence
variable

PREVIOUS UNDERSTANDINGS:

- addition and subtraction are inverse operations
- multiplication and division are inverse operations
- order of operations

GETTING STARTED:

Write the following four sentences on the chalkboard:



8	+	6	=	14
addend		addend		sum

14	-	6	=	8
minuend		subtrahend		difference

63	+	7	=	9
dividend		divisor		quotient

9	x	7	=	63
factor		factor		product

Say, "The addition and subtraction facts shown here are related. The division and multiplication facts are also related. What name is often given to a pair of operations related in this way?" (They can be described as inverse operations. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.) Write the following on the chalkboard:

Addition	Subtraction
9 + 2 = 11	
	17 - 8 = 8
8 + 5 = 13	

Multiplication	Division
9 x 8 = 72	
5 x 4 = 20	
	56 + 7 = 8

Ask a student to come to the chalkboard and write a related fact involving the inverse operation for the first sentence.

GETTING STARTED: (cont'd)

e.g.,

Addition	Subtraction
9 + 2 = 11	11 - 2 = 9

Repeat this procedure until both charts are complete.

Say, "Every addition fact has a related subtraction fact. Every multiplication fact has a related division fact."

Allow students to explore these statements by asking one student to state a fact and then asking other students to express the related, inverse fact.

6.11.01

6.11.01.01A **solve one-step open sentences with one variable by using the inverse operation**

STRATEGY 1:

Write the following on the chalkboard:

$6 + 8 = 14$ $\qquad\qquad$ $14 - 8 = 6$

Say, "The related facts $6 + 8 = 14$ and $14 - 8 = 6$ show us how addition and subtraction are related. They are called inverse operations." Ask a student to come to the chalkboard and demonstrate the similar idea by writing two related number sentences using multiplication and division.

e.g.,

$9 \times 3 = 27$ $\qquad\qquad$ $27 \div 3 = 9$

Using the pair of sentences, say, "What is the product of 9 and 3? (27) What is the result when the quotient, 27, is divided by the factor 3? (the other factor, 9) The related facts $9 \times 3 = 27$ and $27 \div 3 = 9$ show us how multiplication and division are related and why they are called inverse operations." Each of these examples is a complete number sentence. No numbers are unknown. Write the following on the chalkboard:

$3 \times n = 24$

Say, "How is this number sentence different from the other number sentences on the chalkboard? Explain. (There is a missing number. In each of the other number sentences we added, subtracted, multiplied or divided two numbers. In this number sentence, we are multiplying a number times an unknown number, which is represented by the letter 'n'.) This kind of number sentence is called an open sentence because there is some missing information. The letter 'n' is called a variable or a place holder. We can use any letter of the alphabet or a geometric shape such as a square, triangle, or circle as a variable or place holder." Demonstrate on the chalkboard.

e.g.,

$3 \times n = 24$ $\qquad\qquad$ $3 \times \square = 24$

$3 \times c = 24$ $\qquad\qquad$ $3 \times \bigcirc = 24$

STRATEGY 1: (cont'd)

Say, "What number will we use in place of the variable to make each sentence a true statement?" (8) Record this information on the chalkboard.

e.g.,

$$3 \times n = 24$$
$$n = 8$$

Say, "We were easily able to determine that the variable was a place holder for 8 in this number sentence." Write another open sentence on the chalkboard.

e.g.,

$$A \times 38 = 874$$

Say, "Can we easily determine the value of 'A' in this number sentence? (no) How might we go about finding the value of 'A'?" (We must solve a division problem.) Write the following on the chalkboard:

$$A \times 38 = 874$$
$$874 \div 38 = A$$

or

$$A = 874 \div 38$$

Ask a student to come to the chalkboard and calculate the quotient for the division problem while the other students write in their notebooks.

e.g.,

$$\begin{array}{l} A \times 38 = 874 \\ A = 874 \div 38 \\ A = 23 \end{array} \qquad \begin{array}{r} 23 \\ 38\overline{)874} \\ \underline{-74}\downarrow \\ 134 \\ \underline{-134} \\ 0 \end{array}$$

Say, "What is the value of 'A' in the number sentence? (23) How can we check to make sure that we have arrived at the correct solution?" (We can see what happens when we use this value in place of A in the original number sentence. It tells us to multiply 23 x 38. If their product is 874, we have found the correct solution.)

STRATEGY 1: (cont'd)

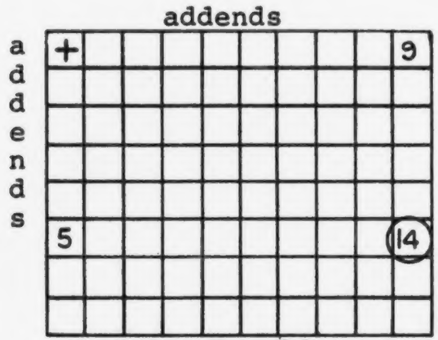
Say, "If our open sentence had involved subtraction, how could we have solved it?" (We would have solved it by using a related addition problem.)

Say, "We can solve any open sentence by using the operation which is the inverse operation in the open sentence." Repeat this procedure to solve other open sentences.

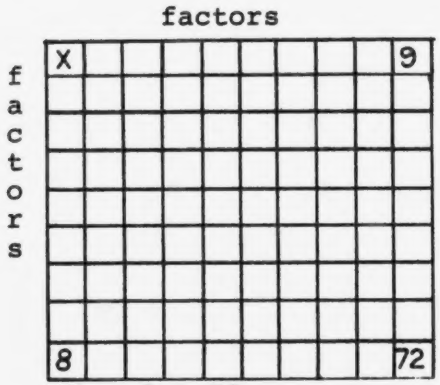
STRATEGY 2:

Materials:
- addition and multiplication tables

Display the addition and multiplication tables. Ask a student to come to the addition and multiplication tables to demonstrate how to find sums and products on the respective tables.
e.g.,



5 + 9 = 14

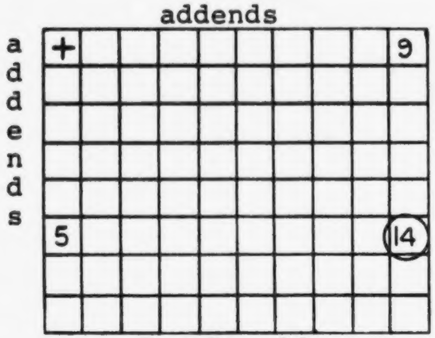


8 x 9 = 72

Say, "Can we use the addition table and multiplication table to solve open sentences with one variable?" (Yes, when the numbers in the problem are not beyond the table. To find the variable in the open sentence 5 + m = 14, we can use the addition table.)

STRATEGY 2: (cont'd)

e.g.



5 + 9 = 14

write

5 + m = 14

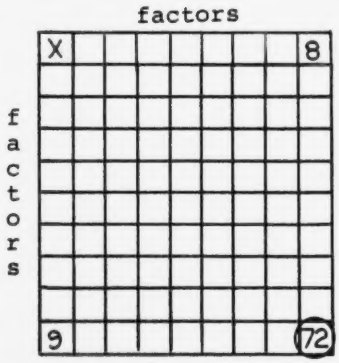
Say, "What is the correct procedure for using the addition chart? (Find the 5 in the column of addends at the left, find the sum 14 in that row.) In which column is the sum 14?" (It is in column 9, which is the correct difference between 14 and 5.)

e.g.,

5 + m = 14 m = 9

Say, "Could we use this chart to solve an open sentence involving subtraction? (yes) How would you use the chart to solve N - 6 = 8?") (We know that the related (equivalent) addition sentence is N = 6 + 8. Therefore, we would use the table to find the sum of 6 and 8.)

Repeat this procedure on the multiplication table to solve open multiplication sentences. Display the following chart on the chalkboard:



Say, "How can we use the chart to solve an open sentence involving division, such as n ÷ 3 = 9 or 72 ÷ r = 9? (In the first case, we know the divisor and the quotient and are looking for the dividend. Since we know the dividend must equal the product of quotient and

STRATEGY 2: (cont'd)

divisor, we just use the table directly to find this product. N = 9 x 3 = 27. In the second case, we know the dividend and the quotient, and need to find the divisor. Since the divisor is 9, we look for 72 in the table in the 9 row. If the product 72 were the dividend and 9 the quotient, you could find the divisor, 8, at the top of the column.) This is possible because multiplication and division are inverse operations."

Repeat this procedure by asking students to explain different examples.

6.11.01.02A write the open sentence that can be used to solve a given word problem

STRATEGY 1:

Write the following word problem on the chalkboard.

> Kim saves the same amount of money each month during the school year. At the end of May, she had $72.00. When the school year ended, on the 1st day of June, she had $81.00 in her bank account. How much did she add to her bank account in June?

Ask a student to come to the chalkboard to read the problem and then to explain it in his/her own words. Say, "Does everyone understand the problem? (Accept reasonable explanations of the problem.) Which strategy will you choose to solve the problem?" (Since I know how much she had at the end of May, $72.00, and how much she had at the end of June, $81.00, we can write an open sentence which will allow us to find the amount of money added in June.) Ask a student to come to the chalkboard to write the open sentence.

e.g.,

$$\square + \$72 = \$81$$

Say, "How can we solve this open sentence? (We can use the inverse operation, subtraction.) What if the numbers are beyond the basic facts? (The use of the inverse operation principle will allow us to

STRATEGY 1: (cont'd)

determine the missing variable even for numbers beyond the basic facts.) How can we explain and solve this open sentence?" (The open box, the variable, is the amount of money we added in June to the $72 saved through May. By the end of June, we had $81. If we use the inverse operation, subtraction, we can solve the problem.)

e.g.,

$\square$ = amount of money added in June

$\square + \$72 = \$81$ $\qquad\qquad$ $\$81 - \$72 = \square$

Say, "What does this model demonstrate? (This model demonstrates how we can use an open sentence to find the difference between the totals at the end of May and June.) Did anyone use a different method to solve this problem?" (Yes, I imagined a chart which shows how much was saved at the end of each month. Since she started saving at the beginning of the school year, the chart would go from September to June. Since there had been 9 months, I knew 72 was 9 times the monthly amount.)

e.g.,

Month	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	March	April	May	June
Amount Saved Each Month										
Total									$72	

Say, "Which word(s) in the problem helped you discover how much was saved monthly?" ("Kim saved the same amount..." This told me I could use S to represent the amount saved in any month. If she had changed the amount from month to month, I would have needed more than one variable in an open sentence.) Ask a student to write the open sentence at the chalkboard.

e.g.,

S = amount of money saved each month

$S \times 9 = \$72$

Say, "Why did you write this open multiplication sentence? (I knew her total savings after nine months but I did not know how much she saved each month.) How can this open sentence help solve the problem?" (We can use division, the inverse operation of multiplication, to find the quotient $8, which is the value of the

STRATEGY 1: (cont'd)

e.g.,

S x 9 = $72 $72 ÷ 9 = 8

Say, "By illustrating this pattern of saving money on the table/chart, we can verify that this is the correct answer."

e.g.,

Month	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	March	April	May	June
Amount Saved Each Month	$8	$8	$8	$8	$8	$8	$8	$8	$8	$8
Total	$8	$16	$24	$32	$40	$48	$56	$64	$72	$81

Say, "Notice that these two approaches looked at the problem in different ways. The variable was thought of differently and different operations were involved. Both are correct. It is good to be able to see it both ways."

P.O. 6.11.01.01-.02

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

I. Write a related subtraction sentence. Then find the difference and check your work by substituting the value for the variable in the original example.

				check
a.	5 + a = 9	9 - 5 = a	a = 4	5 + 4 = 9
b.	9 + c = 14			
c.	m + 3 = 10			
d.	8 + p = 15			
e.	n + 2 = 11			

II. Use the inverse operation to find n and then check your work.

				check
a.	6 x n = 30	30 + 6 = n	n = 5	6 x 5 = 30
b.	n x 8 = 56			
c.	9 x n = 27			
d.	28 + n = 7			
e.	12 + n = 3			
f.	8 + n = 15			
g.	n - 5 = 7			
h.	35 + n = 5			

III. Select the open sentence you could use to solve each problem.

1. Rene has 35 stamps in her U.S. Stamp Collection. How many more does she need to complete the 50 stamp set collection?
   (a) 50 + n = 50 (c) n - 35 = 50
   (c) 35 + n = 85 (d) 35 + n = 50
2. Allie had 72 autographs of baseball stars. She gave some away to her brothers, sisters, and friends. Now she has 50 left. How many did she give away?
   (a) n - 72 = 50 (c) 72 - n = 50
   (b) n - 50 = 72 (d) Not Given
3. The school teams saved $375 for their trip to Yankee Stadium. The trip will cost $500. How much more money is needed for the trip?
   (a) $375 + n = $500 (c) $875 - n = $500
   (b) $375 + n = $875 (d) n - $375 = $500

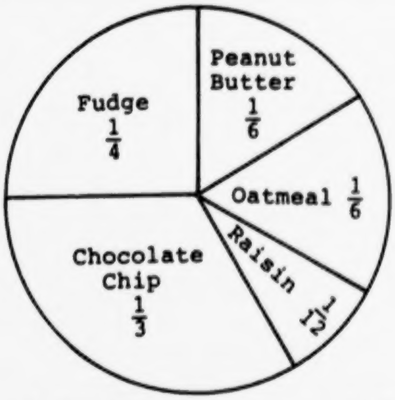
SHARPENING YOUR SKILLS

Units 31, 32, 33

Name: ____________________ **Class:** ____________ **Date:** ________

Use the graph below to answer questions 1 - 3.

BOXES OF SCOUT COOKIES SOLD



Total Number of Boxes Sold: 1200

1. How many boxes of fudge cookies were sold? ______
2. How many more boxes of chocolate chip cookies were sold than peanut butter cookies? ______
3. What fractional part of the cookies sold were raisin cookies? ______

4. $5^2 - 12 + 42 =$ _____

ⓐ 34 ⓒ 18
ⓑ 5 ⓓ Not Here

5. $17 - 2 \times 5 + 3 =$ _____

ⓔ 78 ⓖ 10
ⓕ 120 ⓗ Not Here

6. $(3^2 + 6) \times 2\ 3 =$ _____

ⓐ 10 ⓒ 7
ⓑ 8 ⓓ Not Here

7. $4 \times (5 + 7^2) - 8 =$ _____

ⓔ 208 ⓖ 68
ⓕ 61 ⓗ Not Here

8. $n + 5 = 30$ $n =$ ☐

ⓐ 35 ⓒ 6
ⓑ 20 ⓓ Not Here

9. $a + 9 = 9$ $a =$ ☐

ⓔ 81 ⓖ 72
ⓕ 18 ⓗ Not Here

10. $12 \times m = 132$ $m =$ ☐

ⓐ 120 ⓒ 11
ⓑ 12 ⓓ Not Here

11. Which open sentence can be used to solve this problem?

Jessica had $179 in the bank. She deposited more money this week. Now she has $200 in the bank. How much money did she put in the bank this week?

ⓔ $179 + $200 = N
ⓕ N - $200 = $179
ⓖ $179 + N = $200
ⓗ Not Here

6.13.01 PROBLEM SOLVING: MULTISTEP PROBLEMS

(2 lessons)

PERFORMANCE OBJECTIVES

The student will be able to...

6.13.01.01A choose and sequence operations to solve a multistep word problem;

e.g.,

Don had $350.00. He bought 14 books at $12.95 each and 7 dozen pens at $15.50 a dozen. How much did he have left after his purchase?

Step 1: Find the cost of the 14 books.
Step 2: Find the cost of the 7 dozen pens.
Step 3: Find the total cost of the books and the pens.
Step 4: Subtract the total cost from $350.00.

6.13.01.02A create and solve a multistep word problem.

VOCABULARY

one-step problems
two-step problems
multi-step problems

GETTING STARTED:

Write the following problem on the chalkboard:

> 25 students and 6 adults went on a trip to the circus. The cost of each ticket was $7.50. How much did they spend for the tickets?

Ask students to choose and sequence the operations they would use in their plan to solve this problem. Say, "How would you solve it?" Demonstrate and explain.

e.g.,

Step I Find the total number of people attending the trip by adding 25 + 6.

25 + 6 = 31

Step II Find the total cost of the tickets by multiplying the cost of each ticket ($7.50) by the number of people attending the circus (31).

31 x $7.50 = $232.50

Say, "Did anyone solve this problem another way?" Demonstrate and explain.

e.g.,

Step I Find the cost of the children's tickets by multiplying $7.50 x 25.

$7.50 x 25 = $187.50

Step II Find the cost of the adults' tickets by multiplying $7.50 x 6.

$7.50 x 6 = $45.00

Step III Find the total cost of the tickets for all who were on the trip by adding the two amounts.

$187.50 + $45.00 = $232.50

Say, "As you can see there is more than one way to plan to solve a problem."

6.13.01.01A **choose and sequence operations to solve a multistep word problem**

STRATEGY 1:

Say, "Many problems cannot be solved in one or two steps. Some problems will require many steps in your problem solving plan.

Write the following problem on the chalkboard:

> Mary works in a bakery. From Monday through Friday, she works three hours each day and is paid $4.25 per hour. On Saturday and Sunday, she works eight hours each day and is paid $6.00 per hour. How do her earnings at the end of the week compare with her brother who earns a total of $150 a week at his part-time job?

Ask a student to read the problem aloud. Say, "It is important for all of you to understand the problem. What facts are given in the problem? (The facts given are the amount paid each hour on weekdays and on weekends, the number of hours worked, and the total part-time salary earned by her brother.) Are there questions you must ask yourself while writing out your solution to the problem? (Accept all reasonable responses, e.g., How much does Mary earn altogether for the weekdays?... for the weekend? What am I trying to find? What should I look for first?) What must you do to solve the problem? (I must find Mary's total salary and compare it to her brother's total salary.) What is your strategy for solving this multistep problem?" (I will determine what result I need to get and how they will be combined. After I have planned the whole solution, I will perform the calculations.) Ask another student to come to the chalkboard to write out his/her plan for solving the multistep problem while the students work at their desks.

STRATEGY 1: (cont'd)

e.g.,

	Plan	Calculation
Step I	Find out how much Mary earns each weekday by multiplying $4.25 by 3.	$4.25 x 3 = $12.75
Step II	Find out how much Mary earns from Monday through Friday by multiplying $12.75, the amount she earns each weekday, by 5.	$12.75. x 5 = $63.75
Step III	Find out how much Mary earns each Saturday by multiplying $6.00 by 8.	$6.00 x 8 = $48.00
Step IV	Find out how much Mary earns each Sunday by multiplying $6.00 by 8.	$6.00 x 8 = $48.00
Step V	Find out how much Mary earns in a whole week by adding $63.75, the amount she earns during the weekdays, $48.00, the amount earned on Saturday, and $48.00, the amount earned on Sunday.	$ 63.75 earned during week 48.00 earned Saturday + 48.00 earned Sunday $159.75 total earned by Mary in a week
Step VI	Compare the amount earned by Mary ($159.75) with the amount earned by her brother ($150).	$159.75 Mary's earnings > $150.00 Mary's brother's earnings

Say, "Let's look back at the plan and review the strategy 1 plan. Is it a reasonable approach to finding the solution? (yes) Now let's work out the calculations and see what the answer is. Notice that writing the plan in advance helps you remember where you're going and what to do next. How does this plan compare with your plan?" Allow students opportunities to present their alternative plans.

STRATEGY 1: (cont'd)

e.g.,

	Plan	Calculation
Step I	Find out how many hours Mary works during the week by multiplying 3 hours a day by 5.	3 x 5 = 15
Step II	Find out how much she earns during the week by multiplying 15 by $3.75.	15 x $3.75 = $56.25
Step III . . .		
Step IV		

Say, "What would we do if we wanted to know how much more than her brother Mary earned?" (We would add another step to find the difference, if any, between how much Mary earned and how much her brother earned.)

Repeat this procedure with other multistep problems.

6.13.01.02A create and solve a multistep word problem

STRATEGY 1:

Record the following information on the chalkboard:

> Jason lives in Brooklyn, New York. He and his family plan to visit his grandmother in San Francisco, California. The round trip airfare for people under the age of 12 is $292.50. The round trip airfare for anyone 12 years or older is $456.75.

Ask a student to read the information aloud. Say, "What facts are we given? (where Jason lives, where his grandmother lives, his family's plans to visit his grandmother, the cost of round trip airfare for people 12 years and over and for people under the age of 12) Are we asked to solve a problem with this information? (No) Can we create and solve a problem with this information? (Yes, we can solve a problem but in order to solve a multistep word problem, we would need to provide some additional information.) Ask the students to add some

STRATEGY 1: (cont'd)

facts to the information already given and to use all the information to create a problem that will be solved by the other students. Ask one student to come to the chalkboard and record his/her additional information.

e.g.,

> Jason is 11 years old. His family consists of his mother, his father, his 14 year old brother and 8 year old twin sisters. Will $2000 be enough for the entire family to make the trip?

Say, "What must we do to solve this problem?" (We must find the cost of airfare for family members 12 years old or older and add this to the cost for family members under the age of 12.) Ask a student to come to the chalkboard and write a sequence of steps that will lead toward solving the problem.

e.g.,

	Plan	Calculation
Step 1	Find the cost of airfare for family members 12 years old or older.	$ 456.75 x 3 $1370.25
Step 2	Find the cost of airfare for family members under the age of 12.	$ 292.50 x 3 $2247.75
Step 3	Add the two amounts.	$1370.25 + 877.50 $2247.75
Step 4	Compare the total cost of airfare for the family to $2000.00.	$2247.57 > $2000.00

Say, "What is the answer to the problem?" Explain. (The answer is 'no'. Two thousand dollars will not cover the cost of airfare for the entire family.)

Repeat this procedure by allowing other students to add additional information to the original information and create and solve problems utilizing all of the information.

P.O. 6.13.01.01-.02

NAME: ______________________ **CLASS:** ________ **DATE:** ______________

1. Four sixth grade classes are planning an end-term spring trip to a museum in Philadelphia, Pennsylvania. There are 29 children in each of the classes going on the trip. In addition to the four classroom teachers, eight parents will accompany the students. The cost of renting a bus for the trip is $140. They are renting two buses. The cost of admission to the museum and lunch is $6.00 for each adult and $4.00 for each student. How much will this entire trip cost?

   Here is my plan and solution to the problem:

2. Create and solve a multistep problem using the following information:

   Steven's father gave him $50 for his birthday and told him he could purchase anything of his choice with it. He went to the sporting goods store and saw the following sign:

   ATTENTION -- BOYS and GIRLS
   SALE!
   NeeNok Athletic Shoes -- $29.95 a pair
   Jogging Shorts -- $2.99 each or 2 for $5.00
   Tank Tops -- $1.99 each or 2 for $3.00
   Tube Socks -- $1.99 per pair or 6 pair for $5.00

   Problem: ______________________________

   Solution: ______________________________

Unit 35 **6.13.02 USE TRIAL AND ERROR TO SOLVE PROBLEMS**

(1 lesson)

PERFORMANCE OBJECTIVES

The student will be able to...

6.13.02.01A use trial and error to solve a problem; e.g.,

A bakery sells double dutch chocolate cakes for $6.00 each, and fresh strawberry shortcakes for $7.00 each. Joan purchased 8 cakes for her club's party and paid a total of $53.00 for them. How many of each did she buy?

Note: The student may investigate various cake combinations before discovering one with the specified total cost.

*6.13.02.02A create and solve a word problem which may be solved by using trial and error.

VOCABULARY

trial
error

PREVIOUS UNDERSTANDINGS:

- guessing and checking to solve problems
- making tables or charts to solve problems

GETTING STARTED:

Write the following on the chalkboard:

> As part of his exercise program, Howard wants to train himself to do 80 sit-ups a night. If he starts by doing 5 sit-ups a night the first week and doubles the amount each succeeding week, how many weeks will it take him to reach his goal of 80 sit-ups?

Ask a student to read the problem aloud. Say, "Sometimes it is helpful to create a chart to solve a problem." Write the following on the chalkboard and ask the students to write it in their notebooks:

Week	Number of Sit-ups

Say, "How many sit-ups will Howard do in the first week? (5) According to the information we are given in the problem, how many sit-ups will Howard do the second and third weeks? Explain." (If he doubles the amount each week, he will do 10 sit-ups the second week and 20 sit-ups the third week.) Ask a student to record this information on the chalkboard as the other students do the same in their notebooks. e.g.,

Week	Number of Sit-ups
1	5
2	10
3	20

GETTING STARTED: (cont'd)

Ask the students to complete the chart in their notebooks as another student completes it at the chalkboard.
e.g.,

Week	Number of Sit-ups
1	5
2	10
3	20
4	40
5	80

Say, "How many weeks will it take Howard to achieve his goal?" (5)

6.13.02

6.13.02.01 use trial and error to solve a problem

STRATEGY 1:

Write the following on the chalkboard:

> The Tart 'n' Tasty Company specializes in preparing fresh fruit drinks which it sells to various restaurants. Two of its specialties are a lemonade which sells for $4.80 per gallon and an orangeade which sells for $3.75 per gallon. On Saturday, the company sold 14 gallons of the two drinks for a total of $61.95. How many gallons of each fruit drink were sold?

Ask a student to read the problem aloud. Say, "What are we being asked to find? (How many gallons each of lemonade and orangeade were sold.) What information are we given? (Lemonade sells for $4.80 a gallon, orangeade sells for $3.75 a gallon, 14 gallons were sold for a total of $61.95.) What operations will we use to solve the problem? (multiplication and addition) Can we easily solve this problem? Explain. (No, we do not know how many gallons each of lemonade and orangeade were sold, therefore we don't know the numbers to be multiplied by $3.75 and $4.80 that will give us a total of $61.95.) Let's examine a simpler problem that will help us to develop a strategy for solving the original problem." Write the following:

> Three gallons of fruit drinks were sold for a total of $13.35. How many gallons each of lemonade and orangeade were sold?

Say, "What is a possible combination of lemonade and orangeade in this problem?" (1 lemonade and 2 orangeade) Ask a student to come to the chalkboard and calculate the total.

STRATEGY 1: (cont'd)

e.g.

1 lemonade x \$4.80 = \$4.80
2 orangeade x \$3.75 = \$7.50

Total = \$12.30

Say, "Is this the total we want? (no) What question should we ask next? (Is there another way to combine lemonade and orangeade for a total of three gallons?) What is the answer to this question?" (yes, 2 gallons of lemonade and 1 gallon of orangeade) Ask a student to compute the value of this combination.

e.g.

2 lemonade x \$4.80 = \$9.60
1 orangeade x \$3.75 = \$3.75

Total = \$13.35

Say, "Is this the total we want? (yes) What was the correct combination? (2 gallons of lemonade and 1 gallon of orangeade) The strategy we used to solve this problem is called trial and error. Can we use this strategy to solve the original problem?" (Yes, we can investigate the possible combinations of 14 gallons of lemonade and orangeade and use trial and error to solve the problem.) Write the following on the chalkboard and ask the students to write it in their notebooks:

Trial	Gallons of Lemonade x \$4.80	Gallons of Orangeade x \$3.75	Total

Say, What would be a good combination of lemonade and orangeade for our first trial?" (Since there are 14 gallons, we can begin with an equal number of gallons each, 7 lemonade and 7 orangeade.) Ask a student to perform the calculation at the chalkboard as the other students work in their notebooks.

e.g.,

Trial	Gallons of Lemonade x \$4.80	Gallons of Orangeade x \$3.75	Total
1	7 x 4.80	7 x 3.75	33.60 + 26.25 = 59.85

STRATEGY 1: (cont'd)

Say, "Is 7 gallons of lemonade and 7 gallons of orangeade the correct combination? (No, the total cost does not equal $61.95.) What would be a good combination to try next? Explain." (Since $61.95 is more than $59.85 and the lemonade costs more than the orangeade, we should try a greater number of gallons of lemonade than orangeade.) Ask the students to continue the calculations for each trial in their notebooks while individual students do the same at the chalkboard.

e.g.,

Trial	Gallons of Lemonade x $4.80	Gallons of Orangeade x $3.75	Total
1	7 x 4.80	7 x 3.75	33.60 + 26.25 = 59.85
2	8 x 4.80	6 x 3.75	38.40 + 22.50 = 60.90
3	9 x 4.80	5 x 3.75	43.20 + 18.75 = 61.95

Say, "How many gallons of each type of fruit drink were sold?" (9 gallons of lemonade and 5 gallons of orangeade)

Repeat this procedure to solve other word problems using trial and error.

6.13.02.02A create and solve a word problem which may be solved by using trial and error

STRATEGY 1:

Display the following chart:

Dairy King

Today's Specials	
Banana Barge	$1.25
King Fudge Sundae	$2.25
King Banana Split	$1.95
Strawberry Frappe	$1.80
Rainbow Sherbet Boat	$1.50

Bonnie went to the Dairy King with some of her friends.

STRATEGY 1: (cont'd)

Ask the students to use the chart and the information beneath it to create problems that may be solved using trial and error. Ask individual students to write their problems on the chalkboard so that the other students can solve them. e.g., Bonnie went with two friends. They purchased three specials at a total cost of $6.15. Which specials were purchased by Bonnie and her two friends?

Remind students that it may be helpful to create and solve a simpler problem and/or to make a table or chart when solving problems of this type.

P.O. 6.13.02.01-.02

NAME: ____________________ **CLASS:** ________ **DATE:** ____________

Use trial and error to solve the following problem.

> Five members of Joseph's family took a trip from New York City to Miami, Florida. They paid a total of $784 for round trip airfare. If the cost of roundtrip airfare was $196 for adults and half price for children under the age of 12, how many members of Joseph's family paid adult airfare? How many children under 12 went on the trip?

Trial 1: ____________________

Create and solve a word problem using trial and error.

Problem: ____________________

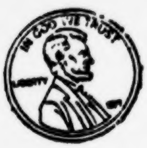
Trial 1: ____________________

CHALLENGE!

Select digits between 1 and 9 to create numbers that complete the following number sentence. No two digits can be alike in your final solution.

$$(5 + \square) \times (\square + 4) = 80$$

RESOURCE MATERIAL - Money (1)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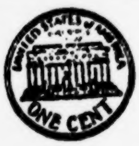
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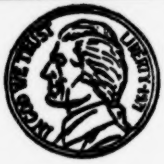
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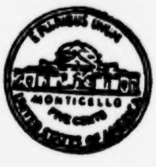
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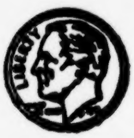

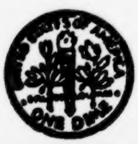
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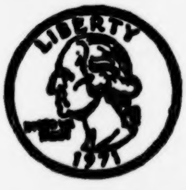
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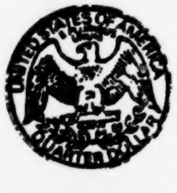
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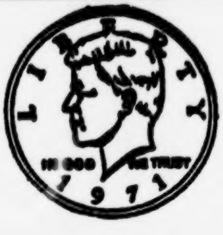
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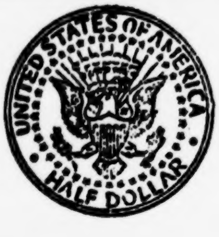
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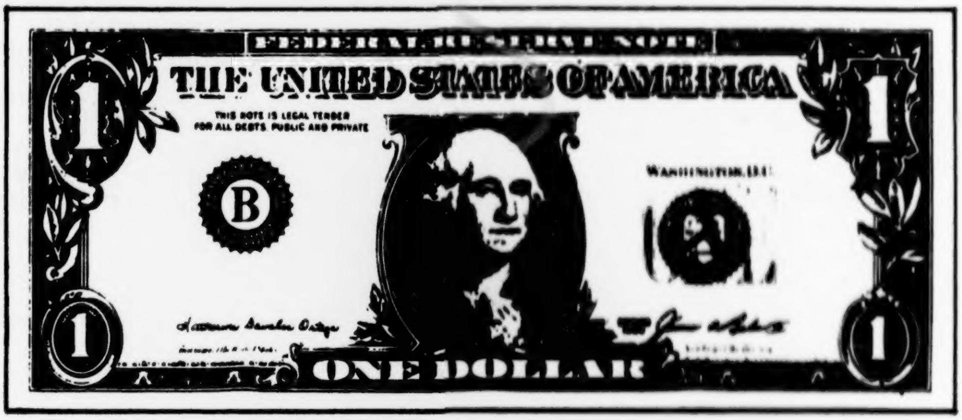
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THE UNITED STATES OF AMERICA
THIS NOTE IS LEGAL TENDER FOR ALL DEBTS, PUBLIC AND PRIVATE
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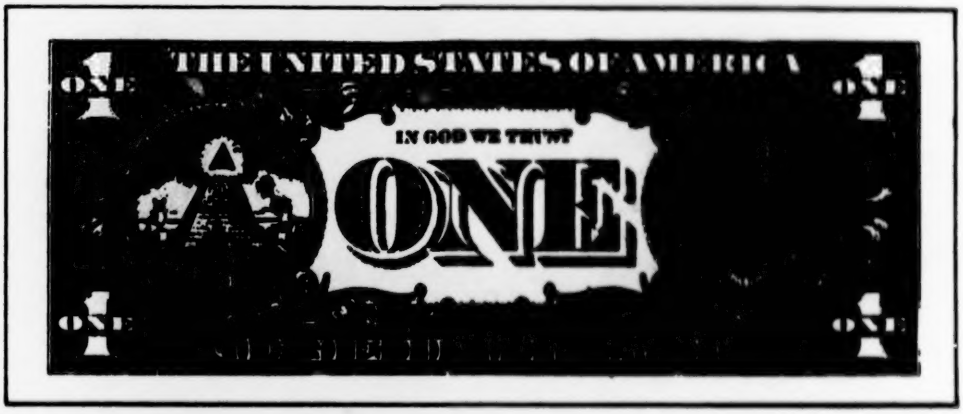
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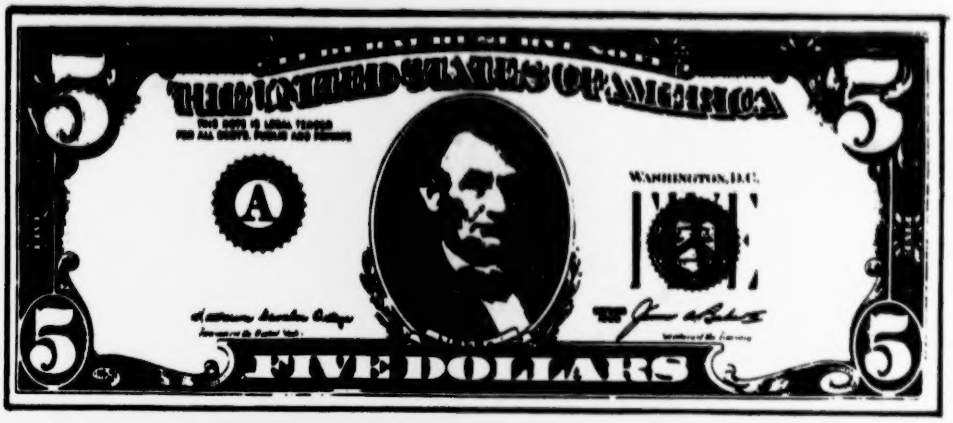
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THIS NOTE IS LEGAL TENDER FOR ALL DEBTS, PUBLIC AND PRIVATE
A
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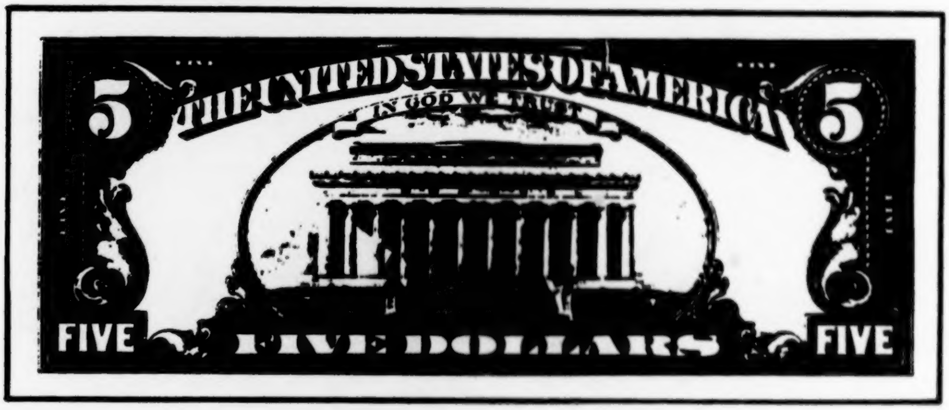
5
FIVE DOLLARS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IN GOD WE TRUST
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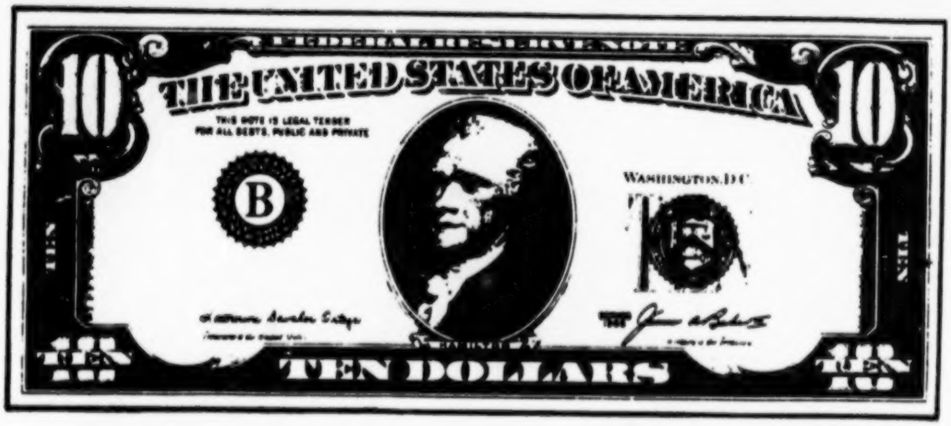
FIVE DOLLARS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THIS NOTE IS LEGAL TENDER FOR ALL DEBTS, PUBLIC AND PRIVATE
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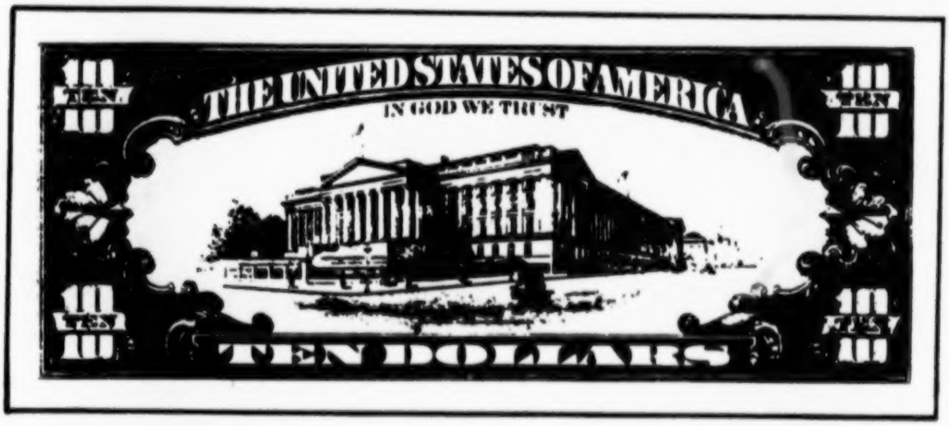
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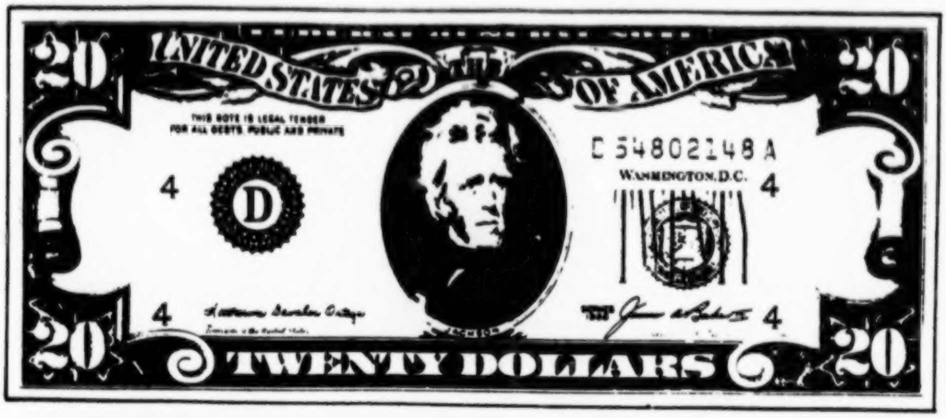
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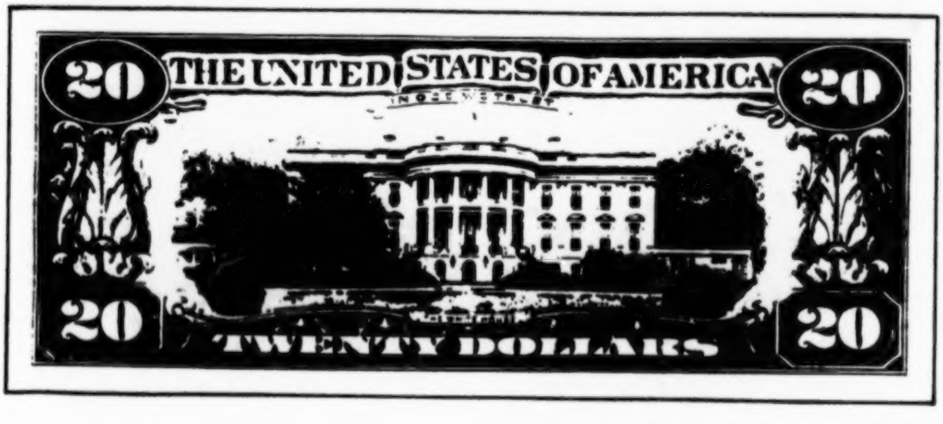
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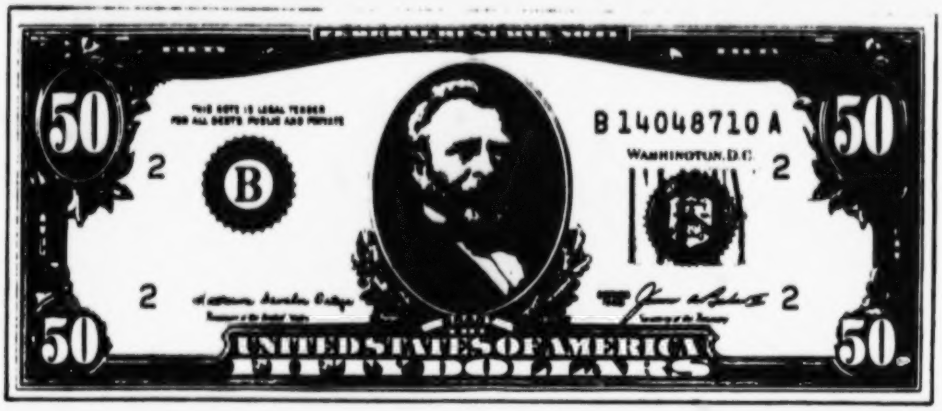
IN GOD WE TRUST
TWENTY DOLLARS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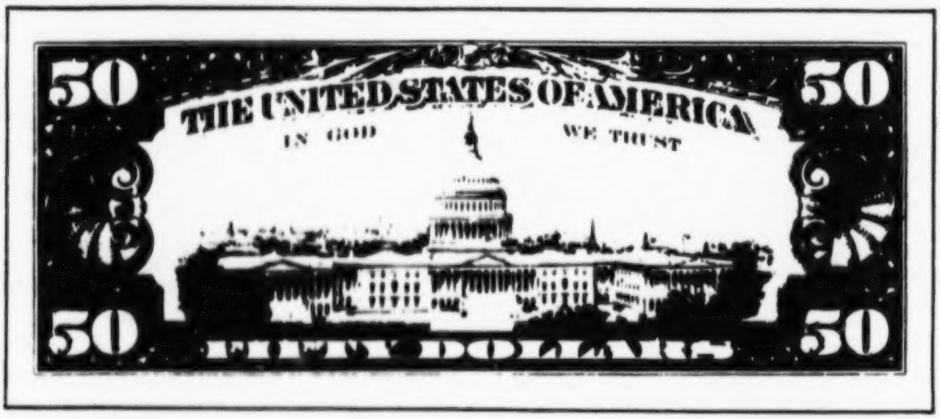
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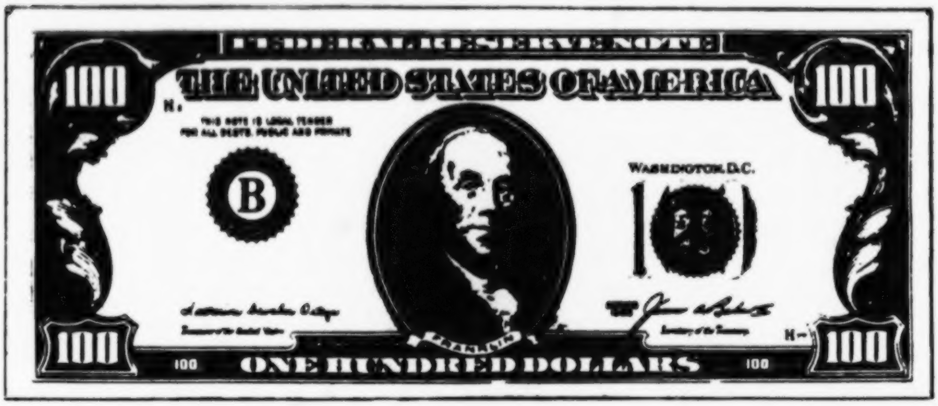
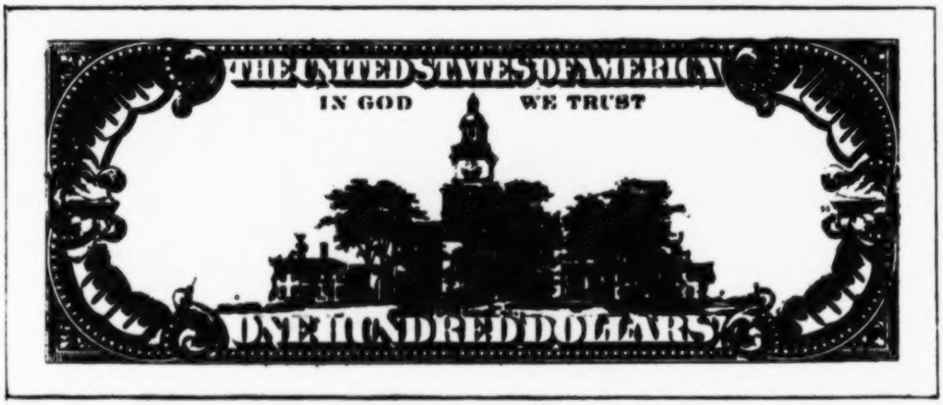
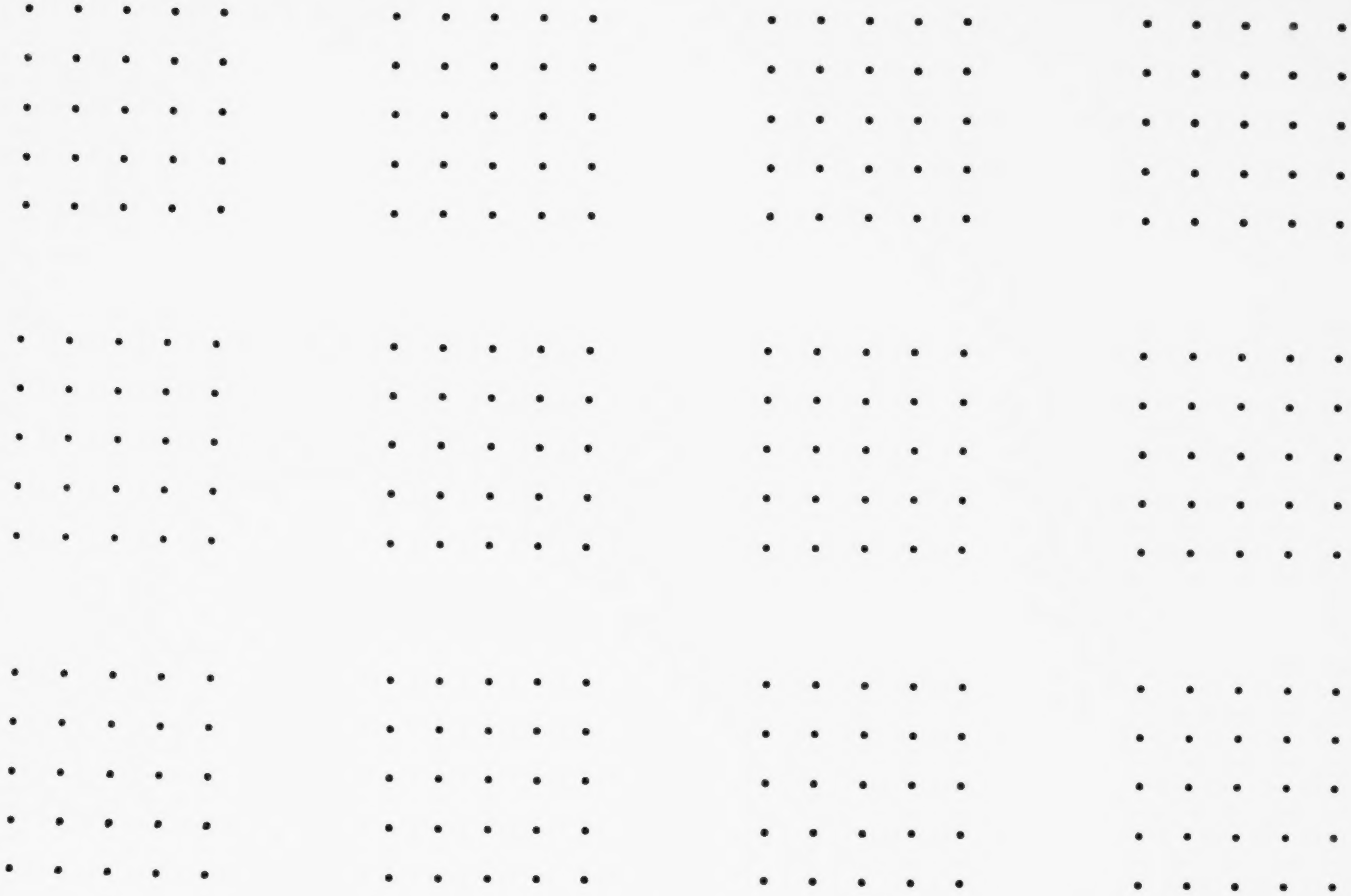
IN GOD WE TRUST
FIFTY DOLLARS


FEDERAL RESERVE NOTE
100
THE UNITED STATES OF AMERICA
THIS NOTE IS LEGAL TENDER FOR ALL DEBTS, PUBLIC AND PRIVATE
B
WASHINGTON, D.C.
FRANKLIN
ONE HUNDRED DOLLARS


THE UNITED STATES OF AMERICA
IN GOD WE TRUST
ONE HUNDRED DOLLARS

RESOURCE MATERIAL - Dot Paper (1) Geometry

RESOURCE MATERIAL - Dot Paper (2) Geometry



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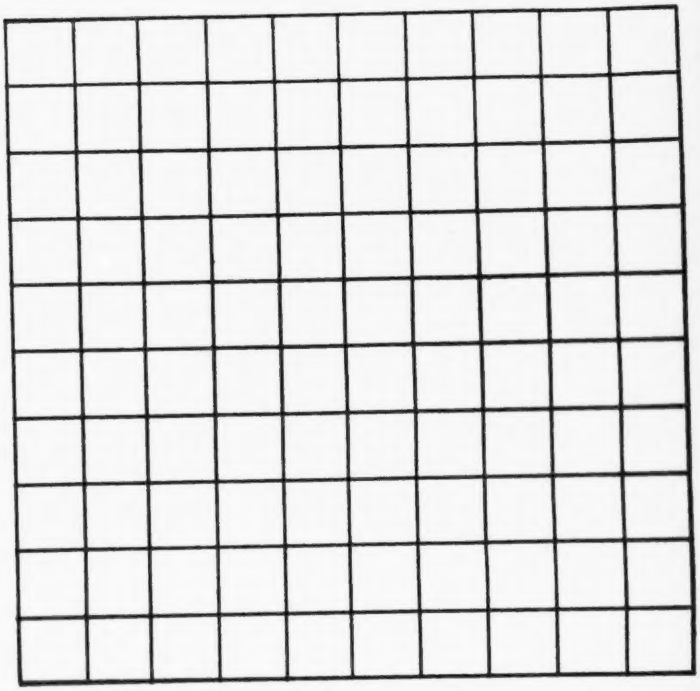
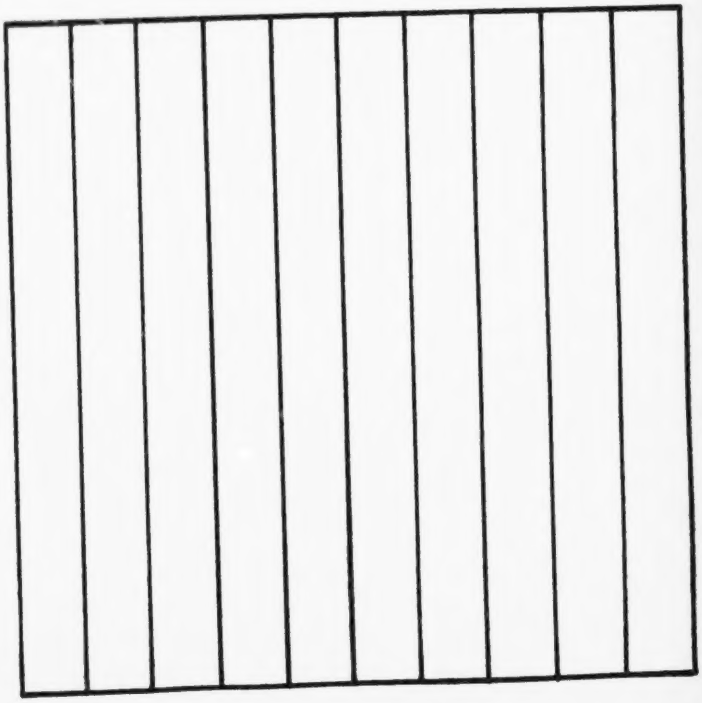
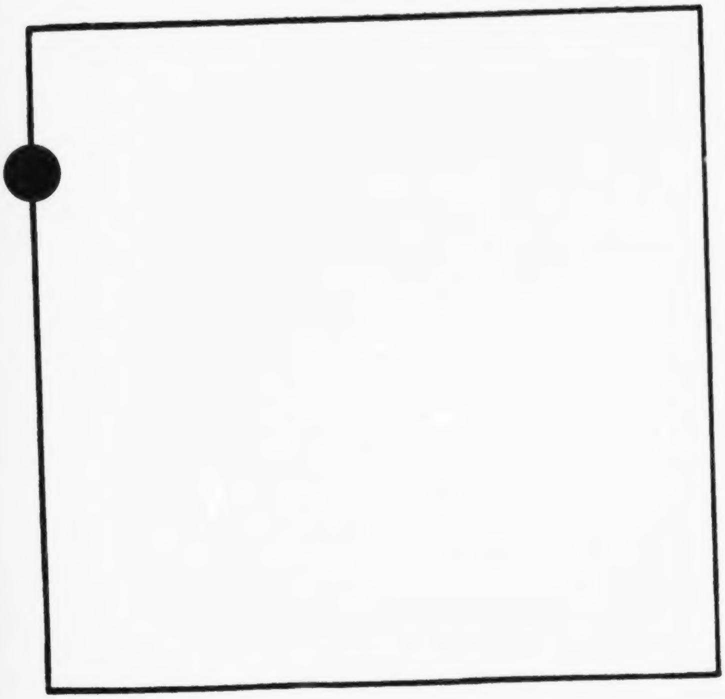
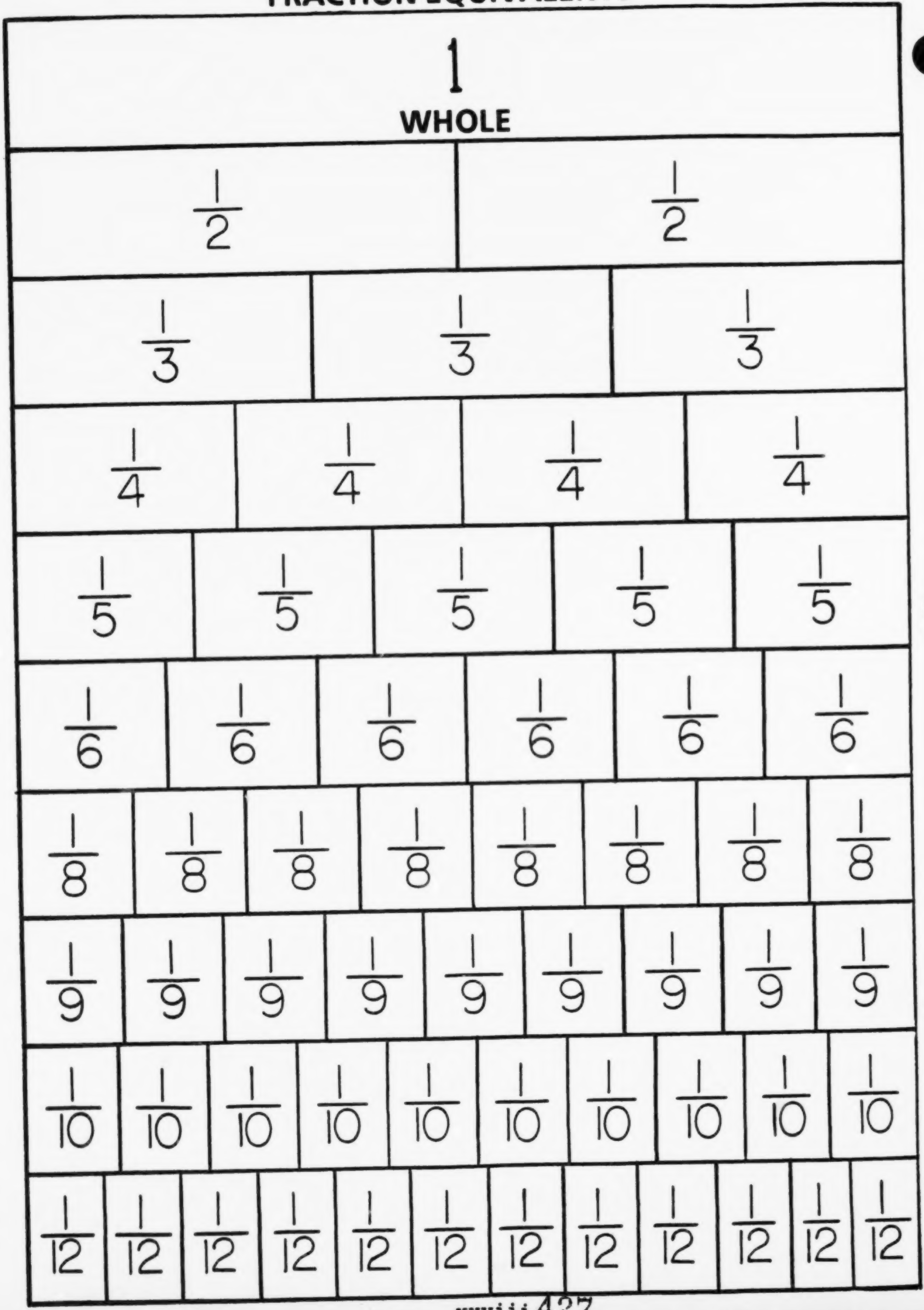






FRACTION EQUIVALENTS



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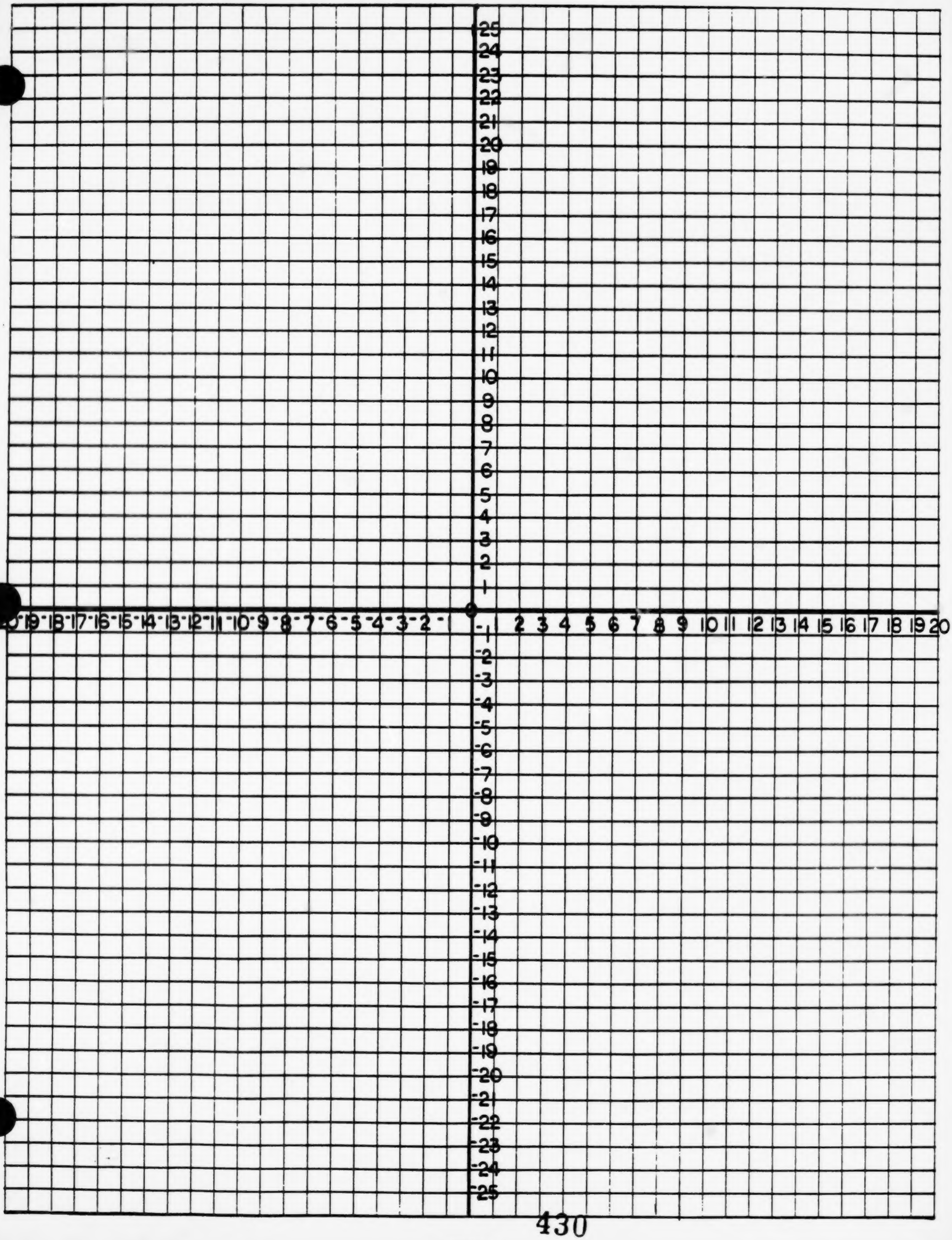
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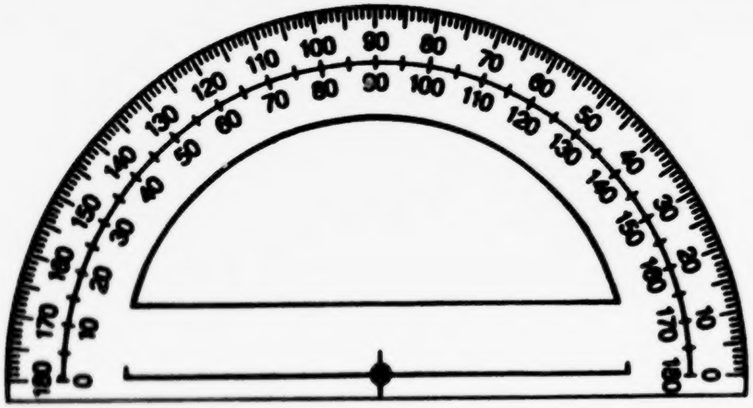

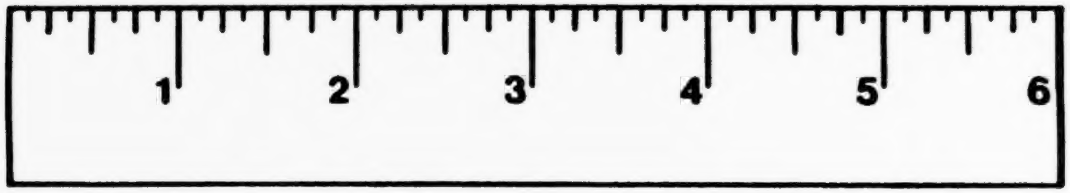

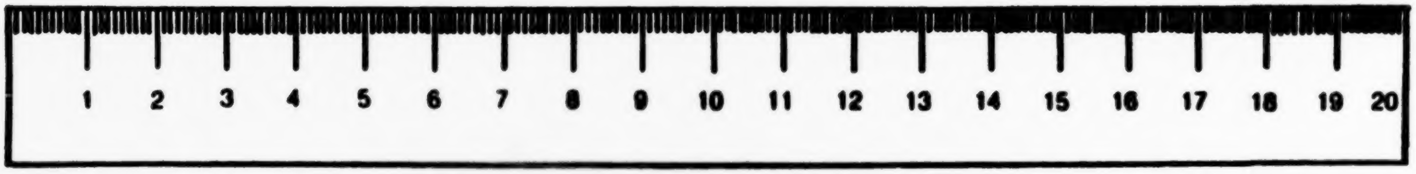

TABLE OF SYMBOLS

DEFINITIONS	SYMBOL	DEFINITIONS	SYMBOL
plus; add; positive	+	line	<——>
minus; subtract; negative	-	ray	——>
times; multiply	· or X	perpendicular lines	⟂
divide	)‾‾‾ or ÷	parallel lines	⇉
angle	∡	degree	°
between two numerals or letters or at the end of a series or sequence; and so on, continues without end	...	is perpendicular to	⊥
		is parallel to	//
		is equal to	=
		is not equal to	≠
probability of an event	P(E)	is less than	<
		is greater than	>
line segment (as $\overline{AB}$)	•——•	is similar to	~
degrees Fahrenheit	°F	is congruent to	≅
degrees Celsius	°C	encloses a set when numbers are included: empty set when empty	{ }
percent	%		
foot	'		
inch	"	subset	⊂
arc	◯	intersection	∩
		union	∪

METRIC TABLE

PREFIX	VALUE	LENGTH (LINEAR)	CAPACITY	WEIGHT (MASS)
KILO	1000	kilometer* (km)	kiloliter (kL)	kilogram* (kg)
HECTO	100	hectometer (hm)	hectoliter (hL)	hectogram (hg)
DEKA	10	dekameter (dkm)	dekaliter (dkL)	dekagram (dkg)
	1	meter* (m)	liter* (L)	gram* (g)
DECI	$\frac{1}{10}$ or 0.10	decimeter (dm)	deciliter (dL)	decigram
CENTI	$\frac{1}{100}$ or 0.01	centimeter* (cm)	centiliter (cL)	centigram (cg)
MILLI	$\frac{1}{1000}$ or 0.001	millimeter* (mm)	milliliter (mL)	milligram* (mg)

* most frequently used

BASIC CLASSROOM EQUIPMENT CHECKLIST

MATERIALS:

Abacus - Class size/indiv....... ____
Attribute blocks................ ____
Balance beam.................... ____
Calculators..................... ____
Calendar - class size........... ____
Clock - large/individual........ ____
Clock - stamp................... ____
Coin stamps..................... ____
Cubic counting blocks........... ____
Color rods...................... ____
Decimal cards - hundredths, tenths....................... ____
Felt cut-outs................... ____
Flannel board................... ____
Flash cards..................... ____
Fractional parts (assorted)..... ____
Geoboards....................... ____
Geometric shapes - Plane/Solid.. ____
Height Chart.................... ____
Hundred charts - class size/ind. ____
Interlocking cubes.............. ____
Laminating paper................ ____
Magnetic board.................. ____
Magnetic discs.................. ____
Measuring tools-English, Metric (rulers, scales, compasses, protractors, beakers, cups, spoons, thermometers)......... ____
Number lines -class/indiv...... ____
Numerals -tactile.............. ____
Plastic discs.................. ____
Play money..................... ____
Powers of Ten blocks .... ____
Protractor - teacher/student... ____
Screen (pull-down type)........ ____
Spinners....................... ____
Squared place value material... ____
Standard playing cards......... ____
Tangrams....................... ____
Tens frame..................... ____
Timers .... ____
Tokens (various sizes/colors).. ____
Transparencies................. ____
Trundle wheel.................. ____
Yardstick...................... ____

SUPPLIES:

Adding machine tapes (for classroom/ind. number lines)........ ____
Chalk.......................... ____
Construction paper............. ____
Elastic bands.................. ____

BASIC CLASSROOM EQUIPMENT CHECKLIST

SUPPLIES: cont'd

Envelopes (all sizes).......... ____
Erasers, board/individual...... ____
Felt pens, markers............. ____
Ditto, Rexograph paper......... ____
Glue........................... ____
Gram/customary weights......... ____
Graph paper (1 cm, ½ cm, 1")... ____
Grease pencils................. ____
Oak tag........................ ____
Paper clips.................... ____
Paste.......................... ____
Plastic discs.................. ____
Pointer........................ ____
Primary pencils................ ____
Rubber bands................... ____
Scissors - primary, left hand.. ____
Spinners....................... ____
Tape (transparent, opaque)..... ____

COLLECTIBLES:

Beads and beans................ ____
Bottle caps.................... ____
Buttons........................ ____
Cans........................... ____
Cardboard (all sizes).......... ____
Cereal boxes................... ____
Detergent boxes, shoe boxes, trays, and plastic containers..................... ____
Egg cartons.................... ____
Felt for desk mats............. ____
Ice cream sticks............... ____
Juice containers............... ____
Macaroni....................... ____
Magazines...................... ____
Menus.......................... ____
Metrically measured cartons.... ____
Milk containers................ ____
Newspapers..................... ____
Paper plates................... ____
Reclosable plastic bags........ ____
Spools......................... ____
String, yarn................... ____
Styrofoam...................... ____
Tongue depressors.............. ____
Wire hangers................... ____

ACKNOWLEDGMENTS - 1986-87

The Comprehensive Instructional Management System (CIMS) was developed as a project of the Division of Curriculum and Instruction, Charlotte Frank, Executive Director. Bernard Mecklowitz, Community School Superintendent CSD 1; Peter Negroni, Community School Superintendent CSD 12; Harvey Garner, Community School Superintendent CSD 18; Carlos Edwards, Community School Superintendent CSD 23, and Joseph Pacheco, Community School Superintendent CSD 32 supervised the overall development of the CIMS program within their respective districts.

Morris Freedman, Director of the Office of Curriculum Development and Support; Irwin Kaufman, Director of Technology, Assigned to the Chief Executive for Instruction; Gerald Elgarten, Director of Mathematics, 1984-85; Lorraine Largmann, Director of Mathematics 1985-86; Susan Zakaluk, Director of Mathematics; Marjorie McAllister, Director of Early Childhood Education; Kenneth Bates, Director of CIMS; Maria DeSalvio, Assistant Director of Mathematics; Arthur Flug and Doreen D. Genkin, Assistant Directors of CIMS and Gene Menchini, Assistant to the Director for Systems Planning and Coordination, ODPC provided overall supervision of the project.

Overall supervision of curriculum in addressing those students with special needs was coordinated by Edward M. Sermier, Chief Administrator, Division of Special Education.

Gloria Cole, Director of Office of Program Development; and Roslyn Persky, Deputy Assistant Administrator to the Director of the Office of Program Development, Special Education provided guidance and support in addressing these special needs.

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Stuart Goldberg	Asst. to Math Director	Joseph Porzio	CSIP Facilitator
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Lillian Frechtman	P.S. 279K
Merle Glass	P.S. 219K
Howard Mankes	P.S. 219K
Lillian Pecker	J.H.S. 252K
Virginia Pero	J.H.S. 211K
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Arlie Solola	J.H.S. 252K
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Bruce Landau	P.S. 327K
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